THE FORGOTTEN GIFTS OF HERMES: THE LATIN RECEPTION OF PROCLUS' COMMENTARY ON EUCLID'S *ELEMENTS*[*]

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Abstract

The present work offers an account of the Latin reception of Proclus' *Commentary in the first book of Euclid's Elements* (*In Euclidem*). The overall goal of this paper is to offer a unified historical and philological map as a ground-tool for understanding the Latin reception of *In Euclidem* from the early 16th century to the 17th century, which includes manuscript sources hitherto overlooked. I will deal with the three known complete translations produced by Bartolomeo Zamberti (1473–1543), Giovanni Battista Gabia (*c.*1500–1590), and Francesco Barozzi (1537–1604), as well as with five fragmentary translations embedded in the works of Giorgio Valla (1447–1500), Jerónimo Muñoz (1520–1591), Conrad Dasypodius (1532–1600), Federico Borromeo (1564–1631), and Johannes Kepler (1571–1630). Each of these translations and the intentions of their authors are discussed in chronological order. Five of them (following the fragment chosen by Kepler) are partially edited in the Appendix.

Key Words

Proclus; Euclid; Philosophy of mathematics; History of philosophy; History of science

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Καὶ ἡ ἐνέργεια αὐτῆς αὕτη τέλος ἂν εἴη τὸ ἄριστον τῆς περὶ γεωμετρίαν σπουδῆς καὶ ὄντως τῆς Ἑρμαϊκῆς δόσεως ἔργον, ἀπό τινος Καλυψοῦς ἀναγούσης αὐτὴν εἰς τελειοτέραν καὶ νοερωτέραν γνῶσιν καὶ ἀπολυούσης τῶν ἐν φαντασίᾳ μορφωτικῶν ἐπιβολῶν.

This achievement would itself be the perfect culmination of geometrical inquiry, truly a gift of Hermes, leading geometry out of Calypso's arms, so to speak, to more perfect intellectual insight, and emancipating it from the pictures projected in the imagination.

PROCLUS, *In Euclidem*, p. 55, l. 18–23.

The purpose of this paper is to provide a comprehensive historical and philological tool for retracing the reception in the Latin West of Proclus' *In Euclidem* and show how this work became part of the general mathematical and philosophical culture, sometimes as a hidden source. Expanding the index of witnesses previously known to us,[1] I will analyse eight cases going from the early 16th century to the 17th century. This index of sources will include complete or partial translations into Latin of Proclus' commentary, some of which have been hitherto overlooked. I will provide information and direct access (when possible) to the digitised manuscripts in the footnotes. The works under consideration will be the complete translations of *In Euclidem* produced by Bartolomeo Zamberti (1473–1543), Giovanni Battista

[1] For previous scholarship dealing with the reception of Proclus' *In Euclidem*, cf. FILIP KARFÍK, PETER ADAMSON, « Proclus' Legacy », in PIETER D'HOINE, MARIJE MARTIJN (eds.), *All From One: A Guide to Proclus*, Oxford University Press, Oxford 2016, p. 290–232, especially section 15.2; GUY CLAESSENS, « Proclus in the Renaissance », in MARCO SGARBI (ed.), *Encyclopedia of Renaissance Philosophy*, Springer, Cham 2018, online at: <https://link.springer.com/referenceworkentry/10.1007/978-3-319-02848-4_412-1>; ID., « Het denken verbeeld. De vroegmoderne receptie (1533–1650) van Proclus' Commentaar op het eerste boek van Euclides' Elementen », Ph.D. Diss. KU Leuven 2011; DAVID RABOUIN, « Le rôle de Proclus dans les débats sur la 'mathématique universelle' à la Renaissance », in ALAIN LERNOULD (ed.), *Etudes sur le Commentaire de Proclus au premier livre des Eléments d'Euclide*, Presses Universitaires du Septentrion, Villeneuve D'Ascq 2007, p. 217–235; SHIN HIGASHI, « Giorgio Valla et Alessandro Piccolomini. Quelques aspects de la réception de Proclus ê la Renaissance », *Bulletin of Liberal Arts Education Center*, 27 (2007) p. 31–52; MARIO. O. HELBING, « La fortune des commentaires de Proclus sur le premier livre des Eléments d'Euclide à l'époque Galilée », in GERALD BECHTLE, DOMINIC J. O'MEARA (eds.), *La philosophie des mathématiques de l'Antiquité tardive*, Actes du colloque international Fribourg, Suisse 1998, p. 173–193; and LUIGI MAIERU, « La diffusione di Proclo, commentatore di Euclide, nel Cinquecento », in *11° Annuario del Liceo Scientifico 'B.G. Scorza'*, Calabria Letteraria Editrice, Soveria Mannelli 1999, p. 49–68. For some details regarding Proclus' influence in the printed mathematical texts during the Renaissance cf. RICHARD J. OOSTERHOFF, *Making Mathematical Culture: University and Print in the Circle of Lefèvre d'Étaples*, Oxford University Press, Oxford 2018 (Oxford – Warburg Studies). Two remarkable monographs partially treating the reception of Proclus' commentary are ANNA DE PACE, *Le matematiche e il mondo. Ricerche su un dibattito in Italia nella seconda metà del Cinquecento*, Francoangeli, Milan 1993 (Filosofia e scienza – Studi), and SHIN HIGASHI, *Penser les mathématiques au XVIe siècle*, Classiques Garnier, Paris 2018 (Histoire et philosophie des sciences, 17).

Gabia (*c.*1500–1590), and Francesco Barozzi (1537–1604), and five fragmentary translations used both with pedagogical and scientific purposes contained in the works of Giorgio Valla (1447–1500), Jerónimo Muñoz (1520–1591), Conrad Dasypodius (1532–1600), Federico Borromeo (1564–1631), and Johannes Kepler (1571–1630).[2] I shall also dwell (as far as the scope of this study allows it) on some aspects related to early modern philosophy of mathematics and science, showing how Proclus' doctrines were received and interpreted in each case in topics such as the classification of mathematical sciences, the epistemology of mathematics (mostly as a form of innatism), metaphysics, and theology.

By signaling some significant episodes in the Latin re-discovery and spread of Proclus' commentary, I attempt to provide a general picture which, I hope, may serve as a guide for further research.

I. *The First Entry of Proclus' Greek Text in the West*

The first obscure hints pointing to the possible recovery of some Greek manuscripts of Proclus' *In Euclidem* appear in the *carteggio* of the Sicilian scholar Giovanni Aurispa (1376–1459). According to Paul Lawrence Rose,[3] Aurispa can be considered one of the principal restorers of the Greek mathematical tradition in Italy through his acquisition of several codices from Byzantium during the first decades of the 15th century.[4] Proclus' name is mentioned twice in Aurispa's letters to the Florentine monk Ambrose Traversari (1386–1439). It first appears in a letter dated the 27th of August of 1424, where Aurispa talks about all of Proclus' works in Greek:

> [...] quidquid scripsit Plotinus, quidquid Proclus, viri Platonici [...].[5]

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[2] The fragment translated by Kepler and chosen for the collation in the Appendix corresponds to the Greek text from p. 12, l. 2 to p. 18, l. 4 in PROCLUS, *Procli Diadochi in Primum Euclidis Elementorum Librum Commentarii*, ed. GODFRIED FRIEDLEIN, B. G. Teubneri, Leipzig 1874.

[3] Cf. PAUL LAWRENCE ROSE, *The Italian Renaissance of mathematics: studies on humanists and mathematicians from Petrarch to Galileo*, Libraire Droz, Genève 1975, p. 28 (Travaux d'humanisme et renaissance CXLV).

[4] Between 1418 and 1423, Aurispa was in Constantinople, engaging both in scholarly and diplomatic activities, and acquired a collection of around 238 Greek codices, many of them containing works ignored in the Latin West. Cf. GIOVANNI AURISPA, *Carteggio di Giovanni Aurispa*, ed. REMIGIO SABBADINI Istituto storico italiano, Roma 1931 (Fonti per la storia d'Italia), Prefazione, p. XVII; CHARLES, L. STINGER, *Humanism and the Church Fathers: Ambrogio Traversari (1386–1439 and the Revival of Patristic Theology in the Early Italian Renaissance,* University of New York Press, New York 1977, p. 36–37.

[5] Cf. AURISPA, *Carteggio di Giovanni Aurispa*, p. 12.

The second mention of Proclus' name appears in another letter to Traversari dated 1427. Here Aurispa asks Traversari to return certain manuscripts, among which he mentions a work of Proclus on mathematics:

> Puto te habere Proclum τὸν Πλατωνικὸν καὶ Ἀθηναῖον *Περὶ ὀργάνων πολεμικῶν* et nescio quid aliud in mathematicis.[6]

It is difficult to determine whether Proclus' *In Euclidem* was among the manuscripts Aurispa brought to Italy. Aurispa's indication about 'whatever Proclus wrote' (*quidquid Proclus* [*scripsit*]) may include his *In Euclidem*. Nevertheless, it is (grammatically) unclear whether the second clause from Aurispa's 1427 letter refers to the mathematical works of Proclus or to works on mathematics in general. The fact that the spurious astronomical work *De sphaera* was at that time attributed to Proclus, and given that a certain mechanical work Περὶ ὀργάνων πολεμικῶν was falsely attributed to Proclus by Aurispa in the same letter, only add further uncertainties to the matter.

More solid evidence of the diffusion of the Greek text of Proclus' *In Euclidem* in Italy in the second half of the 15th century can be drawn from the recorded presence of manuscripts, many of which are still extant. The Byzantine émigré cardinal Bessarion (1403–1472) owned a 11th-century manuscript of Proclus' *In Euclidem*, now preserved in the Marciana Library, MS Gr. Z. 306.[7] According to Kibre, a manuscript of *In Euclidem* is also listed as part of the library of Giovanni Pico della Mirandola (1463–1494).[8] The manuscript at Biblioteca Estense of Modena α. T. 9. 11[9] (fol. 67–241v) was copied, according to Rose,[10] by Giorgio Valla (1447–1500), who also produced a partial translation of Proclus' *In Euclidem* (as shown below). The Vatican manuscript Urb. Gr. 71,[11] formerly owned by the Urbino cardinal Giulio della Rovere (1533–1578), also preserves some fragments of Proclus' text. From an epistolary exchange between Pietro Bembo (1470–1547) and

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6 Cf. Aurispa, *Carteggio di Giovanni Aurispa*, p. 51.

7 Elpidio Mioni, *Bibliothecae Diui Marci Venetiarum codices graeci manuscripti. Volumen II: Thesaurus Antiquus. Codices* 300–625, Ministero per i beni culturali e ambientali, Roma 1985 (Indici e Cataloghi, Nuova Serie VI). A digitised version is accessible at <http://www.internetculturale.it/jmms/iccuviewer/iccu.jsp?id=oai%3A193.206.197.121%3A18%3AVE0049%3ACSTOR.240.9963&mode=all&teca=marciana>. This manuscript contains the still unexplored marginal notes by Bessarion to *In Euclidem*.

8 Pearl Kibre, *The Library of Pico della Mirandola*, Columbia University Press, New York 1936, *Proclus on Euclid*, n. 436.

9 Vittorio Puntoni, *Indice dei codici greci della Biblioteca Estense di Modena,* Tipografia dei fratelli Bencini, Firenze – Roma 1896, p. 413. This manuscript has been wrongly quoted by Rose, *The Italian Renaissance*, p. 47, who wrote 46 instead of 76. The MS Est. Gr. 46 now is α. T. 9. 11. Cf. <https://pinakes.irht.cnrs.fr/notices/cote/43443/>.

10 Cf. Rose, *The Italian Renaissance*, p. 47.

11 A digitised version is accessible at <https://digi.vatlib.it/view/MSS_Urb.gr.71>.

the Sicilian mathematician Francesco Maurolico (1494–1575), dated 1536, it is known that the former owned a manuscript of *In Euclidem*,[12] now lost.

In 1533, the German reformer, humanist, and theologian Simon Grynaeus (1493–1541) published one volume in Basel of the first ever edition containing the Greek text of Euclid's *Elementa geometrica* and Proclus' *In Euclidem.*[13] This is a relevant fact for the diffusion of Proclus' work in the Latin West: every scholar acquiring Euclid's *editio princeps* would *ipso facto* gain access to Proclus' commentary, helping thereby to spread some of Proclus' views as a sort of default philosophy of mathematics (I will propose some examples in the next sections). It is precisely this edition that Georg Joachim Rheticus (1514–1574) offered as a gift to his teacher, Copernicus, in 1539.[14]

Grynaeus' edition of Proclus' *In Euclidem* was later ridiculed by the Venetian nobleman Francesco Barozzi as 'more dilacerated than printed' (*dilaniato potius quam impresso*).[15] These deficiencies of the text can be explained by the fact that the edition followed only one manuscript, identified as MS 97 preserved at Corpus Christi College Library, Oxford.[16] As we learn from Grynaeus' preface, he borrowed the manuscript from Corpus Christi with the approval of John Claymond (1468–1537).[17]

MS 97 of Proclus' *In Euclidem* was probably produced at Padua during the first years of the 16th century. Grynaeus' edition of Proclus' commentary does not present auxiliary diagrams (unlike his edition of Euclid in the same volume) and is divided into four books, in so far as each of the two prologues was considered an independent book. The margins contain Greek and Latin titles for distinguishing sections in the continuous text as well as Latin notes describing the contents (e.g., the marginal note on fol. 15 reads: *Hactenus obiectio, respondet ex Platone*).

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12 Cf. Rose, *The Italian Renaissance of Mathematics*, p. 162.

13 The complete title of the 1533 edition was Euclid, Proclus, *Εὐκλείδου Στοιχείων βιβλία 15 ἐκ τοῦ Θέωνος συνουσίων. Εἰς τοῦ ἑαυτοῦ τὸ πρῶτον ἐξηγημάτων Πρόκλου βιβλία 4*, ed. Simon Grynaeus, I. Hervagium, Basel 1533.

14 Cf. Rose, *The Italian Renaissance of Mathematics*, p. 130. Some of the extant copies of Grynaeus' printed edition are located at Roma, Biblioteca Casanatense, M. V. 7; Roma, Biblioteca Angelica G. 9. 22; Rome, Biblioteca Barberina, I. V. 74, and XXI. E. 17; Florence, Biblioteca Riccardiana, E. II. 10224; Leiden University Library, Math. 86; London, British Museum Library 716. I; Oxford Bodleian Library Auct. K 3. 8, C. 3. 3, and A A. 70. Art. For more details on the extant copies here presented and other editions cf. Baldassarre Boncompagni, *Intorno al commento di Proclo sul primo libro degli Elementi di Euclide*, Tipografia delle Scienze Matematiche e Fisiche, Roma 1784.

15 Francesco Barozzi, *Procli Diadochi Lycii philosophi platonici ac matematici probatissimi in primum Euclidis Elementorum librum commentariorum ad uniuersam mathematicam disciplinam principium eruditionis tradentium libri IV*, Gratiosus Perchacinus, Padova 1560, Praefatio ad lectorem, p. 2.

16 Nigel Wilson, *A Descriptive Catalogue of the Greek Manuscripts of Corpus Christi College Oxford*, D.S. Brewer, Oxford 2011. Cf. <https://pinakes.irht.cnrs.fr/notices/cote/48629/>.

17 Simon Grynaeus, *Εὐκλείδου Στοιχείων*, p. 5v: « [...] mihi ipsi Procli commentaria Oxonii Iohann. Claymundus candide suppeditabat [...] ».

Grynaeus, who was a friend of Melanchton (1497–1560), had great interests in the reform of the curriculum and placed great emphasis on mathematical subject. [18] He considered mathematics, and especially geometry, both as propaedeutics for attaining foundational and certain knowledge, and (much like Proclus in the first prologue)[19] as an applied instance of dialectics.[20] Regarding the educational role of mathematics, Grynaeus seems to have valued the rational powers bestowed from mathematical training upon the learners in a way which seems to have been inspired by the reading Proclus' *In Euclidem*. He even declares that true philosophy cannot be exercised if either dialectics or mathematics is lacking, and that the absence of either of them would undermine the scholarly and intellectual life.[21]

II. *The Latin Translations and Uses of Proclus'* In Euclidem

II.1. Giorgio Valla

The earliest Latin translation of parts of Proclus' *In Euclidem* known to us are included in the monumental work *De expetendis et fugiendis rebus* by Giorgio Valla.[22] As mentioned in the previous section, Valla was not only familiar with Proclus' Greek text but also copied by himself the work and translated into Latin the two prologues.

Valla's *De expetendis*, published one year after his death (in 1501), consists of forty-nine books devoted to various topics leading to universal wisdom, including mathematics, astronomy, astrology, metaphysics, logic, medicine, and ethics.

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[18] Cf. Eckhard Kessler, « Clavius entre Proclus et Descartes », in Luce Giard (ed.), *Les jésuites à la Renaissance, Système éducatif et production du savoir*, PUF, Paris 1988 (Bibliothèque d'histoire des sciences), p. 291.

[19] Cf. First prologue, chapter XIV, Proclus, Glenn r. Morrow (ed.), *Proclus: A Commentary on the First Book of Euclid's Elements,* Princeton University Press, Princeton, NJ 1970, p. 35–36.

[20] Grynaeus, *Εὐκλείδου Στοιχείων,* Praefatio, p. 5v.

[21] Grynaeus, *Εὐκλείδου Στοιχείων,* Praefatio, p. 5r: « Ac si quis me quidem audiat, sic iudico, ad philosophandum vere, cum neutra sufficiat sola [dialectica et mathematica], rite coniunctas omnem protinus difficultatem profligaturas, tum siquidem alterutra carendum sit, aegrius hac nos carituros ».

[22] Giorgio Valla, *De expetendis et fugiendis rebus opus*, Manutius, Venice 1501. For an account of Valla's activity as a restorer of mathematical and scientific texts cf. Amadeo Raschieri, « Giorgio Valla editor and translator of ancient scientific texts », in Paula Olmos (ed.), *Greek Science in the Long Run: Essays on the Greek Scientific Tradition (4th c. BCE–16th c. CE)*, Cambridge Scholars Publishing, Cambridge 2012, p. 127–149; cf. also Paul L. Rose, « Bartolomeo Zamberti's Funeral Oration for the Humanist Encyclopaedist Giorgio Valla », in Cecil H. Clough (ed.), *Cultural Aspects of the Italian Renaissance: Essays in Honor of Paul Oskar Kristeller*, Manchester University Press, Manchester 1976, p. 299–310. For Valla's reception of Proclus' *In* Euclidem, cf. Higashi, « Giorgio Valla et Alessandro Piccolomini. Quelques aspects de la réception de Proclus ê la Renaissance ».

Book I and book X of Valla's *De expetendis* include large sections, in Latin, of Proclus' two prologues to *In Euclidem*. However, Proclus is never mentioned as the author of these fragments. In Valla's book I, a translation of Proclus' first prologue is tacitly inserted between chapter XIV (*De tota mathematica*) and chapter XXIII (*Unde mathematices nomen*). I present in Appendix 1 an index containing, on the one hand, the *incipit* and *explicit* of each chapter from XIV to XXIII in the Greek text of Proclus (in Friedlein's edition) and, on the other hand, Valla's corresponding translation of the same *loci*. From this analysis, we conclude that chapters XIV to XXIII consist of an almost complete translation of Proclus' first prologue.

Some of the chapters present changes. Chapter XIX, for instance, contains three alterations: an omission[23] of Proclus' explanation of the ways towards perfection expounded in Plato's *Phaedrus*; secondly, a mention[24] of the Pythagorean Philolaus and the divine mysteries is replaced by the clause *cui trinus et unus influxerit Deus, atque adeo illuxerit*; thirdly, the five pages (in Friedlein's edition)[25] of Proclus' enumeration of the benefits and utilities of mathematics if applied to other arts and human affairs are replaced by a succinct summary.[26]

In some places, Valla's translation adds some Latin phrases for the sake of clarity or as a matter of philological zeal. In other sections, Valla introduces apologetic interjections for the Latin expressions he is forced to use in translating Proclus. Sometimes he either abbreviates sentences by skipping parts and clauses or even omits entire sentences. For example, when reading διάνοια in cap. XIV, l. 10, Valla adds *quam interdum intelligentiam a Cicerone video appellatam*. In chapter XVII, Valla attributes to Cicero Proclus' doctrine on the projection of mathematical contents of the soul onto the imagination (or *phantasia*) and the reflection in the cosmos of the mathematical order: *ut secutus etiam Platonem Cicero de universitate docet.* In the beginning of cap. XVI, Valla preserves the Greek word: *quam vocant graeci criterium*. In cap. XVII, when discussing the origin and the ontological status of mathematics,[27] Valla skips Proclus' mention of Plato and the order emanation: [...] ὡς ὅ τε Πλάτων ἀξιοῖ καὶ ἡ τῶν ὅλων ἐπιδεικνύει πρόοδος. In the same cap. XVII, Valla omits one of the questions about the possibility of identifying in the sensible world pure geometrical properties, such as actually right angles or totally exact line equality.[28] In chapter XVII, we find four tokens of Valla's apologetic remarks concerning the unusual Latin he is adopting, regarding

[23] Corresponding to Proclus, *In primum Euclidis*, p. 21, l. 4–l. 19.
[24] Corresponding to Proclus, *In primum Euclidis,* p. 22, l. 14–l. 16.
[25] Corresponding to Proclus, *In primum Euclidis,* p. 23, l. 12–28, l. 23.
[26] Valla, *De expetendis,* Liber I, cap. XIX: « Quanta praeterea scientiis, et artibus omnibus mathematica perveniat utilitas mihi pluribus hic disserendum existimarem, nisi suus singularum disciplinarum desideraret, ac patefaceret locus ».
[27] Corresponding to Proclus, *In primum Euclidis,* p. 12, l. 8–9.
[28] Corresponding to Proclus, *In primum Euclidis,* p. 12, l. 23–26

the non-Ciceronian terms *intellectus*, *exemplariter*, *substantiatur*, and *essentiales*.[29] Valla also adds four lines of his own at the end of the last chapter (XXIII) of book I as a final exhortation to mathematical studies.[30]

It is perhaps noteworthy that Valla omitted Proclus' references to the name of Aristotle in the first prologue (in chapter XIX), but not in the second. In chapter XX[31] Valla also omitted Proclus' words φησὶν Ἀριστοτέλης, deleting thereby the authoritative source of the quotation from *Eth. Nic.* 1094b, l. 28 or *De Part. An.* 639a, l. 1–5: *nam qui in iis rationibus plane fuerit eruditus, is recti examinis ferre poterit sententiam*. One likely way of explaining this absence might be that Valla, following the humanist anti-Aristotelian trend of finding ancient authorities alien to the university curricula, skipped the reference to Aristotle, rather dwelling on Cicero, Plato, and the Pythagoreans.

The translation of the second prologue of Proclus' *In Euclidem* is inserted and adapted by Valla as the first chapter, entitled *De geometria* (see table 2 in Appendix 1), of book X of his *De expetendis*. After a short introduction, Valla translates, still without mentioning Proclus' name, the entire prologue with the exception of the initial paragraph.[32]

A gap of five folia[33] provides the occasion for Valla's own writing. Valla uses the opportunity to insert an excursus of his own on the practical uses of mathematical disciplines and their gnoseological relations to each other: optics, catoptrics, scenographics, geodesy, and even architecture are discussed. This section introduces authorities foreign to Proclus' text, such as Vitruvius and Damianus of Larissa. It is not rare to see Valla referring to authorities that are absent in the

[29] Valla, *De expetendis,* Liber I, cap. XVII: « [...] intellectus siquidem, sive mentem libet appellare [...]; exempli in modum, seu malis exemplariter dicere [...]; [...] dixerim substantiatur nisi verear mihi acclamitari; [...] ut sic loquar, essentiales [...] ». Cf. Appendix 2, 1 for the entire transcription of the section.

[30] Valla, *De expetendis,* Liber I, cap. XXIII: « Cum igitur etiam agitari humanos animos mathematica cognitio et acui ingenia et celeritate percipiendi inde venire, concedant omnes inde non sine causa summi viri etiam impensam huic scientiae operam dederint, nec magnus olim unquam habitus vir quisquam sine hac sit. Cum, inquam, tot tantisque nos bonis cumularit, quaenam causa fuerit cur ipsam toto pectore amplecti non debeamus? ».

[31] Corresponding to Proclus, *In primum Euclidis,* p. 32, l. 24.

[32] Corresponding to Proclus, *In primum Euclidis,* p. 48, l. 9–81, l. 22.

[33] Corresponding to Proclus, *In primum Euclidis,* p. 70, l. 14–75, l. 6

Greek original;[34] moreover, he even introduces the name of Proclus among the authorities.[35]

To the last sentence of the second prologue, Valla adds a fragment of Proclus' commentary on theorem III, proposition VI.[36] Proclus refers there to Porphyry's views on mathematical composition from first principles, resolution through syllogisms, and demonstration *ad impossibile*.[37] After this implicit quotation, Valla finishes chapter I of his tenth book with two lines of his own announcing the topic of the next section, namely the definition of a point.

Interestingly, whereas in his classification from the beginning of the prologue Proclus situates geometry just after arithmetic (καὶ ὅτι δευτέραν ἔχει τάξιν μετὰ τὴν ἀριθμητικήν), Valla argues that geometry is also *post musicam*. This is an interesting epistemological shift: instead of considering music as a subalternate science, Valla concedes its priority over geometry, and so ranks it second in simplicity and, thus, primacy, only after arithmetic.

In sum, although Valla did not mention Proclus as the author either of chapters XIV–XXIII of the first book of *De expetendis*, or as the source of most of chapter I of book X, *De expetendis* is a remarkable piece of evidence that the two prologues of *In Euclidem* (containing a powerful anti-abstractionist and innatist philosophy of mathematics) were available in Latin and disseminated in a printed edition at the very beginning of the 16th century. Valla's choice to assimilate them to his own text, without mentioning the name of Proclus, would require a more detailed explanation, which is beyond the scope of this paper.

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[34] Corresponding to Proclus, *In primum Euclidis,* p. 69, l. 4. Valla's version reads as follows: « Fuit etiam Conon, cuius Ad Dosifeum de tetragonismo meminit utpote sui amici Archimedes, Apollonius, Heron, Philon, Bizantius, Diocles, Porus, Nicomedes, Menealeus, Theon, qui ante Ptolemaeum ».

[35] In the section corresponding to Proclus, *In primum Euclidis,* p. 78, l. 13, and p. 76, l. 4. Valla's text indicates : « [...] tam Speusippeis, quam Menechmatis, Heron et Proclus aestipulantur; [...] Heronis Alexandrini, Anatoli, Proclique ».

[36] The addition corresponds to Proclus, *In primum Euclidis,* p. 255, l. 11–256, l. 8.

[37] I shall deal with this topic in a future work. For now, it is worthy to note that Proclus' discussions on demonstration and mathematical analysis as presented in the commentary were read within the context of the northern Italy long-held discussions on *regressus demonstrativus*. Cf. Olivier Dubouclez, *Descartes et la voie de l'analyse*, Presses Universitaires de France, Paris 2013 (Épiméthé essais philosophique), Ch. IV *passim*; John H. Randall, *The School of Padua and the Emergence of the Modern Science*, Editrice Antenore, Padova 1961; Jaakko Hintikka, Unto Remes, *The Method of Analysis: Its Geometrical Origin and Its General Significance*, Springer, Dordrecht 1974 (Boston Studies in the Philosophy of Science, 25); cf. also Jaakko Hintikka, « Method of Analysis: A Paradigm of Mathematical Reasoning? », *History and Philosophy of Logic*, 33/1 (2012), p. 49–67.

II.2. Bartolomeo Zamberti

The Venetian Bartolomeo Zamberti, probably inspired by his master Valla in the value he placed on Proclus' *In Euclidem*, accomplished one of the earliest complete Latin translations of the text. Zamberti also edited Euclid's *Elementa* in Latin (1505, Venice), and showed a remarkable philological sensitivity in his approach to the editing of mathematical works. His trade was the restoration of texts (and their contents) from the inaccuracies and gaps of preceding editors and translators to their ancient, original text so that an understandable Latin translation could then be accessible to readers.[38]

Zamberti's translation of Proclus' *In Euclidem* is preserved in the manuscript Munich, Bayerische Staatsbibliothek, Clm 6, fol. 1r–154v. The *incipit* of the first prologue (named *liber primus*) reads (fol. 1r):

> Procli Lycii diadochi Platonici in primum librum *Elementorum* Euclidis Megarensis mathematici praestantissimi, commentariorum liber primus, Bartholameo Zamberto Veneto interprete.

It cannot be determined with precision the date when Zamberti began the translation of Proclus' commentary, but at the end of the preface to his 1505 edition of Euclid's *Elementa*, he made clear his intention of providing a Latin translation of Proclus' text.[39] On the other hand, the closing lines after the colophon of book four, which closes his Latin version of *In Euclidem*, state the details of the date of its completion. It reads (fol. 154v):

> Procli Lycii diadochi Platonici in primum librum Elementorum Euclidis megarensis Platonici et mathematici praestantissimi, commentariorum libri quarti et ultimi finis Bartholameo Zamberto Veneto interprete. Immortali Deo gratias Anno Salutis MDXXXVIIII, Mense septimo. Anno interpretis VI et LX absoluto.

Thus, Zamberti had the intention of producing a translation of Proclus' *In Euclidem* at least since 1505, and was eventually finished in July of 1539, four years before

[38] On Zamberti's contributions to the mathematical culture of the 16th century and his restoration of mathematical texts (mostly regarding Euclid's *Elementa*), see Robert Goulding, *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History*, Springer, Dordrecht 2010 (Archimedes, New Studies in the History and Philosophy of Science and Technology, 25), chap. 5, *passim* and chap. 6, p. 150–154.

[39] Bartolomeo Zamberti, *Euclidis Megarensis philosophi platonici mathematicarum disciplinarum ianitoris habent in hoc volumine quicumque ad mathematica substantia aspirat Elementorum libros XIII*, Venice, Giovanni Tacuino 1505, p. 6v: « Efficiemus ut nostris laboribus Proclus Platonicus ipsius Euclidis interpres et Nicomachus Pythagoreus hucusque Latinis ignoti, Graecia relicta inter Italiae nitescentia vireta resplendeant et scholas Italiae vagari non vereantur ».

his death. This raises the question of the Greek source(s) used by Zamberti for his translation into Latin. If Zamberti initiated his planned translation soon after his edition of the *Elementa*, he would have used Greek manuscripts available at the time in northern Italy.

If Zamberti began the translation after Grynaeus' *editio princeps* of 1533, he might have used this version of the text, but also another source. Indeed, the Zamberti's translation is more complete that the Grynaeus' edition, such as for the case of the famous anecdote of King Ptolemy asking Euclid for a more expedient way of learning geometry, shows an important gap.[40] In Grynaeus' edition,[41] this line reads as:

> [...] ἤρετο ποτὲ αὐτὸν εἰ τίς ἐστι περὶ γεωμετρίαν.

This sentence, as it stands, lacks the main nominative as well as the genitive complements of the noun. Friedlein's restored text reads (p. 68, l. 14–15):

> [...] ἤρετό ποτε αὐτόν, εἴ [*sic*] τίς ἐστιν περὶ γεωμετρίαν ὁδὸς συντομωτέρα τῆς στοιχειώσεως.

And Zamberti translates this passage as:

> [...] numquid ad geometriam capessendam aliquod extaret brevius iter elementari. (Clm 6, fol. 24, l. 1–3)

Zamberti's Latin text is accompanied by a sequence of numbered titles written on the margins, most likely intended for an *index rerum*. For example, at fol. 4v, l. 4, a marginal note reads: *38 Genera et formae mathematicae unde essentiam obtineant?*, whereas l. 9 is assigned: *40 Id propter quod unumquodque tale etiam magis est huiusmodi.* The marginal titles accompany the entire translation, although their placement does not follow any regular pattern. For instance, whereas fol. 5v and 6r do not show any titles, fol. 6r contains eight of them in a few lines (l. 6–34, titles numbered as 43–51).

Some margins show diagrams of geometrical figures drawn freehand. For instance, early in the manuscript, fol. 5r contains three diagrams of the three triangles (isosceles, equilateral, and scalene) mentioned by Proclus in that passage. Later folia contain diagrams illustrating the commented propositions, which are clearly traced by using compass and rule.

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[40] For more details regarding this locus cf. Stephen P. Rigaud, *On the Arenarius of Archimedes*, S. Collingwood, Oxford 1837, p. 28–29.

[41] Grynaeus, *Εὐκλείδου Στοιχείων*, p. 20 l. 4–5.

The Latin of the translation, if compared to that of Zamberti's other texts (e.g., the prologue of his 1505 edition of the *Elementa*) does not seem extremely polished. A comparison of Zamberti's *In Euclidem* and Valla's translation of the prologues makes clear that Zamberti did not draw from the latter. Some details on a particular coincidence will be commented upon in subsequent lines.

It seems that Zamberti's manuscript was faulty in some areas. Ex. gr., where Proclus declares that the superior nature of the mathematical reasons embedded in the soul reveals a pathway to being, the Greek text states « [...] εἰς τὸ εἶναι πάροδον [...] »,[42] and Zamberti translates as « [...] ex se ipsa semita, ut sit gressus » (Clm 6, fol. 6r, l. 10–11).

It seems that Zamberti's source contained two errors in this section: ἰέναι ('to go') was written instead of εἶναι ('to be'); πάροδου instead of πάροδον – both are common errors. It is likely that, trying to make sense of the passage, Zamberti read εἰς as ἐκ, getting thereby 'ἐκ τὸ ἰέναι παρόδου' ('from the pathway to go'), and then rightly took the verb to be a nominal form substantivized by the neuter article (τὸ).[43] Zamberti's Latin translation of Proclus' *In Euclidem* was not printed.

Finally, a point of intellectual history is worth noting here. Already in his 1505 prologue to Euclid's *Elementa* Zamberti was familiar with the doctrines of Proclus' prologues (perhaps through Valla's influence) and saw some of their contents as theories of great importance. There he states:

> Nam disciplinae mathematicae in anima positae, ut Proclus inquit Diadochus, si intellectu cogitentur, tunc omni prorsus materia carent, at si volumus ipsas sensibus percurrere, materiae subiaceant necesse est. Hoc igitur medio a physicis, hoc est a naturalibus, usque ad metaphysicam, hoc est transnaturalia, penetrare possumus.[44]

Remarkably, characteristic features of Proclus' *In Euclidem* are presented in these lines. Firstly, we find the doctrine of mathematical innatism, a doctrine which was destined to play such a great role in early modern philosophy from Descartes onwards. Secondly, a version of *medietas mathematicarum*, presenting mathematics as both an epistemological and ontological hinge between physics and metaphysics. This shows that already at the beginning of the 16th century, Proclus' teachings contained in his commentary were already understood as an

[42] Cf. PROCLUS, *In primum Euclidis,* p. 16. l. 7–8.

[43] Yet this seems to have been a hard-to-translate locus in Proclus' *In Euclidem*, for both Valla and Gabia also independently misread the line in a similar way. VALLA, *De expetendis,* Liber I, cap. XVII: « [...] ipsaque per sese gignit, ad id ut sint accessus ». Gionavvi Battista Gabia, MS Ambrosiana P.51 sup., fol. 15v, l. 19–20: « [...] ut sint, progressum sortitae fuerint ». Barrozzi and Kepler translation interpreted this reading τὸ εἶναι. Cf. Appendix 2.

[44] ZAMBERTI, *Euclidis Megarensis*, p. 4r.

exhortation or apology for mathematics in printed mathematical texts and as default philosophy of mathematics. The influence of Proclus' two prologues to *In Euclidem* as introduction to mathematics become common place in the literature and philosophy of mathematics that extended to the editions of Euclid's *Elementa* of Federico Commandino (1509–1575),[45] Christopher Clavius (1538–1612), and beyond.[46]

II.3. Giovanni Battista Gabia

The second complete Latin translation of Proclus' *In Euclidem* to consider is that of Giovanni Battista Gabia, preserved at the Ambrosiana Library, MS P. 51 sup. This translation has been confusingly catalogued under the name of Francesco Barozzi's. Gabia's translation comprises the entire volume, namely from fol. 1r to fol. 343r, and was never printed.

The precise date of Gabia's birth is unknown. In 1553, he gained the chair of Greek at the University of Rome. He died in 1590. Gabia was mostly famous as a humanist devoted to translating literary and theological works from Greek. In 1543 he published a Latin translation of all of Sophocles' tragedies, in 1562–1563 two theological commentaries of Theodoret of Cyrus but also ancient mathematical texts, most notably, Hero's *Pneumatica*[47] and Proclus' *In Euclidem*.[48]

According to Rose,[49] it is highly plausible that Gabia was encouraged to translate at least these two former mathematical texts at the suggestion of an acquaintance, the renowned Neapolitan astrologer Luca Guarico (1475–1558). In addition to his own astrological and mathematical works, Guarico was dedicated to a project of translation and editing of mathematical texts in collaboration with

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45 For an account of the philosophy of mathematics of Commandino (in whose edition of the *Elementa* the influence of Proclus' *In Euclidem* can be found) cf. Guy Claessens, « The Drawing Board of Imagination: Federico Commandino and John Philoponus », *Journal of the History of Ideas*, 76/4 (2015), p. 499–515.

46 Cf. Sabine Rommevaux, *Clavius, une clé pour Euclide au XVIème siècle*, Vrin, Paris 2006 (Collection Mathesis); Guy Claessens, « Clavius, Proclus, and the Limits of Interpretation: Snapshot-idealization versus Projectionism », *History of Science*, 47/3 (2009), p. 317–336. Vincenzo De Risi, *Leibniz on the Parallel Postulate and the Foundations of Geometry, The Unpublished Manuscripts*, Birkhäuser, Berlin 2018 (Sciences Networks Historical Studies 51), ch. 2.1; Kessler, « Clavius entre Proclus et Descartes »; Nicholas Jardine, « The Forging of Modern Realism: Clavius and Kepler against the Skeptics », *Studies in History and Philosophy of Science*, 10/2 (1979), p. 141–173; Luigi Maierù, « Metafisica ed enti geometrici: Benito Pereyra, Pedro Fonseca, Francisco Suárez », in *Sciences et religions. De Copernic à Galilée* (1540–1610) Actes du colloque international, Roma 1996 (Publications de l'École française de Rome, 260), p. 47–67.

47 Cf. Rose, *The Italian Renaissance,* p. 50, cf. MS Vat. Lat. 4575, fol. 1r–19r.

48 Cf. Gabia's biography by Elena Del Gallo in *Dizionario Biografico degli Italiani*, Volume 51, 1998 at <https://www.treccani.it/enciclopedia/giovan-battista-gabia_%28Dizionario-Biografico%29/>.

49 Cf. Rose, *The Italian Renaissance*, p. 50.

(or in honour of) other scholars. Some of the fruits of Guarico's mathematical publishing activity include fragments of Archimedes' *De quadratura circuli* and *De quadratura parabolae* (1503), the *Perspectiva communis* (1504) of John Pecham, and an edition of the Latin version of the *Almagest* made by Trapezuntius (1528). There is strong evidence that Gabia and Guarico maintained a cordial relationship. The complete works of Guarico[50] opens with a poem composed by Gabia in his honour.

If Rose's suggestion is right and Gabia translated mathematical works encouraged by Guarico, it is likely that he completed them in his youth. Moreover, Luca Guarico died in 1558, and his interest in ancient mathematical texts was more intense in the early period, between 1503 and 1528. It is likely that Gabia translated Proclus' *In Euclidem* before becoming professor at Rome in 1553. However, as things presently stand, it is hard to tell whether Gabia's translation preceded Zambia's (1539). The collation made by Professor Carlos Steel has revealed that Gabia was using MS Venice, Marciana VI.11 (coll. 1409)[51] copied by the Paduan professor Matteo Macigni (1507–1583) as the base for his translation.[52]

Gabia's translation from the manuscript Milan, Ambrosiana P. 51 sup., fol. 1r, opens with the following line:

> Johanne Baptista Gabio interprete, Procli Diadochi in primum *Elementorum* Euclidis liber primus.

Gabia's translation has titles which divide the text of the two prologues into chapters: *capitulum primum* (on fol. 1r, l. 1), *capitulum secundum* (on fol. 3r, l. 19) etc. The text does not contain diagrams and presents a translation which is interrupted by a line-break whenever a new chapter is introduced.

Gabia's translation seems to have been corrected in many sections by an unknown person. The difference in the ink, the Latin style, and the very understanding of the text makes it very likely that the translation was not modified by Gabia himself but by a second hand. It seems that Gabia firstly translated Proclus' *In Euclidem* by adopting a scholastic (and less elegant) Latin for various concepts; a second hand deleted or modified many of these scholastically flavoured terms and replaced them with those of a more Ciceronian prose. Below some of the passages before and after the correction:

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[50] Luca Guarico, *Opera omnia quae quidem extant Lucae Gaurici*, Tomus I, Ex Officina Henricpetrina, Basel 1552, p. 5r.

[51] Ciro Giacomelli, « I libri greci di Matteo Macigni. Contributo allo studio di una biblioteca umanistica », *La parola del passato*, 407/2 (2019), p. 361–420. p. 377, 378 (tav. 2), 396, 417. Digitised at <http://www.internetculturale.it/jmms/iccuviewer/iccu.jsp?id=oai%3A193.206.197.121%3A18%3AVE0049%3ACSTOR.241.10806&mode=all&teca=marciana>.

[52] I want to thank Professor Carlos Steel for this helpful information.

MS Milan, Ambrosiana P. 51 sup.	Gabia's translation	Altered text
fol. 11r, l. 6–9	Sequitur ut inspiciamus quaenam mathematicarum specierum generumque **substantia** merito dicatur, et utrum **ab sensibilibus** [...]	Sequitur ut inspiciamus quaenam mathematicarum formarum generumque **natura** merito dicatur, et utrum **ab iis quae sensu percipiuntur** [...]
fol. 13v, l. 9–15;	Atque enim si quis, ut aiunt, demonstraret **aequicrurium triangulum** habere tres angulos aequales duobus rectis et **aequilaterum** et **scalenum** similiter, hic rectum scit [...]	Atque enim si quis, ut aiunt, demonstraret **ex duabus aequalibus lineis compositum triangulum**, **quem aequicrurium vocant**, habere tres angulos aequales duobus rectis et **triangulum tribus aequalibus lineis contentum, quem aequilaterum appellamus**, et similiter **tribus inaequalibus, quae dicuntur scalenum**, hic rectum scit [...]
fol. 15r, l. 13–17	[...] neque prorsus posteriore loco, id est ex **sensibilibus** genita, necesse sane est **animam**, aut ab se, aut ex **anima** ipsas accipere, aut et a se et ab illa.	[...] neque prorsus posteriore loco, id est, ab **iis quae sub sensum cadunt**, genita, necesse sane **est animum**, aut ab se, aut ab **mente** ipsas accipere, aut et a se et ab illa.

As can be easily seen, this stylistic zeal at times makes the translation somewhat clumsy or confusing. The term *anima* is systematically replaced by the more Ciceronian *animus* or *mens*; *sensibilia* is always transformed into the lengthy clause *ea quae sub sensum cadunt* or *iis quae sensu percipiuntur*; confusingly, *substantia* is made *natura*, which translates both the Greek οὐσία and φύσις. One of the most blatant instances of this stylistic prodigality relates to the names of geometrical figures and specific mathematical terminology. Indeed, certain technicalities are not part of the Ciceronian lexicon, the anonymous hand adopted lengthy, descriptive relative clauses; hence, instead of *aequilaterum*, the anonymous author has *triangulum tribus aequalibus lineis constantem quem aequilaterum appellamus*. Such is the anonymous hand's linguistic care and superficial understanding of the philosophical and mathematical contents that it seems legitimate to argue that

more emphasis was put on the Latin style than the doctrinal accuracy and clarity of the version.

Because of the anonymous author's stylistic predilections, this translation of Proclus' *In Euclidem* is a case of great interest: it might be the first (and perhaps the only case) of an attempt to render Proclus' highly abstract and philosophically technical language into a Latin which (with great pains and sacrifices of precision!) aspires to that Ciceronian prose that many Renaissance scholars so cherished and effected.

II.4. Francesco Barozzi

Francesco Barozzi's translation is better known to scholars, as it was the first printed complete Latin version of Proclus' *In Euclidem* (in 1560). Its full title reads:

> Procli Diadochi Lycii philosophi Platonici ac mathematici probatissimi in primum *Euclidis Elementorum librum commentariorum* ad universam mathematicam disciplinam principium eruditionis tradentium libri IV, a Francisco Barocio patritio Veneto summa opera, cura ac diligentia cunctis mendis expurgati, scholiis et figuris, quae in graeco codice omnes desiderabantur, aucti, primum iam Romanae linguae venustate donati, et nunc recens editi, cum catalogo deorum, et virorum illustrium, atque auctorum, elencho librorum, qui vel ab auctore, vel ab interprete citati sunt, et indice locupleti notabilium omnium in opere contentorum, Patauii, excudebat Gratiosus Perchacinus, 1560.

As expressed in the lengthy title, the edition is embellished with diagrams absent in the Greek codices and includes an index of deities referred to, and of books and authors quoted either by Proclus or by Barozzi himself. The title also establishes (wrongly or perhaps with insincerity) that the present edition is the first ever Latin translation of Proclus' *In Euclidem*. I shall elaborate on this point below.

Barozzi's printed edition consists of 312 folia. The first 14 folia, containing a poem, the dedicatory letter to Vitruvius' translator and ardent mathematical student Daniele Barbaro (1514–1570), the preface, and the catalogues announced in the title are not numerated (fol. 1r–16r). This translation was announced by the Accademia Veneziana della Fama (an erudite society at Venice active from 1557 to 1561) in their *Somma delle Opere* (1558), fol. 13v.[53]

Barozzi was nobleman from Venice educated at Padua with excellent knowledge of Greek (the famous Hellenist Andrea Doni was his teacher) and a scholar with access to many financial and intellectual resources (such as his

[53] Cf. Valeria Guarna, *L'Accademia veneziana della Fama (1557–1561). Storia, cultura e editoria, Con l'edizione della Summa delle Opere (1558) e altri documenti inediti*, Vecchiarelli, Roma 2018 (Cinquecento, testi e studi letter. Ital).

impressive collection of Greek manuscripts, almost entirely preserved in the Bodleian Library).[54] He not only accomplished a Latin translation of Proclus' *In Euclidem*, but also reconstructed a more reliable Greek text than the one used as the source of the 1533 *editio princeps* by collating several manuscripts at his disposition to use as the basis for his Latin version. Concerning this restored text and the details of its manuscript sources, Barozzi states the following in his dedicatory letter to Daniele Barbaro:

> Verumtamen cum divina providentia, propter communem studiorum omnium utilitatem huic meo flagranti desiderio auxiliari maximo suo amore decrevisset, fecit ut, cum essem in insula Creta tertio abhinc anno, quoddam vetustissimum exemplar eorumdem Procli in Euclidem commentariorum, qui iam impressi fuerant, ad manus meas perveniret, quod fuerat Andreae Doni praeceptoris mei [...], ex quo quidem exemplari impressum illud quoad potui diligenter emendavi. Nam illud etiam antiquum pluribus in locis imperfectum erat. Postea vero cum in Italiam reversus essem [...], primum Bononiam profectus sum, ubi inveni duo exemplaria manuscripta, alterum in bibliotheca S. Salvatoris, ut appellant, quod una cum aliis etiam libellis, ut transcriberem, concessum mihi fuit a reverendis viris Floriano Cedroplano Bononiensi, priori tunc illius coenobii, et Raphaele Campiono procuratore [...]; alterum in bibliotheca excellentissimi viri Fabritii Garzoni medicam facultatem publice in Bononiensi Gymnasio profitentis, qui etiam quae maxima fuit eius liberalitas, voluit illud ipsum suum exemplar mecum afferri. [...]. Deinde cum illhinc discessissem, Patavium me contuli, ubi ex iis omnibus exemplaribus quoad fieri potuit unum integrum feci, quod postremo e Graeca lingua in Latinam converti [...]. Quod dum imprimebatur duo adhuc vidi Graeca exemplaria, unum Venetiis in bibliotheca Sanctorum Iohannis et Pauli, alterum Patavii ex bibliotheca Iohannis Vincentii Pinelli Genuensis [...].[55]

Thus, from his own testimony, we learn that Barozzi set himself the goal of improving Grynaeus' 1533 edition, which elsewhere he had deplored.[56] To this end, he employed an old manuscript he obtained from his own Greek teacher, Andrea Doni, and two other manuscripts from Bologna, at San Salvatore and Padua, respectively. Later, when the text was already given to the typesetters, Barozzi found two more Greek copies of Proclus' *In Euclidem*: one in Venice, in the library of Saints John and Paul, the other in Padua, in the library of the physician,

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[54] Cf. For some notes on Barozzi's life and intellectual profile cf. HIGASHI, *Penser les mathématiques au XVIe siècle*, p. 277 and forth; DE PACE, *Le matematiche e il mondo. Ricerche su un dibattito in Italia nella seconda metà del Cinquecento*; PAUL L. ROSE, *A Venetian Patron and Mathematician of the Sixteenth Century: Francesco Barozzi: 1537–1604*, Giardini Editori, Pisa 1977.

[55] BAROZZI, *Procli Diadochi in primum Euclidis,* p. 4–5.

[56] BAROZZI, *Procli Diadochi In primum Euclidis*, p. 3: « [...] quanvis nescius non essem, quod impressi fuerant Basileae quatuor Procli Diadochi libri commentarirorum in primum elementorum Euclidis, quod adeo laceros et corruptos inveni, ut nihil boni ex eis elicere potuerim ».

Vincenzo Pinelli. Rose[57] has identified the following manuscripts mentioned by Barozzi: the one borrowed from Vincenzo Pinelli corresponds to MS Milan, Ambrosiana A. 164 sup.,[58] and the one he obtained from Floriano Cedroplano and Rafaele Campiono as MS 2293 at the Bolognian Biblioteca Universitaria (fol. 1–149v).[59] As Rose suggests,[60] some candidates for Andrea Doni's and Fabrizio Garzoni's copies might be found among the manuscripts of Proclus' *In Euclidem* preserved in Italy, such as MSS Vat. Barb. Gr. 101,[61] Vat., Barb. Gr. 145[62] or the manuscript copied by Valla preserved in Estense Library MS α T.9.11 (Puntoni 46);[63] or, more importantly, the MS Oxford, Bodleian Library, Barocci 161,[64] which contains short scholia and corrections in Barozzi's hand. To my knowledge, these scholia have not been edited nor studied. The MS Venice, Marciana VI. 11 (coll. 1409), the one used by Gabia, seems to contain the Greek text that, according to Rose,[65] was emended by Barozzi himself, although further comparative research should be done to determine the parallels with precision.[66]

Barozzi comments in the preface on the Latin style of his translation, which offers interesting insight in his own project and methodology. There he states:

> Primo enim autorem hunc Latinum facere pro virili conatus sum, non ubique Ciceronis dumtaxat verba et formas dicendi sectando, sed Quintiliani etiam et aliorum Latinae auctoritatis virorum, qui de hisce, quae hoc in volumine continentur, scientias pertractarunt. Deinde vocabula scientiarum passim, ut fieri potuit, legitima, synceraque vertere volui. Ambitus praeterea orationis sive circuitus perspicuitatis gratia quandoque immutavi ac ea usus sum figura, quam

57 Cf. Rose, *A Venetian Patron*, p. 127.

58 Aemidius Martini, Domenico Bassi, *Catalogus codicum Graecorum Bibliothecae Ambrosianae*, t. I–II, Hildesheim, New York – Milano 1906. <https://pinakes.irht.cnrs.fr/notices/cote/42251/>

59 Digitised version at <https://amshistorica.unibo.it/286>.

60 Cf. Rose, *A Venetian Patron*, p. 127–128.

61 Valentinus Capocci, *Bibliothecae Apostolicae Vaticanae, Codices Barberiniani graeci*, Tomus I. Codices 1-163, Vaticano 1958. Digitised version at <https://digi.vatlib.it/view/MSS_Barb.gr.101>.

62 Capocci, *Bibliothecae Apostolicae Vaticanae,* 0799.
Digitised at <https://digi.vatlib.it/view/MSS_Barb.gr.145>.

63 Puntoni, *Indice dei codici greci della Biblioteca Estense di Modena*, p. 379–536.

64 Digitised version at <https://digital.bodleian.ox.ac.uk/objects/2c5e0267-3c38-4fbd-8fa6301f79df82ad/surfaces/f8da86c7-25f1-4a1e-adc6-911387d9ffe8/>.

65 Cf. Rose, *A Venetian Patron*, p. 127.

66 MS Paris, Bibliothèque nationale de France, Baluze 281 must also be mentioned. This volume of 329 folia contains an integral hand-written copy of Barozzi's 1560 edition. The quality of the calligraphy reveals that it was most certainly produced by a professional scribe. Marginal numbers indicate to which page of the printed edition corresponds each folium. Further research may reveal more details. From its existence, it can be established that such great value was place in Proclus' work by someone that (a far more expensive) manuscript copy of the Latin translation was commissioned to a professional, perhaps motivated by a personal distaste for printed editions. Digitised version at <https://gallica.bnf.fr/ark:/12148/btv1b90014927>.

> ὕστερον πρώτερον Graeci vocant. Ambiguitates insuper evitavi atque effugi tum geminatione verborum, vel mollioribus loquutionibus, vel participiorum Graecarumque dicendi formularum resolutionibus; tum etiam recte scribendi scientia, ut legenti tibi notum erit. A quibusdam denique dictionibus necessitatis Latinaeque linguae paupertatis causa non abstinui, quae exempli gratia huiuscemodi sunt: identitas, simplicitas, immaterialitas, totalitas, impartibilitas, et alia id genus [...].[67]

Barozzi reveals six relevant points regarding his translation preferences, sources, and style. After claiming that he is the first to translate Proclus' *In Euclidem* into Latin, (1) he declares that he has not only followed the Ciceronian prose, but also Quintilian and other classic authorities. Then, (2) he adds that he has also followed certain unnamed Latin authorities who treated the sciences contained in Proclus' *In Euclidem*. This probably refers to Latin authorities in mathematics. (3) He affirms that Greek scientific terminology has been rendered into Latin as better as possible. (4) Barozzi declares that, for the sake of clarity, changes have been made in the structure of the Greek lengthy sentences, mostly by inverting some of the clauses so that the subject become immediately apparent. As for the terminology, (5) Barozzi points out that he has tried to avoid any reduplication of terms which may cause confusion and has periphrastically rephrased the ubiquitous Greek participles. Finally, (6) he declares in an almost apologetic tone that, given the linguistical poverty of Latin (the infamous Lucretian *locus communis*), it was necessary for him to adopt abstract, scholastic, and artificial terms completely foreign to the golden standard of the Ciceronian Latin, of which he offers a list of examples. This list includes abstract nouns formed with *-itas* (*identitas, simplicitas, immaterialitas*), adverbs ending in *-iter* (*uniformiter, multiformiter, impartibiliter*), scientific terms (*symptoma, quaesitum, praedicatum*), and Latin geometrical terminology (*obtusangulum, acutangulum, quinquangulum*).

Barozzi's translation is arguably the most accomplished Latin version of Proclus' *In Euclidem*. Its merits consist mainly, but not exclusively, in its carefully chosen style, its respect for the integrity of the Greek text, its doctrinal precision, and its expressive clarity. All these merits reflect Barozzi's philological command of Greek, as well as the great work of scholarship presupposed by the reconstruction of a qualitative Greek text, all united with his expertise in mathematical subjects and the fittingness of the thoughtful selection of his Latin terminology.

Nevertheless, Barozzi's translation is doctrinally biased, for the wording of some passages echoes the controversies concerning the philosophy of mathematics in which the Venetian nobleman was involved, namely the 15th- and

[67] Barozzi, *Procli in primum Euclidis, Praefatio ad lectorem*, p. 12.

16th-century iteration of the debate known as *quaestio de certitudine mathematicarum*.[68]

This debate was re-ignited across Europe by the Sienese Alessandro Piccolomini (1508–1579) with a treatise intitled *De certitudine mathematicarum* (1547). Piccolomini, amongst many other issues which cannot be analysed here, defended the controversial position that the certainty of mathematical proofs and conclusions are grounded in the ontological status of mathematical objects, not in a would-be preeminent epistemological quality of mathematical demonstrations. In other words, Piccolomini contends that mathematical demonstrations are not instances of the so-called *demonstratio potissima*. As noted in the seminal works of Giacobbe and De Pace (and confirmed by further scholarship, see last note),[69] Piccolomini abundantly invoked the authority of Proclus' *In Euclidem* as one of his

[68] Regarding the debate on the certainty of mathematics and its historical and logical aspects cf. Dmitri Levitin, *Ancient Wisdom in the Age of the New Science*, Cambridge University Press, Cambridge 2015 (Ideas in Context, 113), part 4.5, p. 313–325; John Longeway, « The Place of *Demonstratio Potissima* in some 16th-Century Accounts of Mathematics », in Joël Biard (ed.), *Raison et démonstration*, Brepols, Turnhout 2015 (Studia Artistarum, 40), p. 223–251; Daniel Cozzoli, « Alessandro Piccolomini and the Certitude of Mathematics », *History and Philosophy of Logic*, 28/2 (2007) p. 151–171; De Pace, *Le matematiche e il mondo*, 1993; Tobias Schöttler, « From Causes to Relations: The Emergence of a Non-aristotelian Concept of Geometrical Proof out of the *Quaestio De Certitudine Mathematicarum* », *Society and Politics*, 6/2 (2012), p. 29–47, Paolo Mancosu, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, Oxford University Press, Oxford 1999, p. 13–35; Giovanni Ferraro, « Dimostrazioni Matematiche e Conoscenza Scientifica in Alessandro Piccolomini », in Howard Burns, Francesco Paolo Di Teodoro, Giorgio Bacci (eds.), *Saggi di Letteratura architettonica da Vitruvio a Winckelmann III*, Olschki, Firenze 2010, p. 215–233; Helbert E. Velilla-Jiménez, « El debate sobre la certeza de las matemáticas en la filosofía natural de los siglos XVI y XVII (*De quaestio* [sic] *de certitudine mathematicarum*) », *Estudios de Filosofía*, 3 (2018), p. 59–93; Paolo Palmieri, « On *scientia* and *regressus* », in Henrik Legarlund, Benjamin Hill (eds.), *Routledge Companion to Sixteenth Century Philosophy*, Routledge, Dordrecht 2017 (Routledge Philosophy Companions), p. 319–349; Higashi, *Penser les mathématiques au XVIe siècle*; Rabouin, « Le rôle de Proclus dans les débats sur la 'mathématique universelle' à la Renaissance »; Giulio Cesare Giacobbe, « Il *Commentarium De Certitudine Mathematicarum Disciplinarum* di Alessandro Piccolomini », *Physis*, 14/2 (1972), p. 162–193; Id., « Francesco Barozzi e la *quaestio de certitudine mathematicarum* », *Physis,* 14/4 (1972), p. 162–193; Luís M. Carolino, « Mathematics and the Late Aristotelian Theory of Science: The *Quaestio de Certitudine Mathematicarum* in Seventeenth-Century Portuguese Universities », in Victor Navarro-Brotons, William Eamon (eds.), *Beyond the Black Legend: Spain and the Scientific Revolution*, Publicacions de la Universitat de València, Valencia 2007, p. 399–412; Nicholas Jardine, « Problems of Knowledge and Action: Epistemology of the Sciences », in Charles B. Schmitt, Quentin Skinner, Eckhard Kessler, Kraye Jill (eds.), *The Cambridge History of Renaissance Philosophy*, Cambridge University Press, Cambridge 1988, p. 685–711; Paul L. Rose, « Certitudo Mathematicarum from Leonardo to Galileo », in *Leonardo nella scienza e nella tecnica*: *Atti del Simposio Internazionale di storia della Scienza*, Giunti Barberà, Firenze – Vinci 1969, p. 43–49.

[69] Giacobbe, « Il *Commentarium De Certitudine Mathematicarum Disciplinarum* di Alessandro Piccolomini »; Id., « Francesco Barozzi e la *quaestio de certitudine mathematicarum* »; De Pace, *Le matematiche e il mondo*.

authoritative sources in *De certitudine*. Twelve years after the publication of Piccolomini's text, Barozzi wrote a response, encouraged by his protector Daniele Barbaro, that vindicated both the superior status of mathematical demonstration and its authority over the interpretation of the contents of Proclus' *In Euclidem.* Barozzi's position is published in *Opusculum in quo una oratio et duae quaestiones, altera de certitudine, et altera de medietate mathematicarum, continentur* (Padua, 1560). Piccolomini and Barozzi are the first key-figures of a great controversy in which the right interpretation of some passages of Proclus' *In Euclidem* played a major philosophical, authoritative, and argumentative role. Moreover, Barozzi dealt with this question in a series of lectures held at the university of Padua (1559–1560) while teaching Proclus' commentary as the main text in the course of mathematics, becoming thereby the first person who taught *In Euclidem* in a Western university.[70] Through the intervention of the Jesuits Clavius and Benito Pereira (1535–1610), the question of mathematical certainty became an internal debate within the Jesuit order and, through the Jesuit intellectual spreading, a controversy all around Europe.[71] This debate would include figures such as (to name only a few) Petrus Ramus (1515–1572),[72] the Jesuit Giussepe Biancani (1566–1624), and, as Velilla-Jiménez has recently shown,[73] the sceptic Francisco Sánchez (1551–1623). The question would become so diffuse and momentous that, even in 17th-century Cambridge, we found Isaac Barrow (1643–1677), Newton's professor of mathematics and intellectual ally, wrestling both with the question and quoting Proclus' *In Euclidem* in his *Lectiones mathematicae XXIII* (Cambridge, 1664).[74]

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[70] As Rose has confirmed through his discovery of a letter dated the 7th of November of 1559 in the Venetian archives, Barozzi held a course on Proclus' *In Euclidem* as professor of mathematics at the university of Padua that year. Barozzi's lectures are preserved in MS Latin 7218 at the Bibliothèque nationale de Paris. These lectures have been transcribed by De Pace as an Appendix to De Pace, *Le matematiche e il mondo*.

[71] Cf. Carolino, « Mathematics and the Late Aristotelian Theory of Science: The *Quaestio de Certitudine Mathematicarum* in Seventeenth-Century Portuguese Universities ».

[72] Regarding the Reception of Proclus by Ramus and its Role in Early Modern History of Mathematics cf. Goulding, *Defending Hypatia*, p. 6–8, and 19–25. Cf. also Robert Goulding, « Pythagoras in Paris: Petrus Ramus Imagines the Prehistory of Mathematics », *Configurations*, 17/1 (2009), p. 51–86.

[73] Velilla-Jiménez, « Francisco Sánchez and the *Quaestio de certitudine mathematicarum*: A Sceptical Approach », *Endeavour*, 46/4 (2022), p. 1–10.

[74] Cf. Dimitri Levitin, *The Kingdom of Darkness: Bayle, Newton, and the Emancipation of the European Mind from Philosophy*, Cambridge University Press, Cambridge – New York 2022, p. 60–80; Levitin, *Ancient Wisdom in the Age of the New Science*, part 4.5, p. 313–325; Ian Stewart, « Mathematics as Philosophy: Barrow and Proclus », *Dionysius*, 18 (2000), p. 151–181; Michael S. Mahoney, « Barrow's Mathematics: between Ancients and Moderns », in Mordechai Feingold (ed.), *Before Newton: The Life and Times of Isaac Barrow*, Cambridge University Press, Cambridge – New York 2021 (Transformations: Studies in the History of Science and Technology), p. 179–249; Niccolò Guicciardini, *Isaac Newton on Mathematical Certainty and Method*, MIT Press Ltd, Cambridge, MA 2011.

Coming back to the origin of this controversy, I argue that Barozzi's personal views, such as expressed in the debate over the certainty of mathematics, are echoed in his choices of rendering Proclus' text into Latin. Thus, his translation is not neutral, but the Latin text contains some biases related to his part in the controversy. For instance, in one of the key-passages of the prologue, Proclus claims that (1) not every mathematical proof reaches the perfection of the demonstration (ἀποδείξεως τελειότης), (2) that the necessity of mathematics derives from its subject matter (τὴν ὑποκειμένην), and, lastly, (3) that many of the mathematical demonstrations have as middle terms a 'sign' (τεκμήριον) instead of real causes; he then summarily provides theorem I.32 of Euclid as an instance of demonstration through sign.[75] Piccolomini adopts and elaborates on this section in his *De certitudine* claiming that, according to Proclus, mathematical sciences do not provide any *demonstratio potissima.* Moreover, Piccolomini defended that there is no causation in mathematics at all. By way of consequence, the certainty of mathematics derives not from its demonstrative procedures, but from its subject matter, namely the more epistemically adequate nature of mathematical entities – an opinion that, again, Picolomini buttresses with the aid of *In Euclidem*.[76]

In his translation of this section, Barozzi's choice for the word τεκμήριον is not *signum* (as in the commonly used Latin syntagm *demonstratio signi*), but rather ἐκ τεκμηρίων is put as *certis notis* and τεκμήριον as *certum signum*. The adjective *certum* appears in both expressions.

Proclus, *In primum Euclidis*, p. 206, l. 12–23	Barozzi, *Procli Diadochi in primum Euclidis Elementorum*, 1560, p. 118
Τὴν δὲ λεγομένην ἀπόδειξιν ὅτε μὲν καὶ τὰ ἴδια τῆς ἀποδείξεως ἔχουσαν εὑρήσομεν ἀπὸ τῶν ὁρισμῶν μέσων τὸ ζητούμενον δεικνύουσαν – αὕτη γὰρ ἀποδείξεως τελειότης – ὅτε δὲ **ἐκ τεκμηρίων** ἐπιχειροῦσαν. καὶ δεῖ μὴ λανθάνειν. πανταχοῦ μὲν γὰρ τὸ ἀναγκαῖον ἔχουσιν οἱ γεωμετρικοὶ λόγοι διὰ τὴν ὑποκειμένην ὕλην, οὐ πανταχοῦ δὲ περαίνονται διὰ τῶν ἀποδεικτικῶν μεθόδων. ὅταν γὰρ διὰ τοῦ τὴν ἐκτὸς τοῦ τριγώνου γωνίαν ἴσην εἶναι δύο ταῖς ἐντὸς καὶ ἀπεναντίας δεικνύηται τὸ	Illa autem, quae demonstratio dicitur, quandoque propria demonstrationi habentem inveniemus, ex definitionibus mediis quaesitum ostendentem; haec enim demonstrationis est perfectio, quandoque vero **ex certis notis** arguentem. Et oportet non latere. Ubique enim geometrici sermones propter subiectam materiam necessarium habent, non ubique tamen demonstrantibus methodis perficiuntur. Quando enim eo quod extrinsecus trianguli angulus duobus intrinsecis et ex opposito existentibus aequalis est, tres intrínsecos duobus rectis

[75] For an account and history of the demonstration through τεκμήριον cf. Dubouclez, *Descartes et la voie de l'analyse*, chapitre III.

[76] Cf. Alessandro Piccolomini, *De certitudine mathematicarum scientiarum,* Antonio Blado, Roma 1547, p. CII–CIXv, where the core of the argumentation is presented.

τρίγωνον ἴσας ἔχον τὰς ἐντὸς τρεῖς γωνίας δυσὶν ὀρθαῖς, πῶς ἀπ'αἰτίας ἡ ἀπόδειξις αὕτη, πῶς δὲ οὐχὶ **τεκμήριόν** ἐστι τὸ μέσον;	aequales habere triangulus ostenditur, quomodo a causa est demonstratio haec? Quomodo enim medium **certum signum** non est?

Barozzi, who claimed that mathematical certainty derives both from its demonstrative procedures and its subject matter,[77] introduces the adjective 'certain' in describing the troublesome demonstrations by sign, suggesting thereby to the reader that Proclus held that certainty is bestowed upon such demonstrations by formal procedures. Barozzi, by deliberately choosing this wording, introduces a doctrinal shift into Proclus' text, a change that may have coloured the entire section, and a reader unaware of the Greek text may be inclined to follow the translator's own views on the philosophy of mathematics.

As a second example, in a section in which Proclus discusses some authoritative opinions on the practice of geometry and whether causal explanation is part of geometry, Barozzi translates:

Proclus, *In primum Euclidis*, p. 202, l. 9–25	Barozzi, *Procli Diadochi in primum Euclidis Elementorum*, 1560, p. 116
Τήν γε μὴν αἰτίαν καὶ **τὸ διὰ τί** πολλοῖς μὲν ἔδοξεν ἡ γεωμετρία μὴ θεωρεῖν – ταύτης γάρ ἐστι καὶ ὁ Ἀμφίνομος τῆς δόξης Ἀριστοτέλους κατάρξαντος. [...] ὅταν μὲν οὖν καὶ ὁ συλλογισμὸς ᾖ δι'ἀδυνάτου τοῖς γεωμέτραις, ἀγαπῶσι τὸ σύμπτωμα μόνον εὑρεῖν, ὅταν δὲ διὰ προηγουμένης ἀποδείξεως, τότε πάλιν, εἰ μὲν ἐπὶ μέρους αἱ ἀποδείξεις γίγνοιντο, οὔπω δῆλον τὸ αἴτιον, εἰ δὲ καθ'ὅλον καὶ ἐπὶ πάντων τῶν ὁμοίων, εὐθὺς καὶ **τὸ διὰ τί** γίγνεται καταφανές.	At causam et **ipsum** [**propter quid**] geometriam minime contemplari pluribus visum fuit. Huiusce enim sententiae est et Amphinomus Aristotele duce. [...] Quando igitur syllogismus geometris per impossibile fuerit, symptoma tantum invenire cupiunt, quando autem per praecipuam demonstrationem, tunc rursus si quidem in particulari demonstrationes fiant, causa nondum manifesta est. Si vero in universali, in omnibusque similibus, continuo et **ipsum** [**propter quid**][78] manifestum fit.

[77] Cf. Barozzi's comment on this section in Barozzi, *Opusculum in quo una oratio et duae quaestiones, altera de certitudine, et altera de medietate mathematicarum, continentur*, E.G.P., Padua 1560, p. 28v: « Dixit quidem Proclus quod geometrici sermones ubique propter subiectam materiam necessarium habent, non ubique autem demonstrantibus methodis perficiuntur, non dixit autem, quod geometrici sermones necessarium habent propter subiectam materiam tantum, et nullo pacto propter naturam demonstrationum mathematicarum, sicut ait recentior ».

[78] The brackets appear in Barozzi's printed text.

In this section Proclus mentions that some ancients, including Amphinomus, did not hold that mathematics engages in causal explanation. The Greek expression used by Proclus is τὸ διὰ τί, i.e., 'the why'. Instead of translating it with *cur* or *per quod*, or just *explanatio*, Barozzi chooses the more unusual syntagm *ipsum propter quid*, interpreting it as though it were the technical Aristotelian logical term διότι. This choice is related again to the *quaestio de certitudine mathematicarum*. As Piccolomini argues, a *demonstratio potissima* would provide both the *quia* (a cause-to-effect, symptomatic account) and the *propter quid* (an effect-to-cause, casual account). Barozzi's counterargument, according to which causal explanation (sc. *propter quid* demonstration) is possible in mathematics, appears in his Latin translation. For, as the passage proceeds, Barozzi makes Proclus say that causal accounts (*propter quid*) are found in mathematical demonstrations which fulfil two conditions: (1) they are not about particulars but about universals, and (2) they are not proven through *ad impossibile* arguments. This would imply that even the controversial theorem I.32 (which universally demonstrates through signs and is not *ad impossibile* and was a common object of dispute in the controversy) is not, according to Barozzi's version of Proclus' text, explanatory (as the original Greek states: τὸ διὰ τί), but it is, nevertheless, causally demonstrative (as the Latin translation suggests: *Si vero in universali, in omnibusque similibus, continuo et ipsum [propter quid] manifestum fit*). In this way Barozzi's Latin inclines the text toward his own opinions: that there is causality in mathematics.

Finally, it is worth mentioning Barozzi's relation to the previous Latin translations of Proclus' *In Euclidem*. Barozzi publishes the text, as indicated above, with a title claiming that it is the first Latin translation of the text. Did Barozzi remain unaware of the existence of other Latin versions of Proclus' text and of the existence of Valla's Latin prologues? That would be surprising. Both Valla and Zamberti were, like Barozzi himself, Venetians, and of great influence in the mathematical circles of their time. As previously mentioned, Zamberti announced his translation already in his edition of Euclid's *Elementa* of 1505, which was the most widely used until the 1572 edition of Commandino. One might assume that Barozzi was so unsatisfied with the quality of either the Greek text and (or) the Latin translations that he decided to do everything from scratch. Or that he aimed to have a more authoritative Greek source with a personal, tailored translation for his polemics with Piccolomini.

II.5. Jerónimo Muñoz

The Spaniard mathematician and philosopher Jerónimo Muñoz is a token of how the contents of Proclus' commentary influenced 16th-century mathematics and philosophy of mathematics and reached the chairs of mathematics of prestigious universities.

In 1537 Muñoz earned a degree from the Faculty of Arts at the university of Valencia, his hometown, where he acquired expertise in Greek and Hebrew.[79] Two figures that might have influenced his mathematical education, namely Juan de Celaya (c. 1490–1558), educated amongst the *calculatores parisienses* and the reformer of the curricula at that university, and Miguel Cerves (fl. c. 1520), professor of mathematics and astronomy during the time Muñoz studied at Valencia. Muñoz also received education outside Spain under other mathematicians and technicians of renown: he was instructed by Oronce Finé (1494–1555) at Paris, and by Gemma Frisius (1508–1555) at Leuven. As for his career as a university professor, it is known that Muñoz firstly taught Hebrew at the university of Ancona (although the extant time and length of his teaching is unknown), and then he was granted the chairs of mathematics (from 1563) and Hebrew (from 1565) at the university of Valencia. In 1578, he was appointed by Emperor Philipp II to a professorship of mathematics in Salamanca.

Muñoz published four works during his life: *Institutiones arithmeticae ad percipiendam astrologiam et mathematicas facultates necessariae* (1566), *Libro del nuevo cometa* (1573), *Suma del pronostico del cometa y de la eclipse la Luna* (1577), and *Alphabetum Hebraicum cum ratione legendi cum punctis* (1585).

Some of his mathematical works are still extant in manuscripts.[80] The presence of Proclus' *In Euclidem* is noticeable in two of these unpublished texts: the *Astrologicarum et geographicarum institutionum libri sex* (MS Vat. Lat. 6998, fol. 3r–116r),[81] written between 1565 and 1578; and *Adnotationes in commentaria Procli super Euclidem* (MS Vat. Lat. 6996, fol. 2r–159r), written after 1578, when he already was teaching at Salamanca. The former is mainly focused on cosmography and spherics, disciplines of great usefulness to 16th-century Spain for navigation and

[79] For the details on the life, works, and intellectual profile of Jerónimo Muñoz cf. Víctor Navarro-Brotons, *Jerónimo Muñoz: Matemáticas, cosmología y humanismo en la época del Renacimiento*, Publicacions de la Universitat de València, Valencia 2019; Id., *Disciplinas, saberes y prácticas: Filosofía natural, matemáticas y astronomía en la sociedad española de la época moderna*, Publicacions de la Universitat de València, Valencia 2014; Víctor Navarro-Brotons, Enrique Rodríguez Galdeano, *Matemáticas, cosmología y humanismo en la España del siglo XVI: Los Comentarios al segundo libro de la Historia natural de Plinio de Jerónimo Muñoz*, Instituto de Estudios Documentales e Históricos sobre la Ciencia, Valencia 1998; and Víctor Navarro-Brotons « The Cultivation of Astronomy in Spanish Universities in the Latter Half of the 16th Century », in Mordechai Feingold and Víctor Navarro-Brotons (eds.), *Universities and Science in the Early Modern Period*, Springer, Dordrecht 2006 (Archimedes, New Studies in the History and Philosophy of Science and Technology, 12), p. 83–98.

[80] *Theonis Alexandrini commentaria in primum librum mathematicae constructionis Ptolomei* (MS Biblioteca Nazionale di Napoli, VIII, 33, fol. 21r–300r), *In Isagogas principis Abdilasi servi gloriosi dei Alcabiciis de astrorum apotolesmatis elucubratio* (MS Biblioteca de la Universidad de Salamanca, 2320 fol. 3v–133v), or *Utrum sint plures orbes necne* (MS Munich, Bayerische Staatsbibliothek, Clm 10674, fol. 337r–340v).

[81] Digitised at <https://digi.vatlib.it/view/MSS_Vat.lat.6998>.

cartography purposes. The latter contains Muñoz teachings on the books I–VI of Euclid's *Elementa*. I will analyse in this paper two passages from *Astrologicarum et geographicarum institutionum libri sex*, [82] which opens with a discussion on mathematical ontology and the classification of mathematical disciplines. These are not two isolated themes for Muñoz, for he entertains (although this position is not fully explained) that given a determined mathematical ontology, a particular division of the mathematical sciences will follow from it. Therefore, an ill-conceived metaphysics of the mathematical objects would produce a defective classification of the mathematical disciplines.

Before establishing his views on such matters, Muñoz expounds and criticises the Pythagoreans' and Geminus' ontologies and classifications of mathematical sciences straightforwardly taken from Proclus' *In Euclidem*. Regarding the Pythagoreans, Muñoz offers a Latin paraphrase of Proclus' passage on the topic. Then, he proceeds to criticise what he regards as an unsatisfactory account:

Proclus, *In primum Euclidis*, p. 35, l. 22–36, l. 4	Muñoz, *Astrologicarum et geographicarum*, MS. Vat. Lat. 6998, fol. 4v, l. 11–29
Τοῖς μὲν οὖν Πυθαγορείοις ἐδόκει τετραχὰ διαιρεῖν τὴν ὅλην μαθηματικὴν ἐπιστήμην, τὸ μὲν αὐτῆς περὶ τὸ ποσόν, τὸ δὲ περὶ τὸ πηλίκον ἀφορίζουσι καὶ τούτων ἑκάτερον διττὸν τιθεμένοις. Τό τε γὰρ ποσὸν ἢ καθ'αὐτὸ τὴν ὑπόστασιν ἔχειν, ἢ πρὸς ἄλλο θεωρεῖσθαι κατὰ σχέσιν, καὶ τὸ πηλίκον ἢ ἐστὼς ἢ κινούμενον εἶναι. Καὶ τὴν μὲν ἀριθμητικὴν τὸ καθ'αὐτὸ τὸ ποσὸν θεωρεῖν, τὴν δὲ μουσικὴν τὸ πρὸς ἄλλο, γεωμετρίαν δὲ τὸ πηλίκον ἀκίνητον ὑπάρχον καὶ τὴν σφαιρικὴν τὸ καθ'αὐτὸ κινούμενον.	Pythagorei in quatuor partes universam mathematicen diviserunt: partim enim περὶ τὸ ποσόν; partim περὶ τὸ πηλίκον distinguunt, id est partim circa quantum; partim circa magnum, et unumquodque horum in duo secant. Nam ipsum quantum aut per sese consideratur habere subsistentiam, aut ad aliud refertur secundum habitudinem. Similiter, ipsum magnum aut statum aut motum consideratur. Unde quatuor partes mathematices fluunt, arithmeticam quidem aiunt considerare ipsum quantum per sese; musicam vero ratione alterius; geometriam vero considerare ipsum magnum ut immotum existens; sph<a>ericam vero scientiam magnitudinem considerare per se motam. (*Muñoz's criticism*) Ex qua divisione Pythagoricorum solum erunt arithmetica, musica, geometria et sph<a>erica scientia partes mathematices. Quae divisio imperfecta quidem est.

[82] I shall treat Muñoz's *Adnotationes* in a forthcoming publication.

	Omittitur enim optica et cat[h]optrica et astronomia et ceterae aliae partes, nisi velis referre opticam ad geometriam et astronomiam ad sph<a>ericam facultatem.

It is to be noted that Muñoz, like Valla, does not mention Proclus' name, and introduces his translation from *In Euclidem* as if it was his own text. Muñoz attempted to preserve the general meaning without producing a *de verbo* translation. For instance, in the first sentence the main Greek verb ἐδόκει is substituted in his translation with the secondary verb διαιρεῖν/*diviserunt*. Muñoz decided to preserve some Greek phrases from the original and then to translate them into Latin, such as περὶ τὸ ποσόν and περὶ τὸ πηλίκον/*circa quantum, partim circa magnum* (in the passage cited above).

The Pythagorean view of mathematics, as described by Proclus in his *In Euclidem*, establishes a classification of mathematical disciplines pondering whether their objects of study are quantity or magnitude considered for themselves or applied to other entities. Hence, whereas arithmetic is the science of quantity and geometry the science of magnitude considered in themselves, music is a science in so far as the sound is described as quantity, and spherics is a science in so far as magnitude is applied to the shape and movement of the farthest heavenly sphere. Muñoz considers this division incomplete, arguing that it omits several branches of mathematics, such as catroptics or astronomy.

Geminus' mathematical ontology and disciplinary division are equally presented and then rejected. Muñoz's account fuses two sections of *In Euclidem*, skipping the controversies regarding tactics referred by Proclus and proceeding to some qualifications on the pure geometrical sciences:

Proclus, *In primum Euclidis*, p. 38. l. 4–14 and p. 39, l. 8–9	Muñoz, *Astrologicarum et geographicarum*, MS. Vat. Lat. 6998, fol. 4v, l. 33–5r, l. 9
[…] Γεμῖνος, καὶ ποιοῦσι τὴν μὲν περὶ τὰ νοητὰ μόνον, τὴν δὲ περὶ τὰ αἰσθητὰ ἐνεργοῦσαν καὶ τούτων ἐφαπτομένην, νοητὰ δήπου καλοῦντες, ὅσα καθ'ἑαυτὴν ἡ ψυχὴ θεάματα ἀνακινεῖ, χωρίζουσα τῶν ἑνύλων ἑαυτὴν εἰδῶν. Καὶ τῆς μὲν περὶ τὰ νοητὰ πραγματευομένης δύο τὰ πρώτιστα καὶ κυριώτατα μέρη τίθενται ἀριθμητικὴν καὶ γεωμετρίαν, τῆς δὲ περὶ τὰ αἰσθητὰ τὴν ἐνέργειαν ἐχούσης ἕξ: μηχανικήν, ἀστρολογίαν, ὀπτικήν, γεωδεσίαν,	Geminus dividit mathematicen, unam quae versatur circa [5r] νοετά, id est, res solum mente perceptibiles, cuius duae partes sunt arithmetica et geometria. Rursus, geometriam dividit in ἐπιπεδομετρικήν et in σ<τ>ερεομετρικήν, alteram quae versatur circa αἰσθητά, id est, sensibilia, cuius sex sunt partes: logica seu logistica, geodesia, canonica seu musica, et optica, astrologia et mechanica; alii septimum genus addunt, scilicet τακτικόν, id est facultatem qua quis possit

κανονικήν, λογιστικήν. Τὸ δ'αὖ τακτικὸν οὐκ ἀξιοῦσιν ἔν τι τῶν μερῶν τῆς μαθηματικῆς λέγειν [...]. [Skips p. 38, l. 15–39, l. 7] [...] ἡ μὲν γεωμετρία διαιρεῖται πάλιν εἵς τε τὴν ἐπίπεδον θεωρίαν καὶ τὴν στερεομετρίαν.	invenire quid sit primum, quid secundum ordine, quid tertium et in ordinem h<a>ec redigere.

Muñoz paraphrases Proclus' account about Geminus division of mathematical sciences according to the ontological status of their object of study: some of them deal with intelligible entities (νοητά/*res solum mente perceptibiles*), others with sensible entities (αἰσθητά/*sensibilia*). Hence, geometry and arithmetic are declared to be mathematical sciences engaging with purely intelligible entities (*cuius duae partes sunt arithmetica et geometria*). Geometry is further divided in plane geometry or epipedometry and solid geometry or stereometry (*in* ἐπιπεδομετρικήν *et in* στερεομετρικήν) – an insertion from the later passage. Six, or according to some (*alii*) seven, mathematical disciplines deal with sensible entities: logic, music, geodesy, optics, astrology, mechanics, and tactics. Muñoz preserves the Greek nouns for the two ontologically diverse objects of study (νοητά/ αἰσθητά) adding afterwards a pleonastic Latin translation. He also adopts the adjectival forms of the nouns given in Proclus' text for epipodemetry and stereometry (ἡ μὲν γεωμετρία διαιρεῖται πάλιν εἵς τε τὴν ἐπίπεδον θεωρίαν καὶ τὴν στερεομετρίαν). Lastly, the order of the seven sensible branches of mathematics differs from that of Proclus.

Muñoz equally rejects Geminus' classification on the grounds that it leaves aside judiciary astrology, natural magic (which he seems to consider a mathematical discipline), and spherics: *hactenus ex Gemino excerpsimus, qui imperfectam tradidit astrologiae divisionem, omittens astrologiam iudiciariam et magi[c]am naturalem, et sph<a>erae contemplationem* (fol. 5v, l. 22–25).

Finally, Muñoz presents his own view: his ontology of mathematics is (in tacit) opposition to Proclus, and, thus, anti-Platonic:

> Ego vero cum admi<ttere> nequeam mathematicas formas per se subsistere aut media entia esse inter ideas et res sensibiles et ex mente Aristotelis per abstractionem a motu et materia functione intellectus fieri existimem [...]. (MS. Vat. Lat. 6998, fol. 6r).

The beginning is clearly a reference to *In Euclidem*'s opening,[83] but then Muñoz argues against Proclus and in favour of mathematical abstractionism, presenting himself as a follower of Aristotle. Muñoz' classification of the mathematical sciences is grounded in the Aristotelian distinction between discrete (διωρισμένον) and continuous (συνεχές) quantity.[84] The study of discrete quantity pertains to arithmetic, whereas the study of continuous quantity belongs to geometry. Both types of quantity can be applied to sensible entities, which enables a subordination of other sciences and arts to arithmetic and geometry. Through this flexible criterion Muñoz updates the mathematical divisions of the ancients to the more complex 16th-century state of the art. It also has the advantage of leaving room for additions and developments in the subordination of disciplines. However, Muñoz's account remains unclear about two aspects: firstly, how does this division follow from an abstractionist view of mathematics; secondly, why this classification cannot be consistent with the mathematical ontologies (previously) rejected.

Jerónimo Muñoz' text is a witness to two relevant aspects of the broader Western reception of *In Euclidem*: (1) Proclus' view on the ontological status of mathematical objects and the classification of the mathematical sciences were clearly themes of interest for 16th-century erudite community, regardless of their institutions; and (2) Proclus' doctrines were spread through university teaching, even without mentioning his name explicitly.

II.6. Conrad Dasypodius

Conrad Dasypodius introduces Proclus' *In Euclidem* in the curricula of the University of Strasbourg, where he held the chair of mathematics. A Protestant educator, disciple of the British mathematician Christianus Herlinus (1505–1562), and a promoter of the mathematical sciences as a pivotal part of the education of the youth, Dasypodius organised a four-year curriculum of mathematics through his project of a three-volume work intitled *Mathematicum, complectens praecepta mathematica, astronomica, logistica*. The first volume was published in 1567, the second volume in 1570, and the third never reached the typesetters.[85] The details of this mathematical course can be found in the preface:

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[83] Proclus, *In primum Euclidis,* p. 4, l. 1–4: « Τὴν μαθηματικὴν οὐσίαν οὔτε τῶν πρωτίστων ἐν τοῖς οὖσι γενῶν οὔτε τῶν ἐσχάτων εἶναι καὶ παρὰ τὴν ἁπλῆν διῃρημένων, ἀναγκαῖον, ἀλλὰ τὴν μέσην χώραν [...] ».

[84] Cf. Aristotle, *Categories*, VI, 4b20–23.

[85] Concerning Dasypodius scholarly activity and his reception of Proclus' *In Euclidem* and, particularly the notion of *mathesis universalis*, see Giovanni Crapulli, *Mathesis universalis: Genesi di una idea nel XVI secolo*, Edizioni dell'Ateneo, Roma 1969 (Lessico intellettuale europeo, 3). Parts of

> Unus ergo et primus annus primo tribuatur volumini mathematico, quod qui assequi discendo nequit, is ad βαναύσους redeat. Secundus annus huic secundo volumini concedatur, quo temporis spatio, animus prioribus praeceptis imbutus, omnia, aut saltem praeclaraque quae in eo continentur, facillime adsequetur. Tertio denique volumini tribuimus biennium.[86]

Dasypodius devotes one year to each of the first and second volumes; two years are assigned to the third. It is in the second volume, lengthy sections of Proclus' *In Euclidem* are used as a general introduction to mathematics alongside other Greek authorities: Ptolemy, Cleomedes, Hipparchus, Achilles Tatius, and Theon of Alexandria. These fragments are given both in Greek and, in a later section of the volume, in a Latin translation produced by Dasypodius himself.

In his work *Protheoria mathematica* (where the authority of Proclus' *In Euclidem* is invoked seven times regarding the notion of a *mathesis universalis*), Dasypodius reveals that he had been revising the 1533 *editio princeps.*[87] Dasypodius' handwritten corrections are preserved in a volume at Uppsala university.[88] According to Crapulli's and Friedlein's collatio,[89] Dasypodius' corrections of Grynaeus' edition are based on MS Munich, Bayerische Staatsbibliothek, Cod.

Dasypodius' *Mathematicum* regarding the *mathesis universalis* are presented there in Appendix 2 of Crapulli's work. On Proclus' *In Euclidem* and the Renaissance reception of the *mathesis universalis* (including Dasypodius'); see also David Rabouin, *Mathesis universalis: L'idée de « mathématique universelle » d'Aristote à Descartes*, Presses Universitaires de France (PUF), Paris 2009 (Épiméthé essais philosophique), ch. IV and « Le rôle de Proclus dans les débats sur la 'mathématique universelle' à la Renaissance ».

86 Conrad Dasypodius, *Mathematicum, complectens praecepta mathematica, astronomica, logistica, Volumen II*, J. Rihel, Strasbourg 1570, *Praefatio*, p. aVIv–aVIIr.

87 Dasypodius, *Protheoria Mathematica in qua non solum disciplinae mathematicae omnes, ordine convenienti enumerantur, verum etiam universalia Mathematica praecepta, explicantur*, Iodocus Martinus, Strabourg 1593, in the number 9 on the index of works *Vol. III Pandectarum mathematicarum*, p. 23, Dasypodius writes: « Procli depravatissima commentaria ex graecis exemplaribus correcta ». Cf. Crapulli, *Mathesis universalis*, p. 78.

88 Cf. Pehr Aurivillius, *Catalogus librorum impressorum Bibliothecae Regiae Academiae Upsaliensis,* Stenhammar et Palmblad, Upsala 1814, p. 721, col. 2, l. 34–37: « In Libr. I. Elementor. Euclidis Comm. Libri IV. Graece. Bas. 1533. (Euclides) F. (Exemplar manu Cnr. Dasypodii egregii emendatum & integris quibusdam foliis suppletum) – Lat_c. schol. p. F. Baroccium. Patavii 1560. F ». A copy of the 1533 printed edition which solely contains Proclus' but catalogued as it were Euclid's *editio princeps*, is preserved in Munich (Bayerische Staatsbibliothek, A. gr. b. 954, before A. gr. 1060. According to Friedlein (*In primum Euclidis*, p. 1) it belonged to Pietro Vittorio (1499–1595). As reported by Crapulli, Dasypodius' corrections and those present in A. gr. b. 954 by an anonymous hand coincide for the most. Cf. Crapulli, *Mathesis universalis,* p. 78. A digitised version is accessible at <https://www.digitale-sammlungen.de/en/view/bsb00107029?page=,1>.

89 Cf. Crapulli, *Mathesis universalis,* p. 78, 158.

Graec. 427 (fol. 1–234v).[90] This latter 10th-century manuscript of Byzantine origin, is one of the oldest known copies of Proclus' *In Euclidem*.

The reasons for producing a bilingual edition is provided by the Dasypodius as follows:

> Sed si quis ex me quaerat cur Graeca Latinis coniunxerim, is sciat hoc a me factum esse propter Graecae linguae imperitiores, ut collatione facta, quid tradatur, facilius intelligant. Deinde quia hae scientiae a Graecis praecipue fuerunt excultae, atque idcirco haec non nisi ex ipsorum monumentis petere dignum erat. Maxime vero ut ad fontes harum scientiarum nostri ducerentur discipuli.[91]

The goals of Dasypodius' bilingual text were two. Firstly, to facilitate access to the Greek text for those with an inadequate knowledge of the language. Secondly, to lead the students to the sources of such sciences in their original languages.

The chapters provided by Dasypodius bear titles that reveal his primary interests in the topics under scrutiny. Each of his chapters is a mosaic assembled by Dasypodius: he cut and re-arranged sections, or even single sentences, of Proclus' *In Euclidem* to produce a text that corresponds to the topics that he wanted to emphasise in mathematical studies. In Appendix 1, a table gathers the Greek and Latin titles (and pages) in the edition of Dasypodius' *Mathematicum* and their origins in Friedlein's edition.

From this re-arrangement, it emerges with clarity that Dasypodius was interested in mathematical notions to which Proclus did not necessarily devote specific sections. He gathered in single chapters different fragments from different loci on the same notions in order to produce single chapters. Chapters VI to XI are clear examples of this practice; for example, chapter X treats five notions (λήμμα, πτώσις, πόρισμα, ἔντασις, and ἀπαγωγή) discussed by Proclus in his comments on the axioms and propositions. Dasypodius introduces the first two notions as they appear in the commentary,[92] then for the third and fourth terms he incorporates sections of the later commentary to provide a more detailed and continuous account of the terms.[93] Lastly, Dasypodius returns to Proclus' initial enumeration, and proceeds with the fifth term. Therefore, Dasypodius' composite chapters could be deemed as a collection of dispersed Proclean remarks on a range of technical notions. Other chapters from Dasypodius are simply unmodified sections of Proclus' *In Euclidem*, such as chapters III to V. Chapter II also presents a continuous section of *In Euclidem* with a final addition of six pages (p. 22–28) of *Ex*

[90] A digitised version is accessible at <https://daten.digitale-sammlungen.de/~db/0011/bsb00117589/images/>.

[91] Dasypodius, *Mathematicum, Praefatio*, p. VIIr–VIIv.

[92] Proclus, *In primum Euclidis,* p. 210, l. 25–212, l. 17.

[93] Proclus, *In primum Euclidis,* p. 301, l. 21–302, l. 21; p. 303, l. 5–304, l. 3.

Heronis libro qui de nominibus geometricis inscribitur, as Dasypodius names it in his Latin translation.

Some parts of the text were modified by Dasypodius and transformed into paraphrases, sometimes being only collages of Proclus' text. This is especially noticeable in the very first paragraph of chapter I, where the text concerning the nature of mathematical entities is paraphrased[94] and concludes with a paragraph which is an addition of Dasypodius' own words on universal mathematical sciences (καθόλου ἐπιστήμη μαθηματική).[95] Then the text leaps four folia[96] to another section on the principles of the ὅλη μαθηματική, whereupon the *incipit* is again altered. Similar alterations can be noticed in chapter VIII.[97]

The Latin translation of the sections which remained intact allows a comparison with other translations of Proclus' text. Dasypodius' version, much like that of Barozzi, combines the marks of a deep understanding of the philosophical and mathematical doctrines contained in Proclus' *In Euclidem* (enabling him to render the text with great attentiveness to the original meaning) with a humanistic fondness for Latin elegance.[98]

As a conclusion, it might be added that Dasypodius' *Mathematicum* is a witness to the way some sections of Proclus' *In Euclidem* had become incorporated into the curriculum at the university of Strasbourg by 1570, just eleven years after Barozzi's lectures at Padua. Dasypodius' rearrangement and selection of topics from Proclus' *In Euclidem* in the form of an introductive manual to mathematics for his teaching reveals that some contents of Proclus' text became the default philosophy of mathematics in (at least some) educational contexts.

II.7. Federico Borromeo

Cardinal Federico Borromeo, archbishop of Milan, is mostly known as the founder of the Ambrosiana library (1609), the second public library in Europe (after the Bodleian). A champion of the Catholic reformation, he is one of the historical characters portraited by Alessandro Manzoni in his 1827 novel *I Promessi Sposi*. Borromeo was interested in the Neoplatonic tradition and mathematical sciences,

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[94] It begins with the section corresponding to Proclus, *In primum Euclidis,* p. 1, l. 1–7: (ἡ μαθηματικὴ οὐσία τῶν μέσων χώραν, etc.); then it jumps to the sentence f. 1, l. 14–16, only to break it up in the middle and finish the paragraph with a clause from f. 10, l. 20–21.

[95] Cf. Dasypodius, *Mathematicum,* Praefatio, p. 2, l. 21–3, l. 3, *incipit*: « ἐστι δὲ μία [...] », *explicit*: « [...] ἀναφέρονται ».

[96] Corresponding to Proclus, *In primum Euclidis,* p. 5, l. 14–9, l. 25.

[97] Corresponding to Proclus, *In primum Euclidis,* p. 210, l. 5–25.

[98] Although Crapulli claims that Dasypodius used Barozzi's 1560 edition for his own translation, he does not provide further evidence backing his affirmation. A thorough collation of Barrozi's and Dasypodius' respective Latin translations of the same loci of Proclus' *In Euclidem* is still needed to confirm Crapulli's claim. Crapulli, *Mathesis universalis,* p. 78.

publishing a certain number of original works on these subjects. Witnesses of Borromeo's mathematical interests are Francesco Patrizi's (1529–1597) dedication of the *De numerorum mysteriis* (finished in 1594, but unpublished) and Borromeo's 1613 correspondence with Galileo Galilei (1564–1642), in which the cardinal reveals a profound familiarity with mathematical and astronomical topics.[99]

One of Borromeo's mathematical works is *De Pythagoricis numeris libri tres* (published in 1627), which was part of a wider project and was followed his *De cabbalisticis inventis libri duo* (1627). In short, Borromeo adopted what could be called a rationalistic-naturalistic approach against both recent interpretations of Kabbalah, such as those provided by Johannes Reuchlin (1455–1522) or Pietro Galatino (1460–1530), and to the numerological speculations supposedly following the wisdom of Pythagoras. On Borromeo's account, the traditions of Kabbalah and the Pythagoreans have been corrupted by the arbitrary analogies of alchemists and charlatans. In his works, he tried to dispel such superstitions and, in the case of mathematics, to purify the discourse of his time of what he reckoned as elements of supercherie. In this respect, Borromeo's interpretation seems – in stark contrast with most of his contemporaries – outstandingly modern, in so far as he argues that the main power of mathematics consists in revealing the structures and powers of the natural world.[100] It might be conjecturally affirmed that, perhaps because of that, Borromeo showed great interest on Proclus' commentary and the sections in which Proclus explore the presence of mathematical structure in the natural world.[101]

Borromeo's use of *In Euclidem* comes about in the draft of an intended work on mathematics which was never completed. Brief fragments are preserved in MS Ambrosiana R 181 (16) inf. 1 (fol. 1r–2v).[102] As we learn from a note on folio 2v, the title of this intended work was to be *De floribus mathematicis*. His other mathematical work, *De Pythagoricis numeris libri tres*, was published four years before his death; presumably the *De floribus mathematicis* was also a mature work which was never finished. The beginning of the extant section reads as follows:

> Divisio mathematicarum ex Proclo *In Euclidis Elementa*
>
> Aliae circa intellegibilia, aliae circa sensibilia versantes. Circa intellegibilia duae sunt: geometria et arithmetica, quae et purae ac praecipuae mathematicae

[99] Cf. Federico Borromeo, *De Pythagoricis numeris libri tres*, ed. Manuel Bertolini, Bulzoni, Roma 2017 (Biblioteca Ambrosiana/Fonti e Studi 26), p. XIV–XV; cf. also Jean-Pierre Brach, « Pythagorean Number Mysticism in the Renaissance. An Overview », in Irene Caiazzo, Constantinos Macris, Aurélien Robert (eds.), *Brill's Companion to the Reception of Pythagoras and Pythagoreanism in the Middle Ages and the Renaissance*, Brill, Boston 2021 (Brill's Companions to Classical Reception, 24), p. 457–488.

[100] Cf. Borromeo, *De Pythagoricis numeris,* p. XI–XXXIV.

[101] Cf. for instance Proclus, *In primum Euclidis*, p. 22, l. 17–23, l. 21.

[102] The complete transcription of this text is provided in Appendix 3.

> appellantur. Nam prior magnitudines secundum se et partes suasque passiones speculatur, altera numeros pariter secundum se, hoc est, in abstracto, ut aiunt, et species et accidentia considerat.
>
> Circa sensibilia sex: astronomia, quae et astrologia, perspectiva, quae et optica, geodesia, musica, quae et canonica, su<p>putatrix, quae et arit<h>metica practica, et mechanica. (MS Ambrosiana R 181 (16) inf.; 1, fol. 1r).

Borromeo's Latin shows the elegancies and turns of Renaissance prose: two *trocaichi spondei* meters finishing a sentence with the forms *videantur*/*videatur* can be found in the space of two folia: a recognisable Ciceronian affectation (fol. 1r, l. 29 and fol. 1v, l. 10). Nonetheless, some grammar and declension mistakes suggest that the text was not revised.

Much like Muñoz, Borromeo is interested in Proclus' report of the division of mathematical sciences attributed to Geminus, containing a mathematical ontology and a classification of mathematical disciplines following it. The precise fragment corresponds to Proclus' summary of Geminus' views quoted above, in the chapter on Muñoz (fol. 38, l. 4–14).

Borromeo accepts Geminus' distinction between mathematical sciences whose object of study is intelligible entities (νοητά/*intellegibilia*) and mathematical disciplines studying sensible entities (αἰσθητά/*sensibilia*). However, Borromeo omits Proclus' notes on the psychical activity in the intellectual contemplation of mathematics (νοητὰ δήπου καλοῦντες, ὅσα καθ'ἑαυτὴν ἡ ψυχὴ θεάματα ἀνακινεῖ, χωρίζουσα τῶν ἐνύλων ἑαυτὴν εἰδῶν) as well as Proclus' axiological notes introduced in respect to some disciplines (τὰ νοητὰ πραγματευομένης δύο τὰ πρώτιστα καὶ κυριώτατα). The list of mathematical sciences is preserved untouched, but the order of the second group is altered.

Borromeo adds explanative notes in respect to intelligible disciplines, namely that geometry deals with magnitude, its parts, and passions, i.e., accidental features (*magnitudines secundum se et partes suasque passiones*); arithmetic considers numbers, their classes (*species*), and accidents (*numeros pariter secundum se* [...] *et species et accidentia considerat*). These additions of Borromeo seem to have their origin in Proclus' account of the Pythagoreans who considered that mathematics studies either quantity in itself or quantity in relation to other things.[103]

The rest of the manuscript contains a fragmentary translation and re-arrangement of Proclus' account of the six sensible mathematical disciplines.[104] The order of the disciplines here has been altered: whereas Proclus refers to tactics, geodesy, logistics, optics, music, mechanics and astronomy, Borromeo provides the following list: astronomy (called astrology), optics, geodesy, music, logistic, mechanics, and tactics.

[103] Proclus, *In primum Euclidis*, p. 35, l. 22–36, l. 4.
[104] Proclus, *In primum Euclidis,* p. 38, l. 13–42, l. 8.

In his selected sections sometimes Borromeo updates the information given by Proclus to the 16th-century state of the arts and adds authoritative sources which are not present in Proclus' *In Euclidem*. For instance, when treating music, Borromeo invokes the authority of Boethius to introduce the three genera of octaves, to wit harmonic, chromatic, and diatonic (*Huius tria sunt apud Boet<h>ium harmonicum, chromaticum, et diatonicum*).

When dwelling on logistics, Borromeo introduces, amongst the vulgar arithmetic or operations to practical ends the more recondite algebra (*Huius duae sunt partes: vulgaris et algebra magis recondita*). From this comment it is clear that, notwithstanding its novelty, Borromeo considered algebra as a purely practical, sensible part of mathematics, i.e., a more abstruse, not so evident type of applied arithmetic.

The lengthiest section translated by Borromeo concerns mechanics. Here he follows Proclus on the types of inventions it deals with, proposing warfare machines (*Alia est instrumentorum ef<f>ectrix* [...] *quae bello gerendo sunt apta tam ad oppugnationem quam ad defensionem*), but adds architectonic machines (*quodque sunt organa architectonica*), and what he calls thaumaturgic machines (*alia ταυματοποιετικὴ* [sic][105] *Graecis, id est, mirabilium rerum effectrix*), i.e., machines which operates miraculous things, such as pneumatic, self-moving artifacts. In these passages the name of Vitruvius is added to those of Heron and Ctesibius. Finally, Borromeo offers a rather general definition of mechanical arts: every art that possesses the power for moving matter, can be said to be mechanical (*omnis ars quae materiam movendi vim habet, mechanica dici potest)*. From the numerous additions in this passage perhaps some enthusiasm from the part of the Cardinal can be inferred concerning the re-flourishing mechanical arts.

In sum: although in the absence of more fragments of Borromeo's incomplete *De floribus mathematicis* it cannot be deduced to which extent Proclus' *In Euclidem* was destined to be the guiding thread through in the project, the analysis of the former paraphrased and modified sections cast further light (much as those of Muñoz') upon the momentum of Proclus' commentary on the 16th- and 17thcentury mathematical culture, both regarding the classification of mathematical disciplines and mathematical ontology.

II.8. Johannes Kepler

Johannes Kepler translated parts of Proclus' *Commentary* and integrated them in his work *Harmonices mundi libri V* (*De Harmonice*), published in 1619. The name of

[105] Borromeo himself writes τ instead of θ. He was probably led to error because of his way of pronouncing the same Greek-originated terms in vernacular Italian. This sort of misspelling due to the confusion between Latin and Italian are often found in the manuscript and provide further evidence that it was not revised.

Proclus is explicitly mentioned thirty-six times in the five books of *De harmonice*.[106] By comparison, Plato is mentioned fifteen times, and Plotinus only once.[107] In all likelihood, Kepler became familiar with Proclus' *In Euclidem* by 1599 (the year he began the composition of *De harmonice*) through Grynaeus' 1533 edition.[108] Leaving aside the ubiquitous mentions in *De harmonice* and the later work *Pro suo opere harmonices mundi apologia* (1622), Proclus' commentary is referred four times in Kepler's writings. *In Euclidem* is referred as early as 1599, in a letter to Johann Georg Herwart (1553–1622).[109]

[106] For an analysis of Kepler astronomical theories and their philosophical aspects cf. Rhonda Martens, *Kepler's Philosophy and the New Astronomy*, Princeton University Press, Princeton, NJ 2000; Judith V. Field, *Kepler's Geometrical Cosmology*, Athalone, London 1988 (Bloomsbury Academic Collections: Philosophy). For a developmental account of Kepler's epistemology from his early *De quantitatibus libelli* (c. 1596–1611) to *Harmonices mundi* cf. Jorge M. Escobar, « Kepler's Theory of the Soul: A Study on Epistemology », *Studies in History and Philosophy of Science Part A*, 39/1 (2008), p. 15–41. For a biography of Kepler focused on his notion of harmony, cf. Wolfgang Osterhage, *Johannes Kepler: The Order of Things*, Springer, Dordrecht 2020 (Springer Biographies).

[107] Regarding Kepler's reception of Proclus' theory of imagination and the history of mathematics, cf. Guy Claessens, « Imagination as Self-knowledge: Kepler on Proclus' Commentary on the First Book of Euclid's Elements », *Early Science and Medicine*, 16/3 (2011), p. 179–199 and Id., « Reception and the Textuality of History: Ramus and Kepler on Proclus' History and Philosophy of Geometry », in André Lardinois, Sophie Levie, Hans Hoeken, Christoph Lüthy (eds.), *Texts, Transmissions, Receptions: Modern Approaches to Narratives*, Brill, Leiden – Boston 2015 (Radboud Studies in Humanities, 1), p. 281–294.

[108] Johannes Kepler, *De harmonice mundi libri V*, Ioannes Plancus, Linz 1619, I, p.2: « Proclus Diadochus libris quattor in primum Euclidis editis, philosophum theoricum in mathematico subiecto ex professo egit [...] ».

[109] Kepler, *Opera omnia*, ed. Christian Frisch, vol. 2, Heyder & Zimmer, Frankfurt and Erlangen 1859, p. 20–21: « Ingrediamur autem rem philosophice. Proclus super Eucl. I. pulcherrimam sententiam ponit, puto ex Platonis philosophia, τὰ μαθηματικὰ ἀπογεννᾶν μετ'ἀλλήλων ἐξαρκοῦσι τοὺς μέσους διακόσμους τῶν ὄντων καὶ τὴν ἐν αὐτοῖς ποικιλίαν. Nam etsi non crescit ex quinquangulo flos, ut ex radice, tamen concurrit id ad causam formalem, quam spectavit creator ». The second mention of Proclus' *In Euclidem* is to be found in Kepler's 1604 work *Astronimae pars optica*, c. X: Kepler, *Opera omnia*, vol. 2, p. 337: « Equidem sic censeo, nisi alia habuissemus argumenta, quibus huius Copernicanae sententiae probatur antiquitas, vel solum hunc locum sufficere potuisse, ad Copernicum Pythagorae ex solido vindicandum. Primum constat cum per se, tum ex Proclo interprete, totam Euclidis geometriam esse Pythagoricam et directam in quinque schematum regularium, quae mundana dixere, cognitionem: Euclides igitur Pythagoraeus fuit [...] ». The third one is in Kepler's *Dissertatio cum nuncio sidereo*, written as a commentary on the Galilean work in 1610; see Kepler, *Opera omnia*, vol. 2, p. 489: « Itaque meditatus mecum sum, qui possit aliqua fieri accessio ad planetarum numerum salvo meo Mysterio Cosmographico, quod ante annos 13 in lucem dedi, in quo 5 illae Euclidis figurae, quas Proclus ex Pythagora et Platone cosmicas appellat, planetas circa Solem non plures sex admittunt ». The last references to *In Euclidem* is in a lately added footnote to chapter XII of his *Mysterium cosmographicum* in which Kepler refers to book IV of *De harmonice;* see note g, ch. XXII, in Johannes Kepler, *Opera omnia*, vol. 2, p. 184: « Itaque spero te iocum meum lib. IV. (cap. 7.) Harm. de meis imaginibus ex Procli paradigmatibus delapsis non iniqua censura flagellaturum ».

Several lines from the first prologue of Proclus' *In Euclidem* are quoted directly in Greek at the beginning of *De harmonice*, book I (*De figurarum regularium, quae proportiones harmonicas pariunt, ortu, classibus, ordine, et differentiis, causa scientiae et demonstrationis*).[110]

This invocation of Proclus is highly significant. Kepler takes from Proclus the idea that it is possible to analyse through mathematics the hidden harmony binding both the largest and the smallest parts of the world.[111] Kepler refers again part of this locus in a Latin translation as the heading quote of book IV. In this occasion, some lines quoted in Greek before are omitted, and new loci from Proclus' *In Euclidem* added. In the following table, I provide in italics the part coincident with the first quotation:

Proclus, *In primum Euclidis*, p. 22, l. 17–23, l. 21 with two omissions: p. 22. l. 26–28 and p. 23, l. 16–19.	Kepler, *De harmonice,* IV, p. 105.
	Proclus diadochus Liber I commentariorum in I Elementorum Euclidis De mathematices usu in physiologia et politica, qua potissimum partem illius harmonicam de radiationibus concernunt.

[110] Proclus, *In primum Euclidis,* p. 22, l. 17–26: « Πρὸς δὲ τὴν φυσικὴν θεωρίαν τὰ μέγιστα συμβάλλεται, τήν τε τῶν λόγων εὐταξίαν ἀναφαίνουσα, καθ'ἣν δεδημιούργηται τὸ πᾶν, καὶ ἀναλογίαν τὴν πάντα τὰ ἐν τῷ κόσμῳ συνδήσασαν, [...] καὶ τὰ ἁπλᾶ καὶ πρωτουργὰ στοιχεῖα καὶ πάντη τῇ συμμετρίᾳ καὶ τῇ ἰσότητι συνεχόμενα δείξασα, δι'ὧν καὶ ὁ πᾶς οὐρανὸς ἐτελεώθη, σχήματα τὰ προσήκοντα κατὰ τὰς ἑαυτοῦ μερίδας ὑποδεξάμενος [...] ». (In my own translation: « [Mathematics] immensely contributes to physical speculation, revealing the order of the reasons according to which the entirety of things (τὸ πᾶν) was constructed, and [reveals] the proportion that holds together everything in the cosmos [...], and shows the simple and first-made elements arranged together in altogether symmetry and equality, through which [symmetry and equality] the entire heaven was perfected and completed (ἐτελεώθη), by assuming the fitting figures to its region [...] ».

[111] The same fragment equally serves as a token of Alexandre Koyré's thesis regarding the so-called scientific revolution, namely that the origins and philosophical justification of the use of mathematics in doing natural philosophy during the 16th and 17th centuries is to be found in the revival of Platonic sources. Cf. Alexandre Koyré, « Galileo and Plato », *Journal of the History of Ideas*, 5 (1943), p. 400–428; and Id., *Metaphysics and Measurement: Essays in Scientific Revolution*, Harvard University Press, Cambridge, MA 1968. For an analysis of this historiographical controversy and arguments in favour of Koyré's thesis (on the Platonism of Galilei), cf. Mario De Caro, « Galileo's Mathematical Platonism », in Johannes Czermak (ed.), *Philosophy of Mathematics*, Holder – Pichler – Tempsky, Wien 1993, p. 1–9; De Caro, « Sul platonismo di Galileo », *Rivista di filosofía*, 82 (1996) p. 25–40; Id., « Galileo e il platonismo fisico-matematico », in Riccardo Chiaradonna (ed), *Il platonismo e le scienze*, Carocci, Roma 2012 (Colloquium philosophicum, Nuova serie, 3), p. 119–138.

Πρὸς δὲ τὴν φυσικὴν θεωρίαν [*ἡ μαθηματικὴ*] *τὰ μέγιστα συμβάλλεται, τήν τε τῶν λόγων εὐταξίαν ἀναφαίνουσα, καθ'ἣν δεδημιούργηται τὸ πᾶν, καὶ ἀναλογίαν τὴν πάντα τὰ ἐν τῷ κόσμῳ συνδήσασαν*, ὥς που φησὶν ὁ Τίμαιος, καὶ φίλα τὰ μαχόμενα καὶ προσήγορα καὶ συμπαθῆ τὰ διεστῶτα ποιήσασαν [...] δι'ὧν τάς τε εὐγονίας ἑκάστων καὶ τὰς ἐναντίας φορὰς συλλογίζεσθαι δυνατόν. Ταῦτα γὰρ οἶμαι καὶ ὁ Τίμαιος ἐνδεικνύμενος πανταχοῦ διὰ τῶν μαθηματικῶν ὀνομάτων ἐκφαίνει τὴν περὶ τῆς φύσεως τῶν ὅλων θεωρίαν καὶ τὰς γενέσεις τῶν στοιχείων ἀριθμοῖς καὶ σχήμασι κατακοσμεῖ καὶ τὰς δυνάμεις αὐτῶν καὶ τὰ πάθη καὶ τὰς ποιήσεις εἰς αὐτὰ ἀναφέρει, τῶν τε γωνιῶν τὰς ὀξύτητας καὶ τὰς ἀμβλύτητας καὶ τῶν πλευρῶν τὰς λειότητας ἢ τὰς ἐναντίας δυνάμεις, τό τε πλῆθος καὶ τὴν ὀλιγότητα τῶν στοιχείων αἰτιώμενος τῆς παντοίας μεταβολῆς.	Ad contemplationem naturae omnia suppeditat [mathematica], declarans rationum ordinem pulcherrimum, secundum quem fabricatum est hoc universum, proportionumque analogiam, quae omnia mundana inter se connectit, ut loquitur alicubi Timaeus, quaeque amicitiam inter pugnantia, responsum et mutuam affectionem inter longissime dissita conciliat [...]. Inde et angulationes commodas possibile est ratiocinando venari. Rursum, hoc opinor, et Timaeus significare voluit dum passim per voces mathematicas tradit contemplationem de natura totius universi ortumque elementorum numeris et figuris depingit, facultatesque et affectiones illorum etiamque effectus his *(figuris)* acceptos fert, angulorum acuta vel obtusa, laterumque aspera vel lenia, et cet., causas constituens omnivariarum mutationum.
Πρός γε μὴν τὴν πολιτικὴν καλουμένην φιλοσοφίαν πῶς οὐχὶ φήσομεν αὐτὴν πολλὰ δὴ καὶ θαυμαστὰ συντελεῖν, τούς τε καιροὺς τῶν πράξεων ἀναμετρουμένην καὶ τὰς ποικίλας περιόδους τοῦ παντὸς καὶ τοὺς προσήκοντας ἀριθμούς [...] τούς τε ἐναρμονίου ζωῆς χορηγοὺς καὶ τοὺς τῆς ἀναρμοστίας παρεκτικοὺς καὶ ὅλως φορᾶς καὶ ἀφορίας οἰστικούς.	Ad politicam vero dictam doctrinam qui negari possit illam plurima et mirabilia conferre, dum opportunitates rerum gerendarum dimetitur, variosque circuitus totius universi *etc*., numerosque harmonicos, vitae moderatores, aut incongruentiae authores, et in universum impetus aut remissionis opitulatores *etc*.

Here it is again asserted that mathematics provides insight on the proportions binding the entire cosmos by adding Proclus' mention of Plato adoption of mathematical means in explaining the cosmic structures in the *Timaeus*. In the second part, Kepler relays Proclus' rationale of the usefulness of mathematics in political affairs, pointing principally towards astrology and its capacity for revealing adventitious or ominous times for political action. Kepler would introduce a discussion on similar topics in a *digressio politica* at the end of book III,

commenting on the opinions of Jean Bodin (1530–1596).[112] Kepler would remark that principles of proportionality and harmony can be adopted even in imparting justice and maintaining friendships.

Kepler provides another composed Latin translation from Proclus' *In Euclidem* as the front-door of book III. It reads as follows:

Proclus, *In primum Euclidis*, p. 22, l. 1–16 and p. 24, l. 4–14	Kepler, *De harmonice,* III, p. 1
	Proclus diadochus Liber I commentariorum in I Elementorum Euclidis Cum philosophia multas sit complexa facultates, multas et mathematica, de una quidem huius parte harmonice dicta, deque numeris, harmoniarum principiis creditis, ha<e>c scribit:
[…] [ἡ μαθηματικὴ] καὶ ὅτι θεολογίᾳ μὲν προευτρεπίζει τὰς νοερὰς ἐπιβολάς. Ὅσα γὰρ τοῖς ἀτελέσι δυσθήρατα καὶ ἀνάντη φαίνεται τῆς περὶ τῶν θεῶν ἀληθείας εἰς διάγνωσιν, ταῦτα οἱ τῆς μαθηματικῆς λόγοι πιστὰ καὶ καταφανῆ καὶ ἀνέλεγκτα διὰ τῶν εἰκόνων ἀποφαίνουσι. Τῶν μὲν γὰρ ὑπερουσίων ἰδιοτήτων ἐν τοῖς ἀριθμοῖς τὰς ἐμφάσεις δεικνύουσι, τῶν δὲ νοερῶν σχημάτων ἐν τοῖς διανοητοῖς τὰς δυνάμεις ἐκφαίνουσιν. Διὸ καὶ ὁ Πλάτων πολλὰ καὶ θαυμαστὰ δόγματα περὶ θεῶν διὰ τῶν μαθηματικῶν εἰδῶν ἡμᾶς ἀναδιδάσκει καὶ ἡ τῶν Πυθαγορείων φιλοσοφία παραπετάσμασι τούτοις χρωμένη τὴν μυσταγωγίαν κατακρύπτει τῶν θείων δογμάτων. Τοιοῦτος γὰρ καὶ ὁ ἱερὸς σύμπας λόγος καὶ ὁ Φιλόλαος ἐν ταῖς Βάκχαις καὶ ὅλος ὁ τρόπος τῆς Πυθαγόρου περὶ θεῶν ὑφηγήσεως. […]	Ad theologiam praeparat [mathematica] mentis conatus. Nam ea quae non initiatis circa veritatem rerum divinarum videntur esse captu difficilia, et sublimiora, illa mathematicis rationibus demonstrantur esse fida, manifesta et sine controversia per quasdam imagines. Nam proprietatum superessentialium evidentiam ostendunt in numeris, et quae sint intellectualium formarum potestates in ratiocinativis clarum efficiunt. Itaque Plato multa mirabilia de natura deorum nos per species rerum mathematicarum edocet, et Pythagorica philosophia ceu velis obnubit institutionem de rebus divinis. Huius enim generis est universus ille sermo sacer, et Philolaus in Bacchis, et tota Pythagorae ratio docendi de Deo. […]

[112] Kepler, *De harmonice mundi libri V*, p. 86 and forth.

Πρὸς δ'αὖ τὴν ἠθικὴν φιλοσοφίαν ἡμᾶς τελειοῖ, τάξιν καὶ ἐναρμόνιον ζωὴν ἐντιθεῖσα τοῖς ἤθεσιν ἡμῶν καὶ σχήματα πρέποντα τῇ ἀρετῇ καὶ μέλη καὶ κινήσεις παραδίδωσιν, ἀφ'ὧν δὴ καὶ ὁ Ἀθηναῖος ξένος τελειοῦσθαι βούλεται τοὺς τῆς ἠθικῆς ἀρετῆς ἐκ νέων μεταληψομένους, τῶν τε ἀρετῶν προτείνει τοὺς λόγους, ἄλλως μὲν ἐν τοῖς ἀριθμοῖς, ἄλλως δὲ ἐν τοῖς σχήμασιν, ἄλλως δὲ ἐν τοῖς κατὰ μουσικὴν συμφώνοις καὶ τῶν κακιῶν τὰς ὑπερβολὰς καὶ τὰς ἐνδείας παραδείκνυσι, δι' ὧν ἀποτελούμεθα μέτριοι τὸ ἦθος καὶ κεκοσμημένοι.	Rursum ad moralem philosophiam nos perficit, implantans nostris moribus ordinem, decentiam, et conversationem harmonicam; tradit etiam quae figurae, quae cantilenae, qui motus virtutem deceant; qua doctrina etiam Athenaeus excoli et perfici vult eos qui virtutibus moralibus ab adolescentia daturi sunt operam. Quin etiam proportiones numerorum, virtutibus familiares, explanat, alias quidem arithmeticas, alias geometricas, alias harmonicas: ostendit et vitiorum excessus defectusque, quibus omnibus dirigimur ad mediocritatem morum et decentiam.

Kepler quotes the sections immediately preceding and following the fragment on the contributions of mathematics to physics, given as the opening of book I. In this case, Kepler selected Proclus' elaborations on the contributions of mathematical sciences both to the study of theology and to ethics. The Latin translation is faithful, although Kepler tries here to conceal some of the notes most imbibed of pagan expression: the fragment referring to « the revelation of the truth of the gods » (φαίνεται τῆς περὶ τῶν θεῶν ἀληθείας) is translated as « the truth of divine things » (*circa veritatem rerum divinarum*). The same can be observed below, when Pythagoras' « method of teaching on the gods » is translated as « method of teaching about God » (ὁ τρόπος τῆς Πυθαγόρου περὶ θεῶν /*et tota Pythagorae ratio docendi de Deo*). Nevertheless, Kepler preserves intact the main themes of both sections, namely that mathematics is a pathway to the knowledge of the properties of the supra-essential divine realm (*Nam proprietatum superessentialium evidentiam ostendunt*), and that studying the ordered structure of mathematics improves the character itself and contributes to gain ethical insight (*implantans nostris moribus ordinem, decentiam, et conversationem harmonicam* [...]; [...] *ostendit et vitiorum excessus defectusque quibus omnibus dirigimur ad mediocritatem morum et decentiam*).

The lengthiest section of *In Euclidem* translated by Kepler corresponds to the fragment of the first prologue where Proclus discusses whether mathematical entities are derived via abstraction from senses (in an Aristotelian manner), from the soul, or from a common participation of intellectual powers.[113] After ruling out the opposing views, Proclus' conclusion is that mathematical entities proceed both from the mind and the soul, through the soul's unfolding of the superior noetic

[113] Proclus, *In primum Euclidis,* p. 12, l. 2–18, l. 4. See the transcription in Appendix 2.

forms bestowed upon her.[114] Kepler seems to be interested in Proclus' conclusion that mathematics (pure or archetypical harmony, in Kepler's understanding) is not learned through abstraction from sense experience, but it is an innate structure of the human soul.

Kepler's section opens with a short introduction in which he admits following Proclus' teaching that mathematical species (and most emphatically pure harmonical proportions) do not proceed from sensible things:

> Huius igitur Procli philosophiam de speciebus rerum mathematicarum, quas ego terminos profiteor proportionis harmonicae purae et secretae a sensilibus, operae pretium est ex eius libro I *In Euclidem* huc transcribere de verbo ad verbum.[115]

Kepler did not use the previous translations, and he even admits choosing to produce a *de verbo* translation of his own. The features of Kepler's Latin translation are made evident when one compares it with the Greek text and with Barozzi's:

Proclus, *In primum Euclidis*, p. 12, l. 9–19	Barozzi, *Procli Diadochi in primum Euclidis elementorum*, p. 6	Kepler, *De harmonice,* p. 114
Πρῶτον μὲν οὖν εἰ ἀπὸ τῶν αἰσθητῶν τὰ μαθηματικὰ **εἴδη** λέγομεν ὑποστῆναι, τῆς ψυχῆς ἀπὸ τῶν ἐν ὕλῃ **τριγώνων** ἢ κύκλων τὸ εἶδος τὸ κυκλικὸν ἢ τὸ τριγωνικὸν ὑστερογενῶς ἐν ἑαυτῇ μορφούσης, πόθεν ἡ **ἀκρίβεια καὶ τὸ ἀνέλεγκτον ὑπάρχει τοῖς λόγοις**; ἀνάγκη γάρ, ἢ ἀπὸ τῶν αἰσθητῶν ἢ ἀπὸ ψυχῆς. ἀλλὰ μὴν ἀπό γε τῶν αἰσθητῶν ἀδύνατον, πολλῷ γὰρ ἂν μᾶλλον **ἀκριβείας** τούτοις μετῆν. Ἀπὸ τῆς ψυχῆς ἄρα, **τοῖς μὲν ἀτελέσι τὸ τέλειον,**	Primum itaque si a sensilibus mathematicas **formas** oriri subsistereque dicimus, anima quidem nostra a **triangulis** vel circulis in materia insidentibus, circularem vel triangularem formam postremo in se ipsa formante, unde accurata illa vis et **certitudo illa, quae coargui convincique minime potest**, **rationibus inest mathematicis?** Haec enim aut a sensilibus aut ab anima eruantur necesse est. Atqui a sensilibus haec educi est impossibile. Multo enim maior	Primum itaque, si affirmamus **species** mathematicas a sensilibus constitui, dum anima a **trigonis** aut circulis materialibus speciem circularem aut trigonicam secundaria quadam genitura in se ipsa format, quaero unde veniat **rationibus (seu definitionibus) illa tanta certitudo tantaque accuratio**? Erit enim aut a sensilibus aut ab ipsa anima. At impossibile a sensilibus, multo namque maior subtilitas et **exactio** inest in rationibus istis. Ab anima igitur, quae

[114] « λείπεται δὴ οὖν καὶ παρ'αὐτῆς καὶ παρὰ νοῦ ταῦτα παράγειν καὶ εἶναι πλήρωμα τῶν εἰδῶν αὐτήν [...] », cf. PROCLUS, *In primum Euclidis,* p. 16, 4–6. Friedlein's version has the wrong form αὐτῆς; the spirit should be corrected as αὐτῆς.

[115] KEPLER, *De harmonice,* p. 114.

τοῖς δὲ μὴ ἀκριβέσι τὸ ἀκριβὲς προστιθείσης.	**certitudo** illis concedenda esset. Ab ipsa igitur anima educentur, quae imperfectis quidem perfectionem, iis autem, **quae certa non sunt, quod certum** sit, adhibet.	imperfectis **perfectionem, minime accuratis subtilitatem illam accuratam conciliet**.

Kepler follows Proclus' Greek in a more austere Latin than the stylish version of Barozzi. He preserves some of the original Greek terms (*trigonis, trigonicam*) and, unlike Barozzi, renders some of the Greek participles in indicative verbal forms (*in se ipsa format*). In this particular section, he renders τὰ εἴδη as *species* rather than *formae*. Importantly, the translation of some of the terms seems rather free; for instance, in the passage cited above, Kepler renders τὸ ἀνέλεγκτον as *accuratio*, whereas Barozzi preserves the original sense of "irrefutable" through the lengthy paraphrase *certitudo illa, quae coargui convincique minime potest*. Moreover, Kepler is inconsistent in the translation of the term ἀκρίβεια, rendering it as *certitudo* the first time, and *subtilitas et exactio* the second. Thus, although Kepler's structure of the text is closer to the original than Barozzi's (and other translators, for that matter), his choice of words is not very accurate or consistent. This sheds some light on Kepler's own understanding of Proclus and indicated the theoretical adoption made of him as the philosophical core of *De harmonice*: one may affirm that Kepler's translation is made in such a way that Proclus' words better fit his own theoretical ends (such as distinguishing between two kinds of harmony, namely sensible and archetypical, see below).

Kepler's text also contains additional notes which are either explanatory insertions between brackets or marginal titles and comments.

In his bracketed comments, Kepler adds terms meant to clarify Proclus' text. For instance, when Proclus discusses non-dimensional, one-dimensional, two-dimensional entities and other mathematical properties of natural bodies,[116] several bracketed comments provide the geometrical equivalent of the elements discussed; one reads even a self-reference to his own book I (where he provided an axiomatic exposition of the constant proportions of regular solids):

> Dic enim, ubi inter sensilia inveniatur natura impartibilis [puncti] et latitudine carens [ut linea], et profunditates [ut superficies], aut ubi aequalitas linearum ex centro, aut ubi proportiones semper constantes laterum [materia mei libri I], aut ubi angulorum rectitudines?

[116] Cf. Proclus, *In primum Euclidis,* p. 12, l. 19–23.

Kepler's comments are not always doctrinally neutral. For instance, when Proclus writes about the κοινὸν λόγον[117] and τοῖς λόγοις[118] in mathematics, Kepler translates them both as *rationes* and adds the parenthetical secondary meaning of *definitiones*:

> Concedendumne sit illa a sensilibus subsistentiam accipere, sive per abstractionem, ut mos est loquendi, sive per collectionem eorum, quae sunt per partes dispersa in unam communem rationem [seu definitionem], anne etiam ante haec, ipsis danda subsistentia, ut Plato usurpat, et universitatis rerum progressio demonstrat? [...] quaero unde veniat rationibus [seu definitionibus] illa tanta certitudo tantaque accuratio?

Although in Greek the term λόγος could stand for both 'reason' and 'definition' (among other things), it is nonetheless questionable whether in these particular sections these terms could be used as synonyms.

Some of Kepler's bracketed additions are related to the Christian doctrines of creation and soul. When translating the section where Proclus elaborates on how the mathematical paradigms contained in the soul generate in their unfolding all possible mathematics,[119] Kepler inserted the following comments:

> Atqui si illa continens in se ut prima exempla seu paradigmata secundum essentiam ipsa subsistere facit, ut ita sint generationes [Christianus subaudit: creatio rerum sensilium] nihil aliud quam propagationes specierum, quae in illa prius inerant [rationes creandorum corporum mathematicas Deo coaeternas fuisse, Deumque animam et mentem esse superexcellenter, animas vero humanas esse Dei creatoris imagines, etiam in essentialibus suo modo, id sciunt Christiani] tunc Platoni consentiemus haec dicentes, et vera essentia mathematum nobis inventa erit. Sin autem anima, cum non haberet nec prius alicunde accepisset rationes mathematicas [si, cum non fuissent ipsi concreatae], nihilominus admirabilem hunc ornatum immateriatum texit, pulcherrimam hanc speculationem enititur; quomodo ergo discernit sic genita, sintne subsistentia et constantia [μόνιμα, non γόνιμα lego] an in ventos evanida, et spectra potius quam vera?

To the term γεννήσις mentioned by Proclus to refer to the generative projections of the mathematical reasons contained in the soul,[120] Kepler adds a theological meaning, adding that *Christianus subaudit: creatio rerum sensilium.* In this way, Proclus' doctrine of the generation of mathematics is understood by Kepler not only as an epistemological and metaphysical theory solely concerning

[117] Cf. Proclus, *In primum Euclidis,* p. 12, l. 7.
[118] Cf. Proclus, *In primum Euclidis,* p. 12, l. 14.
[119] Cf. Proclus, *In primum Euclidis,* p. 13, l. 8–18.
[120] Cf. Proclus, *In primum Euclidis,* p. 13, l. 10.

mathematical knowledge and objects, but as a replication of the process through which God generated the world.

Then, when translating τῶν ἐν αὐτῇ προϋπαρχόντων εἰδῶν (l. 10–11), Kepler interprets again Proclus' words in a Christian manner and refers to (1) the co-eternal mathematical reasons through which God created bodies (*rationes creandorum corporum mathematicas Deo coaeternas fuisse*); (2) God, the mind, and the soul belonging to a nobler, superior order of things (*Deumque animam et mentem esse superexcellenter*); and (3) the human souls as images of God (*esse Dei creatoris imagines*), reinforcing thereby the claim that human souls contain mathematical reasons which are able to perceive and understand. Later, Kepler reinforces the view that human souls were created containing mathematics within them through an unreal subjunctive (*si cum non fuissent ipsi concreatae*). Lastly, Kepler declares in this section that he translates *μόνιμα, non γόνιμα*, understanding the term as 'constant' rather than 'fertile'.

In an important fragment, Proclus hypothesises the manner in which mathematical essences come from the soul[121] (ἀνάγκη δήπου τὴν ψυχὴν [...] / *necesse utique fuerit ut anima* [...]). Kepler claims that Proclus speaks here of the *anima mundi*, which is a created god according to Plato and, according to Christians, the mind which is the creator God himself, of whom every created soul is an image:

> Anima hic intellegit mundi praecipue deum creatum Platonis; mentem vero, quam Christiani dicerent ipsum Deum creatorem, cuius imagines sunt omnes animae creatae corporibus vivificandis praefectae.

Kepler comments on the passage where Proclus establishes that the human soul is not a blank slate,[122] but rather contains innate knowledge. To Proclus' claims he adds a further theological point: that not only are souls the images of the creator (and, thus, contain innate knowledge), but that souls are constantly sustained by the irradiation of the divine countenance:

> Christianis et sunt animae exemplaria creatoris et sustentantur etiamnum ab illo per quandam velut irradiationem vultus divini in ipsas.

Finally, when translating the section in which Proclus relates the theory of innate mathematics to the account of the creation of the soul through mathematical and harmonic reasons according to the *Timaeus*,[123] Kepler adds:

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[121] Cf. Proclus, *In primum Euclidis,* p. 15, l. 19.
[122] Cf. Proclus, *In primum Euclidis,* p. 16. l. 4.
[123] Beginning in Proclus, *In primum Euclidis,* p. 16, l. 16.

> In Timaeo, qui est citra omnem dubitationis aleam commentarius quidam in primum caput Geneseos seu Libri I Mosis transformans illum in philosophiam Pythagoricam, ut facile patet attente legenti et verba ipsa Mosis identidem conferenti.

Kepler reveals his belief that Plato's *Timaeus* was in fact a Pythagorean commentary on the book of Genesis, and, moreover, that this can be easily attested by comparison of both texts. Kepler finds the Scriptural authority not only in Plato's own writings, but also in Proclus' *In Euclidem*.

Immediately after the translation, Kepler indicates again the main points taken from Proclus:[124]

> Quod, cum et Plato intellexisset, anima ex omnibus constituit, divisitque secundum numeros, revinxitque analogiis et rationibus harmonicis, inque ipsa contulit principia prima figurarum effectricia, puta rectum et curvum, movetque circulos qui sunt in ipsa intellectualiter. Omnia ergo mathematica primum sunt in anima [...].[125]

The two crucial ideas argue (1) that the soul is constituted by pre-empirical mathematics, and (2) that harmonical reasons among numbers and geometrical figures (including, importantly for Kepler, the Platonic solids) are contained in this pre-empirical mathematics (*et rationibus harmonicis, inque ipsa contulit principia prima figurarum effectricia*).

Kepler would use these Proclean doctrines for his own distinction between *harmonia sensilis* (sensible harmony) and *harmonia archetypalis* (archetypical harmony). [126] Harmony, i.e., numerical proportion, is two-fold: it is to be understood as an archetype in the soul, and as the regular patterns of things in the material world.[127] In the case of the natural world, harmony is represented by the shapes and patterns of bodies, and their movements. In the case of the soul, it corresponds to the abstract, non-particularised mathematical patterns and proportions which are then recognised in the particulars of the natural world (e.g., when analysing music or the movements in the heavens). Kepler claims that innate mathematical knowledge is prior to any recognition of the regularities and proportions in the created world. He declares that such innate knowledge would

[124] Kepler's commentary on Proclus' text spans two pages. Cf. KEPLER, *De harmonice,* IV, p. 118–120.
[125] Cf. KEPLER, *De harmonice,* IV, p. 117. Cf. Proclus, *In primum Euclidis,* p. 16, l. 16–23.
[126] Cf. KEPLER, *De harmonice,* IV, p. 120.
[127] KEPLER, *De harmonice*, IV, p. 120: « Commune enim habent harmoniae sensiles cum archetypalibus, quod terminos requirant, eorumque comparationem, ipsius animae energiam. In hac comparatione utrarumque essentia consistit ».

be a sort of instinct (*instinctus*)[128] through which the structures of the *harmoniae archetypales* are recognised via comparison of sensible patterns. Moreover, for Kepler mathematical proportions are not parts, among others, of the soul. On the contrary, he states, interpreting Proclus' text, that the soul *is* the unfolded whole of all archetypical mathematical proportions.[129] The origin of these innate mathematical (geometrical) is, according to Kepler, the primitive forms of the circle and the arc, from which the rest of geometry is generated.[130] Kepler's interpretation of Proclus' *In Euclidem* introduce a collection of philosophical topics that will become seminal in early modern philosophy, such as the reflection on the role of mathematics in the science of natural world, but also gnoseological innatism, and mathematical apriorism.[131]

III. *Concluding remarks*

The presence of Proclus' *In Euclidem* can been attested amongst northern Italian scholars at least since 1439, when Bessarion moved in that region and brought his library with him. In 1533 Proclus commentary was added as a sort of 'companion book' to Grynaeus' *editio princeps* of Euclid's *Elementa*, making the work widely accessible. As we saw, several partial or complete Latin translations of the text were produced in the 16th and 17th centuries.

Partial Latin translations, such as those provided by Valla, Dasypodius, or Kepler, reveal that the contents of Proclus' *In Euclidem* were accommodated to different purposes. Whereas Valla tacitly inserted Proclus' two prologues, Dasypodius re-arranged several parts of the commentary to produce a handbook of concepts destined to students. Muñoz and Borromeo considered it a source to be reckoned with in dealing with the classification of mathematical disciplines, whereas Kepler adopted (and transformed) the innatist mathematical epistemology of Proclus to defend the inborn nature of harmony (*harmonia archetypalis*). Interestingly, he also drew theological parallels between the generation of the mathematical entities by the human soul and the divine production of the cosmos.

One has to observe that doctrines of Proclus' *In Euclidem* became part of a widespread mathematical culture in the Latin West, although many of the

[128] Kepler, *De harmonice,* IV, p. 129: « Ad hoc respondit supra Proclus, verbis in sua philosophia tritis, nos hodie, ni fallor, vocabulo 'instinctus' rectissime utemur'. And in the margin: 'Anima habet scientiam mathematicam ex instinctu ».

[129] Kepler, *De harmonice,* IV, p. 120: « Ita simplex tantum comparatio, quam instituit anima, suarum ipsius veluti partium inter se absolvit archetypicae harmoniae essentiam omnem ».

[130] Cf. Claessens, « Imagination as Self-knowledge: Kepler on Proclus' Commentary on the First Book of Euclid's Elements ».

[131] I shall treat Kepler's and Proclus' philosophies of mathematics in a forthcoming publication.

contents were not always openly declared to belong to the Diadochus' reflections. Indeed, Proclus' commentary was used as a general introduction to mathematical disciplines and its history, but also as an arsenal of mathematical and logical concepts, or as an authoritative source for a Neoplatonic philosophy of mathematics, as we have seen in Zamberti's prologue to his edition of Euclid's *Elementa*.

Two complete, yet unpublished, translations of Proclus' *In Euclidem* were produced before the better-known version of Barozzi from 1560: one by Zamberti (MS Munich, Bayerische Staatsbibliothek MS clm 6) and the other by Gabia (MS Milan Ambrosiana MS P. 51 sup). A comparison of the selected passage has concluded that later translations have not been influenced by earlier ones. Zamberti's translation does not show signs of revisions, whereas Gabia's manuscript was emendated by a second, anonymous hand. This corrector attempted to re-phrase many of the technical and scholastic Latin expressions of Gabia into Ciceronian prose.

For his part, Barozzi had access to five Greek manuscripts; hence, he could compare the versions and eventually provide a more accurate Latin translation. Although Barozzi's Latin translation can easily be considered the most accurate and elegant of the three, his choices, as I have shown, are biased by his polemic against Piccolomini on the so-called *quaestio de certitudine mathematicarum.* As pointed regarding this questions, Proclus' commentary was destined to become one of the main authorities in the intellectual debate following Barozzi's answer to Piccolomini, extended to all Europe through the influence of the Jesuit order, and even reaching the Cantabrigian Isaac Barrow.

It should be emphasised that some of the topics preferred by these Latin authors using Proclus' *In Euclidem* had strong echoes in early modern philosophy and the so-called scientific revolution, such as: (1) the legitimacy of mathematical means in obtaining knowledge of the natural world; (2) mathematical innatism or apriorism against empiricism; (3) the intermediary status of mathematical entities between the natural world and the divine; (4) the idea of a *mathesis universalis* or general mathematics; (5) the classification of mathematical disciplines. From these it can be concluded that Proclus' *In Euclidem* played a far greater role than scholars have previously assumed in the formation of early modern philosophical and scientific culture in the Latin West.

Nevertheless, it should be emphasised yet again that the original source was not always acknowledged or clearly stated by those making use of the doctrines contained in Proclus' commentary. Some of them (Valla, Muñoz, or Dasypodius) did not even quote Proclus' name, showing thereby that many of the contents of *In Euclidem* had (by the 16th century) tacitly become a source of insight for the general mathematical and philosophical culture.

Appendix 1

TABLE 1					
Valla's *De expetendis*	**Proclus' *In Euclidem* (Friedlein ed.)**	**Proclus' *incipit***	**Valla's *incipit***	**Proclus' *explicit***	**Valla's *explicit***
De tota mathematica, Liber I, cap. XIV	p. 1, l. 1–7, l. 12	Τὴν μαθηματικὴν οὐσίαν οὔτε τῶν πρωτίστων ἐν τοῖς οὖσι γενῶν οὔτε τῶν ἐσχάτων εἶναι καὶ παρὰ τὴν ἁπλῆν διῃρημένων, ἀναγκαῖον, ἀλλὰ τὴν μέσην χώραν ἀπειληφέναι τῶν τε ἀμερίστων καὶ ἁπλῶν καὶ ἀσυνθέτων καὶ ἀδιαιρέτων ὑποστάσεων καὶ τῶν μεριστῶν καὶ ἐν συνθέσεσιν παντοίαις	Mathematicum genus universum, neque primis eorum quae sint generibus, neque postremis prorsus attribuendum, sed medium quondam locum obtinere necesse est inter ea quae substantiarum indivisibilium simpliciumque sunt, quae nec componuntur nec distribuuntur, et ea quae composita inveniuntur, quae divisibilia, variaeque multiplicique sectioni sunt obnoxia.	[...] λόγους καὶ τὰ σχήματα καὶ τὰς μορφάς.	[...] rationes, quoad figuras, quoad formas.

		καὶ ποικίλαις διαιρέσεσιν ἀφωρισμένων.			
De Mathematicis collectim assumptis cognitionnibus, Liber I, cap. XV	p. 7, l. 13–10, l. 14	Ἀλλ’ ὅτι μὲν ἀρχαὶ καὶ τῶν μαθημάτων αὗται προεστήκασιν, αἳ καὶ τῶν ὄντων ἁπάντων [...]	Cum itaque eadem sint mathematicarum quae rerum omnium principia [...]	[...] πάσαις ταῖς μαθηματικαῖς ἐπιστήμαις χορηγοῦσα τὰς ἀρχάς.	[...] cunctis Mathematicis scientiis suggeret principia [...].
De vi mathematicas iudicandi, Liber I, cap. XVI	p. 10, l. 16–12, l. 2	Μετὰ δὲ τοῦτο, τί ποτ’ἂν εἴη τὸ κριτήριον τῶν μαθημάτων θεωρήσωμεν [...]	Nunc quae sit mathematicas iudicandi vis, quam vocant graeci criterium, inspiciamus [...]	[...] τὴν διάνοιαν δόξης μὲν ὑπεριδρύσασαν ἑαυτήν, τῆς δὲ νοήσεως ἀπολειπομένην.	[...] quae opinationem quidem superat, ab intellectu autem superatur.
De genere specieque mathematicarum, Liber I, cap. XVII	p. 12, l. 2–18, l. 4 Omissions: p. 12, l. 8–9; p. 18, l. 23–26.	Ἔπεται δέ που κατιδεῖν ἡμᾶς, τίνα τὴν οὐσίαν προσήκει λέγειν τῶν μαθηματικῶν [...]	At iam mathematicarum considerandam referandamque et formarum et generum arbitramur essentiam [...]	Πάντα γὰρ προείληφεν ἀρχοειδῶς καὶ κατὰ τὴν ἄπειρον ἑαυτῆς δύναμιν ἐκ τῶν προειλημμένων ἀρχῶν	[...] Cuncta siquidem vi primae speciei suaque infinita potentia praecupavit, ex praesumptisque principiis varias, ac multiples atque adeo

				παντοδαπῶν θεωρημάτων ποιεῖται προβολάς.	omnifarias sibi proponendo coniectat inspectiones.
De officio mathematicae, Liber I, cap. XVIII	p. 18, l. 5–20, l. 7	Ἀλλὰ δὴ μετὰ τὴν οὐσίαν τῶν μαθηματικῶν εἰδῶν [...]	Verum postea quod mathematicarum essentiam perspeximus [...]	Τοσαύτα καὶ περὶ τούτων εἰρήσθω.	Ac de his quidem hactenus.
De pertinentibus ad scientiam Mathematicam, Liber I, cap. XIX	p. 20, l. 8–32, l. 20 Omissions: p. 21, l. 4–l. 19; p. 22, l. 14–l. 16; p. 23, l. 12–28, l. 23	Τὸ δὲ ἐντεῦθεν τῆς ἐπιστήμης ταύτης κατίδωμεν [...]	Hinc quae ad mathematicam pertinent scientiam ordinamur [...]	[...] Τοσαῦτα καὶ περὶ τῆς Πλάτωνος γνώμης ὑπὲρ τῶν μαθημάτων εἰρήσθω.	[...] Et haec de Platonis sententia satis.
Quae Mathematicus exquirat, Liber I, cap. XX	p. 32, l. 21–35, l. 16	Τίνα δ'ἄν τις ἀπαιτήσειεν τὸν μαθηματικόν [...]	Quae porro mathematicus anquirat [...]	[...] καὶ πρὸς πάντας συναρμόζει τοὺς ἑαυτῆς λόγους.	[...] quadrat omnibus atque suis assentit rationibus.
De partibus Mathematices, Liber I, cap. XXI	p. 35, l. 17–42, l. 6	Ἀλλὰ τούτων μὲν ἅδην, περὶ δὲ τῶν εἰδῶν τῆς [...]	Hinc de mathematices partibus nobis deinceps disserendum est [...]	[...] ἡλίου καὶ σελήνης καὶ τῶν ἄλλων ἄστρων καταμανθάνουσα διὰ τῶν	[...] solis et lunae, ceterorumque astrorum distantias huiusmodi dinoscens instrumentis.

				τοιούτων ὀργάνων.	
De arte disserendi in mathematicis, Liber I, cap. XXII	p. 42, l. 9–44, l. 23	Εἶεν δὴ οὖν. Πάλιν ἐκεῖνα θεωρήσωμεν, ὅπως [...]	Haec fuere [...]. Reliquum est ut inspiciamus quae [...]	[...] πορείας καὶ τῆς γνωστικῆς ἐνεργείας.	[...] finis denique, tam instituendi viae, quam agitandae cognitionis longe optimus.
Unde Mathematices nomen, Liber I, cap. XXIII	p. 44, l. 25–47, l. 6	Τὸ δ'αὖ ὄνομα αὐτὸ τοῦτο τὸ τῆς μαθηματικῆς [...]	Nunc postremo unde mathematica dicta dicendum putamus.	[...] τοῦ καθαροῦ νοῦ περιάγει πρὸς τὴν μακαρίαν ζωήν.	[...] ac per purae defoecataeque mentis facilem decursum ad beata fruendum vita agit.

TABLE 2					
Valla's *De expeten-dis*	**Proclus' *In Euclidem* (Friedlein's ed.)**	**Proclus' *incipit***	**Valla's *incipit***	**Proclus' *explicit***	**Valla's *explicit***
De geometria, Liber X, cap. I	p. 48, l. 9–81, l. 22 Omissions: p. 48, l. 1–8; fol. 70 l. 14–75, l. 6; p. 81, l. 23–84, l. 23	[...] Ὅτι μὲν οὖν ἡ γεωμετρία τῆς ὅλης ἐστὶ μαθηματικῆς μέρος καὶ ὅτι δευτέραν ἔχει τάξιν μετὰ τὴν ἀριθμητικήν [...]	[...] Quod autem geometriam post arithmeticam musicamque collocare [...]	[...] ὅλον προβλημάτων ἐστί, τὸ δὲ πέμπτον θεωρημάτων.	[...] ut quintus habet liber haec de theoremate ac problemate satis [...]

TABLE 3		
Dasypodius' chapters in *Mathematicum* (Greek) (Dasypodius' Greek titles have no accents; the accentuation is mine)	**Dasypodius' chapters in *Mathematicum* (Latin)**	**Sections from Proclus' *In Euclidem* composing each chapter (Friedlein's edition)**
Τίς ἡ καθ'ὅλον μαθηματικὴ ἐπιστήμη (p. 1–14)	I, Quaenam et qualis sit universalis illa disciplina mathematica (p. 172–176)	p. 1, l. 1–1, l. 7 (altered *incipit*); p. 1, l. 14–16+p. 10, l. 20–21 (altered passage; conflates two sentences); p. 10, l. 21–11, l. 16; Dasypodius' addition (p. 2, l. 21–p. 3, l. 3); p. 5, l. 14 (altered *incipit*) 9, l. 25; p. 10, l. 2–10, l. 16; p. 43, l. 9–46, l. 18
Περὶ τῶν μαθηματικῶν ἐπιστημῶρν (p. 14–22)	II, De scientiis mathematicis (p. 176–190)	p. 35, l. 17–42, l. 9; + A section of Heron's geometry
Τίς ὁ μαθηματικός (p. 28–32)	III, Quinam et sit et appelletur mathematicus (p. 190–195)	p. 32, l. 21–35, l. 16
Περὶ γεωμετρίας καὶ ἀριθμητικῆς (p. 32–40)	IV, De geometria et arithmetica (p. 195–203)	p. 48, l. 9–65, l. 11
Περὶ τῶν στοιχείων καὶ τῆς τούτων οἰκονομίας (p. 40–46)	V, De elementis et eorum institutione (p. 203–209)	p. 71, l. 24–75, l. 26
Περὶ τῆς διαιρέσεως τῶν ἀρχῶν (p. 46–54)	VI, De divisione principiorum (p. 209–217)	p. 75, l. 27–77, l. 6; p. 179, l. 9–182, l. 20; p. 184, l. 11–184, l. 29
Περὶ τῶν μετὰ τὰς ἀρχὰς προτάσεων (p. 54–61)	VII, De propositionibus quae principia sequuntur (p. 217–223)	p. 77, l. 6–81, l. 6; p. 200, l. 6–9; p. 81, l. 6–23
Τίνα τὰ ζητήματα καὶ τὰ τῶν μετὰ τὰς ἀρχὰς μεταπροτάσεων μέρη (p. 61–70)	VIII, Quaenam sint quaestiones et quae sint earum propositionum, quae principia sequuntur, partes vel capita (p. 223–233)	p. 200, l. 22–207, l. 25; p. 210, l. 5–25 (altered)

Τίνες αἱ τῶν μετὰ τὰς ἀρχὰς προτάσεων διαφοραί (p. 70–78)	IX, Quae et quales sint differentiae propositionum earum quae principia sequuntur (p. 233–241)	p. 220, l. 7–223, l. 6; p. 243, l. 11–244, l. 2; p. 244, l. 14–246, l. 12; p. 254, l. 23–260, l. 9; p. 394, l. 11–395, l.2
Περὶ τοῦ λέμματος, πτώσεων, πορίσματος ἐντάσεως καὶ ἀπαγωγῆς (p. 78–84)	X, De lemmate, casu, porismate, instantia, et reductione (p. 241–247)	p. 210, l. 25–212, l. 17; p. 301, l. 21–302, l. 21; p. 303, l. 5–304, l. 3; p. 212, l. 18–213, l. 11
Περὶ ἀντιστροφῆς καὶ ἀπαγωγῆς εἰς τὸ ἀδύνατον (p. 84–89)	XI, De conversione et reductione ad impossibile (p. 247–256)	p. 252, l. 5–255, l. 7; p. 255, l. 12–256, l. 10

Appendix 2

LATIN TRANSLATIONS OF PROCLUS' FIRST PROLOGUE
CORRESPONDING TO FRIEDLEIN'S EDITION, FOL. 12, L. 2 TO FOL. 18, L. 4

Conspectus siglorum

< > – addenda censeo
add. – addidit
corr. – correxit / correctio
del. – delevit
exp. – expunxit
illig. – illigibile
in marg. – in margine
iter. – iteravit
lin. – linea
scrip. – scripsit
sup. – supra

1. Giorgio Valla, *De expetendis et fugiendis rebus*, Manutius, 1501, Liber I, Cap. XVII [s.n.]

Cap. XVII. De genere specieque mathematicarum

At iam mathematicarum considerandam referendamque et formarum et generum arbitramur essentiam, et utrum a sensibilibus manere sit admittendum auferendo, ut dici solet, sive per particularia[132] in communem unam rationem colligendo, an vero ab anima.[133]

Primum itaque, si a sensibilibus mathematicas species animae insidere dixerimus, ut a triangulis vel circulis in materia insidentibus, triangularis vel circularis species in ipsa demum effigietur, unde pensiculatum illud examen idque quod coargui convincique non potest in rationibus inest mathematicis? Haec enim aut a sensibilibus aut ab anima eruantur necesse est. Atqui a sensibilibus id educi impossibile est: in iis siquidem multo maiore opus est examine. Cum ergo a sensibilibus non possint, ab anima igitur educentur, quae imperfectis consumationem examine carentibus, examen acre adhibet. Ubi namque in eis, quae sub sensum cadunt, usquam indivisibile vel latitudinis expers aut crassitudinis percipi potuerit? Ubi porro ex circuli centro linearum illa recepta aequalitas? Ubi semper stabiles laterum rationes? Ubi angulorum rectitudines?[134] Quonam demum pacto obnixissimis rationibus ex mobilibus alio atque alio tempore aliter habentibus stabilem mathematicis tribuemus essentiam? Omne siquidem, quod a mobilibus ortum ducit essentiis, mutabilem habere substantiam convenit. Quonam modo certis et quae minime coargui possint formis ab incertis certitudinem accomodare poterimus? Proinde confessum ac receptum sit formarum atque rationum mathematicarum animam esse genitricem. Quod si qua ad essentiam habet exempla, ea sibi illa substituit, suntque propositae geniturae earum, quae in ipsa praesident, formarum, et perinde Platoni astipulamur, ut haec inveniendo veram mathematicarum asseramus esse essentiam. Quod si minime propositis rationibus tantum subtexit ornamentum materiae expers tantamque gignit inspectionem, quomodo quae genita productaque sunt potest diiudicare sintne a semine orta naturaliaque an secus planeque simulacra pro veris? Quibusnam demum praeceptis insistet ac regulis quo verum suspiciat atque metiatur? Et quo pacto essentiam ipsorum non tenens, tantam rationum producit varietatem, vagam quippe et incertam, ita horum faciemus hypostasin, quam nullo termino concludemus? Quod si anima mathematicas gignit species, neque a sensilibus rationes admittit, quibus obarmetur, et a formis in anima insidentibus

[132] particularia] particularium *ed.*
[133] Follows an omission in respect to Friedlein's edition p. 12, l. 8–9.
[134] Follows an omission in respect to Friedlein's edition p. 12, l. 23–26.

ipsa producuntur sensilia ipsius, utique animae partus foetusque elucescent formis permanentibus sempiternis.

Dein, si sensilibus rationes connectimus mathematicas, quonam modo necesse non fuerit praestantiores perhibere demonstrationes quaecumque a sensilibus instituantur, quod quae a magis semper universalibus formisque simplicioribus? Causas siquidem dicimus in demonstrationibus usquequaque esse praecipuas ad id quod in quaestione positum. Quod si particularia et sensilia universalium et sub animi sententiam cadentium causa sunt, cuiusmodi erit substructio demonstrationis terminum ad universalia potius referendo ac non ad particularia, et intellectilium essentiam prae sensilibus demonstrationibus magis affinem ostenderem? Nam neque siquis, ut dicitur, demonstraverit isosceles duobus rectis aequales habere angulos, ita aequilaterum, ita scalenum, is admodum eius habet scientiam. Verum, qui scit omne triangulum habere duobus rectis aequales uniformiterque demonstrans per sese scientiam habet. Et rursus, quod universale est, id particulari ad demonstrationem longe melius est. Item demonstrationes ex universalibus potius constant atque conflantur, atqui ex quibus sunt demonstrationes, ea priora sunt, singularibusque natura ipsa praecellunt, suntque causa eorum, quae demonstrantur. Demonstrativis igitur scientiis postrema et obscuriora sensilia erunt colligenda in unumque connectenda. Contra quae ab animi sententia comprehensa sunt, quippe quae longe perfectiora, non quae in sensu opinationeque demersa sunt, sibi ducunt eruenda ac inspicienda.

Tertio loco, qui haec aiunt animam formis faciunt inferiorem. Nam, si materia essentialia, quaeque magis esse dicuntur manifestioraque a natura sibi vendicat, <et si> anima, vicissim, secundo loco, ab illis et simulacra et imagines posterius eductas in sese recipit (ac diffingit in essentiam minus honorata<m> a materia, auferens quae natura suapte ab ipsa separari non possunt), quonam pacto animam imbecilliorem multoque inferiorem materia non ostendunt?" Nempe ut silvosarum est rationum locus materia, ita formarum anima conceptaculum, sed primarum altera, altera secundarum, et altera eorum quae opinatione gignuntur. Quomodo igitur anima, mentis, intellectilisque essentiae particeps primo, particeps et cognitione referta a sensilibus totius vitae, obtusiores, hebetiores obscurioresque accipit formas, in quibus ultima est eorum quae sunt sedes et, quatenus sunt, omnium imperfectissima? Adversum itaque hanc a plerisque traditam importune dicimus opinionem.

Quodsi non per ablationem materiatorum mathematicae species, neque per collectionem eorum, quae in singulis communia sunt, neque prorsus postremi sunt generis et perinde a sensibilibus, eas certe anima, aut a sese, aut a intellectu seu mente, aut tum a se tum ab intellectu accipiat sibique vendicet, necesse est. Quodsi a se dumtaxat, quonam modo mathematicae intellectilium erunt formarum imagines? Quomodo inter dividuam individuamque naturam fuerint mediae? Consumationem a primis, ut sint nullam sortientes? Quomodo universi exempla in

mente in primis insederunt? Quodsi non ab anima sunt, sed a mente, quopacto id quo sese anima exercet seque movet stare potuerit, si quae in ipsa sunt rationes in altrinsecus motis suam habentes hypostasin aliunde in ipsam influxere? Praeterea, in quonam demum anima ab ipsa differt materia, quae potentia solum est omnia formarumque materiosarum prorsus gignit nihil. Id ergo, cum ita habeat, reliquum est ut tum ab ipsa anima, tum a mente haec scateant, sitque anima formis referta ab intellectilibus exemplis in ipsa insedentibus ipsaque per sese gignit, ad id, ut sint accessus. Proinde esse qui[135] potest anima tamquam tabula aut liber in quo scriptum sit nihil? Immo equidem est tamquam liber perpetuo scriptus ac suapte natura seseque describens, et ab ipso descriptus intellectu. Intellectus siquidem, sive mentem libet appellare[136], est etiam anima, quod intellectus prae sese obvolvat sitque imago et impraessio illius, quae extrorsus facta. Et perinde, si mens cuncta intellectiliter, et anima cuncta animaliter; et si ea in exempli modum, seu malis exemplariter dicere[137], anima imaginanter ; et si intellectus collectim, anima divisim comprehendit.

Id nimirum est, quod Plato animam inspiciens ipsam ex omnibus mathematicis constituit formis, eamque dividit in numeros, et colligit proportionibus harmonicisque rationibus, principiaque figuras primas constituentia in ipsa defigit, rectam et circularem, movetque in ipsa orbes intellectiliter, ut secutus etiam Platonem Cicero de universitate docet[138]. Cuncta igitur mathematica primum in ipsa sunt anima et, ut Platonici aiunt[139], praecaeteris numeris qui[140] sese movent, et ante apparentes figuras figurae animales, et ante concentus harmonicae rationes, et ante orbes motorum corporum orbes nusquam comparentes creati sunt, rebusque referta est omnibus anima. Et ornamentum aliud ipsum per se perducens suo peractum principio, vita sese perficiens ab universitatis creatore productum omnibus iis refertum, corporis expers sine intervallo, quod ubi suas exerit rationes, tum omnes patefacit scientias atque virtutes. Has itaque formas anima suam induit substantiam, dixerim substantiatur nisi verear mihi acclamitari[141]. Neque in ipsa anima numerum putes qui unitatum est multitudo, neque distinctionum ideam in corporis modum accipere convenit, at omnia vitali intellectilique vi exempla apparentium numerorum figurarumque et rationum ac motuum putanda sunt, Timaeum sequendo, omnem animae ortum atque creationem ex formis complentem mathematicis omniumque in ipsa causas constituentem. Cunctorum sane numerorum limites septem liniamentorum

135 qui] que *ed.*
136 sive mentem libet appellare : *Valla's addition*
137 seu malis exemplariter dicere : *Valla's addition*
138 ut secutus [...] docet : *Valla's addition*
139 ut Platonici aiunt : *Valla's addition*
140 qui] que *ed.*
141 dixerim substantiatur nisi verear mihi acclamitari : *Valla's addition*

principia comprehenderunt, et planorum et solidorum. Rationum autem omnium rationes septem causam in ipsa praestruxerunt. Figurarum porro primordia vim in ipsa creationis erexerunt. Motuum facile princeps caeteros omnes comprehendens et excitans in ipsa constitutus omnium siquidem, quae moventur circulus principium itidem circulus motus. Ergo, ut nunc sic loquar[142], essentiales ac per sese mobiles mathematicarum sunt rationes complentes consumantesque animas, quas inquam animi rationes animi sententia proponens provolvensque, scientiarum mathematicarum omnem constituit varietatem. Nec unquam conquiescit, at gignit illa, ac producit[143] semper aliquid aliaque scrutatur ex aliis, investigatque ac eruit suas partitionum expertes rationes explicando. Cuncta siquidem vi primae speciei suaque infinita potentia praeoccupavit, ex praesumptisque principiis varias ac multiplices atque adeo omnifarias sibi proponendo coniectat inspectiones.

[142] ut sic loquar : *Valla's addition*
[143] producit] procudit *ed.*

2. Bartolomeo Zamberti transl. Procli *In Euclidem*, MS Münich, Bayerische Staatsbibliothek Clm 6, fol. 4v–6v [= M]•

Incipit /fol. 1r/

Procli lycii diadochi Platonici in primum librum Elementorum Euclidis megarensis mathematici praestantissimi, commentariorum liber primus, Bartolomeo Zamberto veneto interprete

[...]

/fol. 4v/ Consequens[144] autem fuerit ut scrutemur qualem essentiam formarum ac generum mathematicarum rationum dicere congruum sit, et utrum ipsam a sensilibus subsistere admittendum sit vel per ablationem, sicuti dicere consueverunt, vel per particularium collectionem in unam communem rationem, vel iis essentiam haec praecedentem assignare conveniat, sicuti Plato arbitratur universorumque processus ostendit.

Et[145] in primis, si mathematicas formas a sensilibus subsistere dixerimus, omnia in semet ipsa posteriori genitura formanti a materiosis triangulis vel circulis, circulosam vel triangularem formam, unde erit certitudo illa et illud quod in rationibus coargui non potest? Necesse enim est quod aut a sensilibus, aut ab anima. Atqui a sensibilibus impossibile est, nam in his longe maiore indagine opus est. Ab anima igitur; nam his quae imperfecta et incerta sunt, perfectionem certitudinemque exhibet. In sensilibus enim ubinam est individuum vel id quod latitudine vel id quod crassitudine caret? Et ubi earum ex centro sunt aequalitas? Et ubi laterum rationes sunt stabiles? Et ubi angulorum rectitudines? Nec video, sicut in alternis omnia divisibilia commisceri, et in his nihil sincerum est, et a contrario expurgatum, sed omnia partitionem, intervallum ac unionem admittunt. Quo igitur pacto his quae immobilibus sunt ex mobilibus et aliquando aliter se habentibus ipsam stabilem essentiam assignabimus? Nam[146] omne quod ex essentiis motis subsistit, essentiam mutabilem etiam ab ipsis obtinet, apud omnes indubitatum est. Et quo pacto illis formis quae certae sunt, et coargui nequeunt, ab his quae incerta sunt certitudinem applicabimus?[147] Nam omne quod est causa eius cognitionis quae semper mobilis est, illud ipsum magis est huiusmodi. Supponendum igitur /fol. 5r/ est animam mathematicarum formarum et rationum genitricem esse. Sed cum siquidem exempla obtineat ipsa secundum substantiam substernit, et generationes sunt integumenta earum formarum quae in ipsa praeexistunt, haec dicentes Platoni astipulabimur, vereque mathematarum

• For the marginal notes and their corresponding numbers, see above, p. 82.

144 Consequens] 38 Genera et formae mathematices unde essentiam obtineant? *add. in marg.* M

145 Et] 39 Si mathematicae formae a sensilibus subsistunt, quid sequatur? *add. in marg.* M

146 Nam] 40 Id propter quod unumquodque tale etiam magis est huiusmodi *add. in marg.* M

147 applicabimus?] Aliter: omne quod causa cognitionis immobilis, id ipsum magis huiusmodi est. *add. in marg.* M

essentiae erimus inventores. Si vero non habet neque rationes prius assumens, tantum materie expertem apparatum sustinet, tantamque contemplationem gignit, quo pacto ea, quae genita sunt, unde quaedam seminaria sint vel vana, et eorum, quae vere sunt simulacra, diuidicare potest? Qualibus vero regulis utens in his veritatem existentem metitur? Quo etiam modo essentiam non habens ipsorum tantam rationum varietatem gignit? Sic enim ipsorum essentiam vagam ad nullumque terminum relatam efficerimus. Igitur si formae mathematicae ab anima ortum habent, nec a sensilibus rationes suscipiuntur,[148] sicut anima substinet, et ab illis has erunt ipsius quoque partus, a manifeste sempiternis formis oriuntur.

Praeterea, si inferut et a sensilibus mathematicarum rationes colligamus, quo modo fieri poterit ut non dicamus quod illae demonstrationes, quae ex sensilibus fiunt, potiores sunt, et non illae quae ab universalibus et simplicioribus formis inferuntur? Dicimus[149] enim quod causae ubique demonstrationibus ad quaesiti venationes sunt peculiares. Igitur si particularia universalium, et sensilia intellectilium sunt causae, quis umquam excogitabit quod terminus demonstrationis ab universalibus assumptus magis conferat, quam a particularibus? Quo erit pacto quippiam ostenderit quod intellectilium essentia sit magis affinis demonstrationibus quam essentia sensibilium? Neque enim siquis, inquunt, demonstraret quod triangulum equicrurium tres habet angulos duobus rectis aequales, et itidem aequilaterum, et scalenum, is id quoddam noscit. Verum qui scit quod omne triangulum, idque simpliciter demonstrat, is scientiam per se obtinet. Et[150] rursus id quod universale est, quoad demonstrandum, particulari melius est. Et insuper quod ipsorum demonstra/fol. 5v/tiones universaliores sunt, illa vero sunt priora et natura ita praecedunt quae singularia sunt, et eorum quae demonstrant sunt causae ex quibus demonstrationes fiunt. Igitur scientiae demonstrativae opus habent, ut ea, quae sensilia sunt et posterius genita ac hebetiora, colligantur. Non autem ea, quae, cum a cognitione percipiantur, his sunt perfectiora, quae sensu et opinione considerantur.

Tertio vero loco, dicimus quod illi, qui haec dicunt, animam formis viliorem efficerint. Nam si materia ea quae substantialia sunt ac magis entia ac integriora a natura suscipit, anima vero ab illis secundum locum obtinet ac sustinet istam, simulacra et imagines posterius genitas effingit ad ignobiliores essentias ac inferioras, quae ex natura a materia inseperabilia sunt, quo pacto animam imbecilliorem materiaque inferiorem esse non ostendunt? Locus is et silva est et si<l>vestrium rationum, et anima formarum, sed alia quidem primarum, alia vero secundarium, et alia eorum quae praecipue entia sunt, alia porro eorum quae inde subsistunt, et alia sunt eorum quae formalem substantiam et alia, demum, eorum

[148] suscipiuntur] subs *add. sed del.* M
[149] dicimus] 41 Causae ad quaesiti indaginem demonstrationibus sunt peculiares *add. in marg.* M
[150] Et] 42 Universale ad demonstrandum particulari efficacius est *add. in marg.* M

quae secundum excogitationem dicunt. Quo igitur pacto illa, quae intellectus et intellectillis essentis primum particeps est, et viose cognitionem implens et totius vitae, hebetiores formas suscipit, quae in his, quae sunt ultima, sedes est et ad essentiam omnibus imperfectior est? Atque huic opinioni, quae multis molestiam exhibuit, occurrere supervacaneum sane fuerit.

Si vero formae mathematicae non sunt per materialium abs<tr>a<c>tionem, neque per illorum collectionem, quae in singularibus sunt communia, neque posterius et a sensilibus genitae, anima vel ex se ipsa, vel ab intellectu, vel et a se ipsa et ab illo, ipsas assevere necessarium est. Atqui se ex semet ipsa solum, hae quo modo intellectilium formarum imagines sunt? Quo aut pacto inter individuam dividuamque naturam medium sunt, nullum a primis supplementum sortita? Quo modo universorum prima exempla /fol. 6r/ in intellectu prius insederunt? Si vero ab illo solum, ille actus et motus per se, qui in anima est, quo modo permanere potest? Quandoquidem quae in ipsa sunt rationes, per essentiam eorum, quae ab alio moventur, in ipsam aliunde influxerunt, etenim quo differt a materia, quae potentia solum omnia est, materiosam[151] autem formam nullam gignit. Relinquitur[152] ergo, quod a se ipsa et ab intellectu haec deducat, et ab ipsa earum formarum sit supplementum, quae ab intelectilium exemplis subsistunt ex se ipsa semita, ut sit gressus. Neque[153] proinde anima tabula non romibus vacua, sed semper descripta, et semet ipsam describens, et ab intellectu descripta. Intellectus enim est et anima, qui cum semet ipsum vertat, illius imago et forma extrorsum fit. Igitur [154] si ille omnia intellectibiliter, anima quoque cuncta animanter; et si ille exemplariter, anima etiam imaginiter; et si ille collectim, anima divisim.

Quo[155] quidem Plato etiam inspirans, animam ex omnibus formis mathematicis constituit, impsamque per numeros dividit, proportionibus rationibusque convenientibus colligit, primaque principia figuras constituentia in ipsam constituit, et rectum et circundans, ac in ipsi orbis intellectiliter movet. Igitur[156] omnia mathematica primo in ipsa sunt anima, et [157] ante numeros se ipsos moventes, et ante figuras apparentes figurae animales, et ante ea quae conceptu conveniunt rationes harmonicae, et ante illa corpora quae circulose moventur, orbes non apparentes creantur, et[158] anima omnium est plenitudo. Et ornamentum huiusmodi alius, idem semet ipsis deducens, et a proprio principio deductum se

[151] materiosam] 43 Materia solum potentia omnia est *add. in marg.* M
[152] Relinquitur] 44 Anima et a se et ab intellectu formas mathematicas deducit *add. in marg.* M
[153] Neque] 45 Anima non est tabula vacua, sed semper diseri ipsa *add. in marg.* M
[154] Igitur] 46 Intellectus et anima qualiter mathematicae percepiunt *add. in marg.* M
[155] Quo] 47 Anima ex omnibus formis mathematicis constitui iuxta Platonem *add. in marg.* M
[156] Igitur] 48 Mathematicae primo insunt anima *add. in marg.* M
[157] Et] 49 Harmonica ratio quod sit Nicomachus docet in secundo libro Aristotelicus
[158] et] 50 Anima est plenitudo omnium *add. in marg.* M

ipsum vita perficiens, et ab opifice refertum, corporis expers est, ac intervallo caret, et qui suas suppeditat rationes, tunc scientias ac virtutes omnes patefacit. Anima[159] igitur in huiusmodi formis essentiam summit, neque admittendus quod numerus in anima sit unitatum multitudo, neque etiam percipiendus est, quod idea eorum quae intervallata sunt, /fol. 6v/ in corporis modum intelligantur, sed omnia exempla numerorum apparentium, figurarum, rationum et motuum, in animato intellectiliterque suscipere debemus, Timaeum sequentes, qui [160] universam ipsius genituram et opificium ex mathematicis formis perficit in ipsaque causas omnium statuit. Omnia enim numerorum septem limites linearum, planorum, ac solidorum principia comprehendunt, septem vero limites omnium rationum, quo ad ipsam causam in ipsa praeexsitunt. Principia vero figurarum in ipsa operando straverunt, ex motibus vero ille praecipuus est omnes qui alios comprehendit ac movet, qui cum ipsam simul constat. Omnium namque[161] eorum quae moventur, circulus motusque circulosus principium est. Mathematicorum igitur rationes substantiales et ex se mobiles animas perficiunt, quas cogitatio suppeditans ac revolvens universam mathematicarum scientiarum varietatem statuit, et semper gignens, numquam desinit aliaque ex aliis investigans, ipsius impartibiles rationes explicat. Omnia enim a primis formis assumpsit, ex suique potestate infinita ex principiis primis assumptis variarum inspectionum diverticula efficit.

[159] Anima] 51 Numerus in anima non est unitatum multitudo *add. in marg.* M
[160] qui] 52 Anima ex formis mathematicis *add. in marg.* M
[161] namque] 53 Circulus eorum omnium quae moventur principium est *add. in marg.* M

3. Giambattista Gabia transl. *Procli In Euclidem*, MS Ambrosiana P51 sup., fol. 11r–17v [= A]

Johanne Baptista Gabio interprete, Procli Diadochi in primum Elementorum Euclidis liber primus.

/fol. 11r/ Capitulum quintum

Sequitur ut inspiciamus quaenam mathematicarum formarum[162] generumque natura[163] ºmerito dicatur, et utrum ab iis, quae sensu percipiuntur[164], ipsam profectam consistere[165], concedendum sit sive secundum ablationem[166], ut quidam dicere consueverunt, sive secundum singularium[167] collectionem in unam communem rationem? An etiam ipsi ante haec status sit tribuendus? Quemadmodum et Plato censet, et universalium demonstrat progressus.

Primum igitur[168], si ab iis, quae suspercipimus[169], mathematicas formas dicamus statum habere atque produci /fol. 11v/ cum anima ab iis triangulis aut circulis, qui in materia sunt, formam circularem aut triangularem posteriore loco in seipsa formet, undenam perfectio ut id quod minime convinci possit rationibus inserit? Necesse est enim ut vel a sensibus vel ab anima. Sed a sensibus fieri non potest, quibus potius hanc[170] perfectionem impartiuntur. Ab anima igitur, quae imperfectis perfectum, incertis certum apponit. Ubi enim in iis quae sensu percipiuntur[171] reperiemus id nos quod sine parte sit, aut id quod careat latitudine, aut id quod altitudine vacat? Et ubinam linearum, quae sint a centro, aequalitas? Et ubi semper permanentes laterum rationes? Atque ubi angulorum rectitudi/fol. 12r/ni<s> horum[172] rationes? Nam videmus inter omnia sub sensu communia[173] commista esse nihilque in iis syncerum aut a contrario vacuum, sed omnia[174] dividua, dimensionibus ac motui obnoxia. Qua ratione igitur[175] immobilibus ab iis, quae moventur, et alias aliter se habent, ipsam manentem stabilemque[176] naturam

[162] formarum *post corr. ex* specierum A
[163] natura *post corr. ex* substantia A
[164] [a]b iis que sensu principiuntur *post corr. ex* sensibilibus A
[165] Pro [...] consistere *post corr. ex* subsistere A
[166] secundum ablationem *post corr. ex* per divisionem A
[167] secundum singularium *post corr. ex* per particularium A
[168] igitur *post corr. ex* itaque A
[169] ab iis que suspercipimus *post corr ex* sensibilibus A
[170] hanc] haec A
[171] sensibilibus *post corr. ex* iis quae quae sensu percipiuntur A
[172] Rectitudini<s> horum *post corr. ex* rectidudines A
[173] Omnia sub sensu communia *post corr. ex* sensibilia A
[174] omnia] *add. illig. sed del.* A
[175] igitur] igiturum *corr. sup. lin ex* itaque A
[176] stabilemque] substantiam et *add. sed. del.* A

tribuimus? Quidquid enim producitur ab iis naturis[177], quae moventur, etiam naturam[178] mutabilem habere omnes fatentur[179]. Et quomodo perfectis ac certis, ita ut coargui nequeant, formis ab incertis perfectionem adhibebimus? Quippe cum omnis causa cognitionis illius, quae semper mobilis est, magis ipsa talis existat. Concedendum igitur[180] nobis est animum[181] esse /fol. 12v/ qui[182] mathematicas formas et rationes gignat et procreet. Et si exempla habens secundum naturam[183], ipsas producit generationesque sunt formarum, quae in ipsa prius insunt, progressus, et [184] cum Platone consentimus [185] eadem dicentes, et veram disciplinarum naturam invenerimus. At si haec non habens, cum neque ante rationes acceperit, tantum subtexit ordinem materia vacantem tantamque speculationem gignit[186], quomodo ea, quae genita[187] sunt, diiudicare potest, utrum naturalia sint an vana, et simulachra pro veris? Quibusve regulis utens eam, quae in iis est, veritatem obstruat? Quomodo item non continens[188] ipsorum naturam[189] tantam rationum /fol. 13r/ gignit varietatem? Ita enim temere et per se natam ipsorum naturam[190] faciemus et quoad nullum terminum refixatur. Si igitur[191] ab animo [192] sunt mathematicae formae oriundae, non ab iis quae sensu percipiuntur [193], rationes habent, quas animus [194] producit, et ab illis ipsae progrediuntur, atque ipsius animi[195] partus ac foetus permanentium formarum ut sempiternarum perspicui[196] sunt.

Deinde, igitur, [197] si ab inferioribus, id est, quae sub sensum cadunt [198], disciplinarum rationes colligamus, qui fieri potest quin praestantiores illas

[177] ab iis naturis *post corr. ex* substantiis A
[178] naturam *post corr. ex* substantiam A
[179] omnes fatentur *post corr. ex* apud omnes est confessum A
[180] igitur *post corr. ex* itaque A
[181] animum *post corr. ex* animam illam A
[182] qui *post corr. ex* quae A
[183] naturam] has *add. sed del.* A
[184] et *post corr. ex* atque ita A
[185] con- *add. sup. lin.* A
[186] gignit *post corr. ex* generat A
[187] genita *post corr ex* generata A
[188] continens *post corr. ex* habens A
[189] naturam *post corr. ex* substantiam A
[190] naturam *post corr. ex* substantiam A
[191] Igitur *post corr. ex* itaque A
[192] genita *post corr ex ex* animo A
[193] ab iis que sensu percipiuntur *post corr. ex* etiam a sensibilibus A
[194] animus *post corr. ex* anima A
[195] animi *post corr. ex* animae A
[196] perspicui *post corr. ex* manifesti A
[197] igitur *post corr. ex* itaque A
[198] Quae sub sensum cadunt *post corr. ex* sensibilibus A

demonstrationes asseramus, quaecumque ab iis, quae sensu percipimus[199], et non illas quae a magis universalibus semper et simplicioribus formis constituuntur? Causas enim ubique demonstrationibus proprias esse asserimus ad rei quae quaeritur inquisitionem. Si igitur[200] particularia /fol. 13v/ universalium, et quae sub sensum cadunt[201], quae cogitatione percipiuntur, causae sunt, quomodo possumus demonstrationis definitionem ad universalia magis quam ad particularia referre? Et eorum quae cogitatione comprehendimus naturam magis quam eorum quae sensu, demonstrationibus cognatam demonstrare? Neque enim si quis, ut aiunt, demonstraret ex duabus aequalibus lineis constantem[202] triangulum quem aequicrurium vocant, habere tres angulos aequales duobus rectis, et triangulum tribus aequalibus lineis contentum, quem aequilaterum appellamus, et similiter tribus inaequalibus, quem dicunt scalenum[203], hic rectum scit. Sed qui de omni triangulo, id et simpliciter demonstravit, hic habet scientiam per se. Rursusque quia praestantius est universale particulari ad demonstrationem, atque item quia /fol. 14r/ demonstrationes ex universalibus magis, ex quibus autem demonstrationes proficiscuntur,[204] haec priora et natura praecedunt particularia, et causae sunt eorum quae demonstrantur. Multum igitur abest ut demonstrativa scientia in iis[205] vere sci<a>tur, ea quae posteriore loco genita sunt, et iis quae sensu percipiuntur[206] hebetioribus[207], et non ea speculentur[208], quae cogitatione comprehendi possunt, perfectiora sane iis, quae sensui nota sunt et opinioni.

Praeterea tertiam rationem afferimus, quod qui haec asserunt, illi ignobiliorem animum[209] formis efficiunt. Nam si materia naturalia, et quae magis sunt atque certiora suscipit a natura, animus[210] vero posteriora ab illis et /fol. 14v/ simulachra et imagines secundo loco genitas in se ipsa confingit,[211] viliorem naturam[212] auferens a materia, videlicet[213] ea quae secundum naturam ab ipsa separari

199 [a]b iis quae sensu percipimus *ex* sensibilibus
200 igitur *post corr. ex* itaque A
201 Quae sub sensum cadunt *post corr. ex* sensibilia A
202 ex duabus aequalibus lineis constantem *post corr. ex* aequicrurium A
203 tribus [...] rectum *post corr. ex* aequilaterum et scalenum similiter hic rectum A
204 proficiscuntur *post corr. ex* procedunt A
205 in iis *post corr. ex* circa ea A
206 iis quae sensu percipiuntur *post corr. ex* sensibilia quae sunt A
207 hebetioribus *post corr. ex* hebetiora A
208 speculentur] que *add. sed del.* A
209 animum *post corr. ex* animam A
210 animus *post corr. ex* anima A
211 confingit propter *add. sed del.* A
212 naturam *post corr. ex* substantiam A
213 videlicet *post corr. ex* scilicet A

nequunt [214] , quomodo non animum [215] imbecilliorem materia deterioremque constituunt? Locus enim etiam materia rationum, quae materia carent, et animus[216]formarum. Sed haec primarum, illa secundarum, et haec quidem eorum quae primo loco insunt, illa rerum quae illinc producuntur, et haec quidem quae sunt secundum naturam, illa eorum quae cogitatione gignuntur. Qua ratione igitur[217] animus[218], qui[219] mentis et intelligentis naturae primus[220] consors est, et cogitationis quae illinc /fol. 15r/ proficiscuntur, totius vitae particeps, incertiorum formarum est receptaculum quae sedem ultimam inter ea quae sunt obtinet, et omnium minime[221] imperfectionem perfectam[222], quatenus ad hoc, ut sint, attinet? Sed huic opinioni, quae multis saepe multas arguendo praebuit occasiones, occurrere supervacaneum est.

Quod si neque per ablationem ab iis quae materia vacant sunt mathematicae formae, neque per collectionem eorum, quae in particularibus communia sunt, neque prorsus posteriore loco, id est, ab iis quae sub sensum cadunt[223], genita, necesse sane est animum[224], aut ab se, aut ab mente[225] ipsas accipere, aut et a se et ab illa. Sed si a se tantum, quomodo hae sunt intelligentium formarum imagines? Et quomodo inter individuam naturam dividuam, quae nullam a primis, in gratia[226] ut sint, perfectio [227] /fol. 15v/nem sortita sunt? Et quomodo prima exempla universalium, quae in mente[228] sunt, ante fuere? Et si ab illo tantum, quomodo id, quod est animi[229] per se operans et per se mobile, permanere potest, siquidem rationes, quae in ipso[230] sunt, secundum eorum naturam[231], quae ab aliis moventur, aliunde in ipsam defluxere? Et quid a materia differet, quae potestate tantum est omnia, nullamque gignit ex iis formis, quae materiae sint immunes? Relinquitur igitur, ut et a se, et a mente has producat, ipsa sit formarum perfectio, id est, ipsas

214 Ab ipsa separari nequuunt *post corr. ex* inseparabilia
215 animum *post corr. ex* animam A
216 animus *post corr. ex* anima A
217 igitur *post corr. ex* itaque A
218 animus *post corr. ex* anima A
219 qui] quae *scrp. sed del.* A
220 primus] primae A
221 omnium minime add. sup. lin.; minime] perfectionis *add. sed del.* A
222 perfectam *post corr. ex* omnibus
223 ab iis quae sub sensum cadunt *post corr. ex* sensibilibus A
224 animum *post corr. ex* animam A
225 ab mente *post corr. ex* intellectu A
226 in gratia *post corr. ex* ad hoc A
227 perfectio *post corr. ex* completio A
228 mente *post corr. ex* intellectu A
229 animi *post corr. ex* animae A
230 ipso *post corr. ex* ipsa A
231 naturam] et substantiam *add. sed del.* A

compleat, quae quidem ab intelligentibus exemplis productae per se[232] ut sint, progressum sortitate fuerint. Et animus[233] non erit tabula rudis[234] a sermoni/16r/bus vacua[235], sed semper scriptus[236] et se ipsum[237] scribens et a mente[238] scriptus.[239] Etenim animus[240] quoque mens est, more illius mentis qui[241] est ante se, se ipsum volvens, et imago illius, et typus extrinsecus effectus. Si igitur[242] ille omnia intelligenter[243], et animus[244] omnia animaliter; si ille exemplariter, animus[245] repraesentative; et si ille coniunctim, anima divisim.

Quod cum animadverterit et Plato, ex omnibus animum[246] producit, et secundum numeros illum[247] dividit, colligitque[248] analogiis et harmonicis rationibus, et prima figurarum principia in ipso[249] construit, rectum scilicet et circumferens, movitque[250] circulos qui in ipsa insunt, intelligendo. Omnia itaque mathematica primum in animo[251] sunt, et ante numeros nu/fol. 16v/meri illi qui per se moventur, ante apparentes figuras animales figurae, et ante ea quae concinnata sunt, rationes harmonicae, et ante corpora quae circulo moventur minime apparentes circuli veluti opificio quodam constructi sunt. Et perfectio omnium animus[252] est, et ordo hic alius ipse seipsum producens et a proprio principio productus, vitaeque seipsum complens, et ab opifice completus sine[253] corpore, et sine[254] dimensionibus, et cum producerit suas rationes tunc etiam disciplinas[255] virtutesque omnes praemonstrat. Naturam igitur[256] in his formis

[232] per se] ad hoc *add. sed del.* A
[233] animus *post corr. ex* anima A
[234] tabula] rudis *add.* A
[235] vacua *post corr. ex* vacuus A
[236] scriptus *post corr. ex* scripta A
[237] ipsum *post corr. ex* ipsa A
[238] a mente *post corr. ex* ab intellectu A
[239] scriptus *post corr. ex* scripta A
[240] animus *post corr. ex* anima A
[241] qui *post corr. ex* quae A
[242] igitur *post corr. ex* itaque A
[243] intelligenter *post corr. ex* intellectualiter A
[244] animus *post corr. ex* anima A
[245] animus *post corr. ex* anima A
[246] animum *post corr. ex* animam A
[247] illum *post corr. ex* illam A
[248] colligi *post corr. ex* colligat A
[249] ipso *post corr. ex* ipsa A
[250] movitque *post corr. ex* movetque A
[251] animo *post corr. ex* anima A
[252] animus *post corr. ex* anima A
[253] sine *post corr. ex* absque A
[254] sine *post corr. ex* absque A
[255] disciplinas *post corr. ex* scientias A
[256] naturam igitur *post corr. ex* substantiam itaque A

habet animus[257] sed neque numerum in ipso[258] imitatum multitudinem opinari debemus, neque eorum, quae sub mensuram cadunt[259], ideam secundum corpus in/17r/ telligere, sed omnia apparentium numerorum exempla et figurarum et rationum et motuum accipere debemus, Timaeum secuti, qui quidem omnem ipsius animi [260] originem, et creationem a mathematicis formis perfecit omniumque causas in ipso[261]constituit. Siquidem numerorum omnium rationes secundum causam in ipso[262] prius constituere, figurarumque item principia quasi per opificium ipso sunt constituta. Et motus qui inter motus caeteros principatum obtinet, quique alios omnes continet, et movet in ipso[263] subsistit. Nam omnium quae moventur principium circulus est, et circularis motus. Naturales[264] igitur etiam illae, quae per se moventur mathematicorum rationes animas perficientes, quas /fol. 17v/ cogitatio proferens atque evolvens omnem producit mathematicarum disciplinarum [265] varietatem. Neque ullo unquam temporum desistet semper procreans [266] aliaque ad alia adinvenire suasque individuas [267] rationes explanare. Nam cum omnia in se ipso[268] praeceperit secundum principem formam, ob infinitam suam facultatem ex praesumptis principiis cuiuscumque generis theoremata producit.

[257] animus *post corr. ex* anima A
[258] ipso *post corr. ex* ipsa A
[259] eorum quae sub mensuram cadunt *post corr.* dimensionum A
[260] animi *post corr. ex* animae A
[261] ipso *post corr. ex* ipsa A
[262] ipso *post corr. ex* ipsa A
[263] ipso *post corr. ex* ipsa A
[264] naturales *post corr. ex* substantiales A
[265] disciplinarum *post corr. ex* scientiarum A
[266] procreans *post corr. ex* generans A
[267] individua s*post corr. ex* indivisibiles A
[268] ipso *post corr. ex* ipsa A

4. Francesco Barozzi transl. *Procli Diadochi Lycii philosophi platonici ac matematici probatissimi in primum Euclidis Elementorum librum commentariorum ad universam mathematicam disciplinam principium eruditionis tradentium libri IIII*, Patauii, excudebat Gratiosus Perchacinus, 1560, Liber primus, Cap. VI, p. 6–10

/fol. 6/ Quae nam sit mathematicorum generum ac formarum essentia et quomodo subsistat, Cap. VI.

Sequitur [269] autem ut consideremus quaenam dicenda sit mathema/fol. 7/ticarum formarum generumque essentia, et[270] utrum a sensilibus ipsam manere in rerumque natura subsistere sit admittendum sive per abstractionem, ut dici solet, sive per collectionem particularium in communem unam rationem, an et ante haec ipsam subsistere fatendum, ut asserit Plato omniumque rerum progressus ostendit.

Primum itaque, si a sensilibus mathematicas formas oriri subsistereque dicimus, anima quidem nostra a triangulis vel circulis in materia insidentibus, circularem vel triangularem formam postremo in se ipsa formante, unde accurata illa vis et certitudo illa, quae coargui convincique minime potest, rationibus inest mathematicis? Haec enim aut a sensilibus aut ab anima eruantur necesse est. Atqui a sensilibus haec educi est impossibile; multo [271] enim maior certitudo illis concedenda esset. Ab ipsa igitur anima educentur, quae imperfectis quidem perfectionem, iis autem, quae certa non sunt, quod certum sit adhibet. Ubi namque in eis, quae sub sensum cadunt, impartibile, vel latitudinis expers, aut crassitudinis percipi potuerit? Ubi porro ex circuli centro exeuntium linearum aequalitas? Ubi semper stabiles laterum rationes? Ubi angulorum rectitudines? Non equidem video. Siquidem omnia, quae sub sensum cadunt, invicem commista sunt nullumque in his syncerum reperitur, quod a contrario purum sit, sed cuncta partibilia, et dimensionum plena, et motui obnoxia existunt. Quonam modo igitur immobilibus rationibus ex iis, quae moventur, et alio atque alio tempore aliter se habent ipsam immutabilitatem firmamque attribuemus essentiam? Quidquid enim ab iis, quae moventur, ortum ducit essentiis, mutabilem ex ipsis habere existentiam nemo est qui non fateatur. Quonam demum pacto certis et quae minime coargui possunt formis a non certis certitudinem adiiciemus? Quicquid enim immobilis cognitionis est causa, magis illud tale est. Confessum[272] igitur ac

[269] Sequitur] Quaestio. *add. in marg. ed.*

[270] et] Prima opinion, quae est Aristotelis. Secunda opinio, quae est Platonis. Primae opiniones confutatio. Argumentum. *add. in marg. ed.*

[271] Multo] Certitudo mathematica ab anima ipsa emanat. *add. in marg. ed.*

[272] Confessum] Conclusio argumenti. Alia quaestio. Prima opinio, quae est Platonis. Secunda opinio, quae est Aristotelis, eiusque confutatio. Primum argumentum. *add. in marg. ed.*

receptum sit animam formarum rationumque mathematicarum esse genitricem. Verum siquidem habens exempla secundum essentiam constituit eas, et sunt huiuscemodi ortus quaedam earum, quae in ipsa praexistebant, formarum emissiones, et Platoni astipulabimur haec dicentes, et vera nobis mathematicarum disciplinarum essentia erit inventa. Si vero non habens neque cum rationes praeoccuparit, tantum subtexit ornatum materiae expertem tantamque gignit contemplationem, quomodo quae genita sunt diiudicare potest, sintne vitalia aut subuentanea et simulacra pro veris? Quibus autem regulis utens veritatem, quae in his est, metitur? Quo demum pacto essentiam ipsorum non habens tantam rationum pro/fol. 8/ducit varietatem? Vagam quippe et incertam ita horum faciemus substantiam quaeque ad nullum terminum referatur. Si[273] igitur anima mathematicas gignit formas, neque a sensilibus rationes habet, quibus eas constituit, ab illis tamen ipsas producit, ipsius utique animae partus ac foetus permanentes aeternasque patefaciunt formas.

Secundo[274], si inferius et a sensilibus mathematicas colligimus rationes, quonam modo necesse non fuerit potiores eas perhibere demonstrationes, quaecunque a sensilibus constituuntur et non eas quae a magis universalibus simplicioribusque formis? Causas enim ubique demonstrationibus esse proprias ad eius, quod quaeritur, venationem dicitur. Si igitur particularia et sensilia universalium et sub cogitationem cadentium causae sunt, quid causae est, quod demonstrationis definitio ad magis universalia vice particularium referatur? Et eorum, quae cogitationi subiciuntur, essentia potius quam sensilium essentia cognatior demonstrationibus magisque affinis ostendatur? Nam neque si quis, ut dici solet, demonstravit aequicrus duobus rectis aequales habere angulos, et aequilaterum, et scalenum, is quodammodo scit. Sed qui omne triangulum et simpliciter demonstravit, per se scientiam habet. Et rursus quod universale est, melius est ad demonstrationem quam particulare, itemque demonstrationes ex magis universalibus constant atque conflantur. Ex quibus autem sunt demonstrationes, ea priora sunt, et singularibus natura praecellunt, suntque causae eorum quae demonstrantur. Multum[275] igitur abest ut quae demonstrandi vim habent scientiae posterius genita obscurioraque sensilia respiciant atque scrutentur, non autem ea contemplentur, quae a cogitatione comprehenduntur, quaeque perfectiora sunt iis, quae a sensu opinioneque cognoscuntur.

Tertio[276] autem adhuc dicimus, quod animam quoque materia ignobiliorem faciunt, qui haec aiunt. Nam si materia quidem essentialia quaeque magis esse dicuntur manifestioraque a natura accipit, anima vero secundo loco ab illis et simulachra et imagines posterius eductas in sese informat in essentiam minus

[273] Si] Conclusio primi argumenti. *add. in marg. ed.*
[274] Secundo] Secundum argumentum. *add. in marg. ed.*
[275] Multum] Conclusio secundi argumenti. *add. in marg. ed.*
[276] Tertio] Tertium argumentum. *add. in marg. ed.*

honoratam, auferens a materia, quae suapte natura ab ipsa separari non possunt, quomodo animam imbeciliorem inferioremque materia non ostendunt? Tum enim materia rationum materialium, tum anima formarum est locus. Sed primarum altera, altera secundarum, et illa quidem earum, quae praecipue sunt, haec vero earum, quae secundum essentiam, haec vero earum, quae secundum excogitationem factae sunt. Quonam pacto igitur anima, quae mentis intelligentisque essentiae primo est particeps, et hinc co/fol. 9/gnitione, totaque vita repletur, obscuriores recipit formas iis, quae ab ultima eorum, quae sunt, et quoad esse omnium imperfectissima recipiuntur sede? Verum[277] enimvero huic quidem occurrere opinioni, quae saepe a plerisque exagitata ac convicta fuit, supervacaneum fuerit.

Quodsi neque per abstractionem materialium mathematicae formae sunt, neque per collectionem eorum, quae in singulis sunt communium, neque prorsus posterius genitae et a sensilibus, necesse[278] est utique animam aut a se, aut a mente, aut et a se et a mente ipsas accipere. At siquidem a se dumtaxat, quonam modo hae intellectilium erunt formarum imagines? Quomodo inter impartibilem partibilemque naturam fuerint mediae, nullam a primis quoad esse perfectionem sortitae? Quomodo demum ea, quae in mente sunt, primaria omnium sunt rerum exempla? Si vero ab illa tantum, quo pacto vis illa exercendi sui, ac movendi sui, quae in anima est, permanere poterit? Siquidem quae in ipsa sunt rationes iuxta eorum, quae ab alio moventur substantiam aliunde in ipsam fluxere? Praeterea, in quonam anima ab ipsa differet materia, quae potentia solum est omnia nullamque prorsus formarum materialium gignit? Reliquum est igitur animam a se, et a mente hasce producere, ipsamque formarum plenitudinem esse, quae ab intelligentibus quidem exemplis oriuntur, ex sese autem ad esse transitum sortiuntur. Non[279] est igitur tabella rationibusque vacua ipsa anima, immo semper scripta, seseque suapte natura describens, cum a mente quoque describatur. Nam [280] anima etiam ipsa mens est, iuxta mentem ipsa priorem seipsam convolvens, imagoque illius et adumbratio extrinsecus facta. Si igitur illa cuncta intelligendo cognoscit, anima quoque cuncta animando; et si illa per exempla, et anima per imagines; et si illa contrahendo, anima distinguendo.

Quod [281] nimirum Plato quoque sciens, animam ex omnibus mathematicis constituit formis, eamque dividit per numeros, et connectit proportionibus harmonicisque rationibus, et primaria figurarum principia in ipsa defigit, rectum

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277 Verum] Conclusio trimembris ex iis quae dicta sunt. *add. in marg. ed.*

278 necesse] Primum membrum. Secundum. Tertium. Primi membri confutatio. Secundi membri confutatio. Primum argumentum. Secundum. Tertii membri confirmatio. Conclusio. *add. in marg. ed.*

279 Non] Digressio contra Aristotelem. *add. in marg. ed.*

280 Nam] Cognitio animae differt a cognitione mentis. *add. in marg. ed.*

281 Quod] Plato in Timeo animam ex omnibus mathematicis formis constituit. *add. in marg. ed.*

inquam, et circulare, et circulos in ipsa existentes ciet intelligenter. Cunctae igitur res mathematicae primum in ipsa sunt anima, et ante numeros numeri, qui per se moventur, et ante apparentes figuras figurae animales[282], et ante ea, quae concinnatae sunt, harmonicae rationes, et ante corpora, quae circulariter moventur, invisibiles circuli producti sunt. Horumque omnium ubertas ipsa est anima, et iste ornatus alius est, qui seipsum producit et a proprio producitur principio, et vita seipsum explet, ab opificeque sine corpore ac sine dimensione expletur. Et quando suas promit ra/fol. 10/tiones, tunc omnes patefacit scientias atque virtutes. His itaque formis anima suam induit essentiam. Nec est numerus in ipsa unitatum multitudo existimandus, neque eorum, quae cum dimensione sunt ideo corporaliter intelligenda, sed vitaliter et intelligenter omnia apparentium numerorum, et figurarum, et rationum, et motuum exempla supponenda sunt, Timaeum sequendo, qui omnem ipsius ornatum atque creationem ex formis complevit mathematicis, omniumque causas in ipsa collocavit. Nam omnium quidem numerorum linearium, et planorum, et solidorum septem termini principia comprehenderunt, rationum vero omnium septem rationes secundum essentiam[283] in ipsa praeextiterunt, figurarum autem principia secundum opificam vim in ipsa collocata sunt. Motuum denique primus, qui caeteros alios comprehendit et movet, una cum ipsa subsistit. Omnium enim eorum, quae moventur, circulus motusque circularis principium est. Essentiales igitur et per se mobiles mathematicarum rerum sunt rationes, animas complentes, quas utique rationes promovens provolvensque cogitatio omnem mathematicarum scientiarum varietatem constituit. Nec unquam quiescet, gignens quidem semper, aliaque post alia inveniens, suas autem individuas rationes explicans. Cuncta siquidem primarie praeoccupavit, et secundum infinitam sui vim ex praesumptis principiis varia producit proponitque theoremata.

[282] animales] vitales *add. in marg. ed.*
[283] essentiam] causam *add. in marg. ed.*

5. Johannes Kepler, *De harmonices Libri V*, excudebat Ioannes Plancus, 1619, p. 114–117

/fol. 114/ Huius [284] igitur Procli philosophiam de speciebus rerum mathematicarum, quas ego terminos profiteor proportionis harmonicae purae et secretae a sensilibus, operae pretium est ex eius libro I in Euclidem hunc transcribere de verbo ad verbum:

Sic ille: *Reliquum est ut videamus quaenam subsistentia seu essentia debeat assignari mathematicis generibus et speciebus. Concedendumne sit illa a sensilibus subsistentiam accipere sive per abstractionem, ut mos est loquendi, sive per collectionem eorum quae sunt per partes dispersa in unam communem rationem* (seu definitionem)*, anne etiam ante haec ipsis danda subsistentia, ut Plato usurpat, et universitatis rerum progressio* [285] *demonstrat?*

Primum itaque, si affirmamus species mathematicas a sensilibus constitui dum anima a trigonis aut circulis materialibus, speciem circularem aut trigonicam secundaria quadam genitura in se ipsa format, quaero un/fol. 115/*de veniat rationibus* (seu definitionibus) *illa tanta certitudo tantaque accuratio? Erit enim aut a sensilibus aut ab ipsa anima. At impossibile a sensilibus, multo namque maior subtilitas et exactio inest rationibus istis. Ab anima igitur, quae imperfectis perfectionem minime accuratis subtilitatem illam accuratam conciliet. Dic enim ubi inter sensilia inveniatur natura impartibilis* (puncti) *et latitudine carens* (ut linea) *et profunditates* (ut superficies)? *Aut ubi aequalitas linearum ex centro? Aut ubi proportiones semper constantes laterum* (materia mei libri I) *aut ubi angulorum rectitudines? Non equidem video, ut sunt commixta et confusa inter se omnia partibilia, nihil sincerum, nihil a contrario suo purum, omnia partibilia et quae locis dissita et quae unita. Quomodo igitur immobilibus ipsam durabilem essentiam ex mobilibus aliasque aliter habentes conciliabimus? Quicquid enim subsistentiam suam habet a motis essentiis, habere eam mutabilem concedunt ipsi. Et quomodo accuratis et certis speciebus a non accuratis impetrabimus accuratiam suam? Omne namque quod causa est notionis semper mutabilis, ipsum multo magis talis est. Supponendum ergo erit animam ipsam genitricem esse mathematicarum specierum et rationum. Atqui si illa continens in se ut prima exempla seu paradigmata* [286] *secundum essentiam ipsa subsistere facit, ut ita sint generationes* (Christianus subaudit: creatio rerum sensilium) *nihil aliud quam propagationes specierum quae in illa prius inerant* (rationes creandorum corporum mathematicas Deo coaeternas fuisse, Deumque animam et mentem esse superexcellenter, animas vero humanas esse Dei creatoris imagines [287], etiam in essentialibus suo modo, id sciunt Christiani) *tunc Platoni consentiemus haec dicentes,*

[284] Huius] Procli philosophia de essentia rerum mathematicarum *add. in marg. ed.*
[285] progressio] Species mathematicas continuas non esse singularibus sensilibus quantis *add. in marg. ed.*
[286] Paradigmata] Vera mathematicarum rerum essentia in anima *add. in marg. ed.*
[287] Imagines] imaniges *ed.*

et vera essentia mathematum nobis inventa erit. Sin autem anima cum non haberet nec prius alicunde accepisset rationes mathematicas (si , cum non fuissent ipsi concreatae), *nihilominus admirabilem hunc ornatum immateriatum texit, pulcherrimam hanc speculationem enititur, quomodo ergo discernet sic genita, sintne subsistentia et constantia* (μόνιμα non γόνιμα lego) *an in ventos evanida, et spectra potius quam vera? Quibus normis usa veritatem eorum metietur? Immo quomodo, cum essentiam eorum non haberet, gignit tantam varietatem rationum? Nam hoc pacto fortuitam illorum subsistentiam faciemus, neque ad ullum scopum vel terminum tendentem. Sunt ergo animae ipsius soboles species mathematicae, nec habet illa rationes quas constituit a sensilibus, quin potius haec ab illis propagantur, suntque hi enixus ipsius et haec puerperia permanentium et perennium specierum evidentia*[288].

Secundo, si ab infra et a sensilibus colligimus rationes mathematum (seu definitiones) *quomodo non meliores erunt demonstrationes quas constituunt sensilia demonstrationibus universaliorum et simpliciorum specierum? Dicimus enim ad quaesitae rei investigationem, ubique principia et propositiones esse demonstrationibus seu conclusionibus cognatas. Si igitur singularia universalibus, si sensilia ratiocinativis sunt causae, quomodo fieri potest ut demonstrationis terminus ad magis universalia, non ad partialia referatur, utque*[289] *intellectualium quam sensilium essentiam cognatiorem demonstrationibus probemus? Sic enim loquuntur: non si quis demonstrat aequicrurum habere angu*/fol. 116/*los duobus rectis aequales, idem aequilaterum, idem scalenum, hoc scire est legitimum. Sed is scientiam proprie dictam habebit qui de omni triangulo simpliciter hoc demonstraverit, item universalia meliora esse ad demonstrationem quam particularia. Amplius, magis universalium esse demonstrationes ex quibus vero constant demonstrationes, illa priora esse et natura praecedere singularia et causas esse eorum quae probantur. Multum igitur absunt scientiae apodicticae ab hoc, ut circa posteriora ortu et obscuriora sensilia propositiones suas mendicato colligant.*

Dicam insuper et hoc tertium: illos animam viliorem facere ipsis speciebus, qui haec dicunt. Nam si materia suscipit a natura illa quae sunt essentialia et magis entia et evidentiora, anima vero posteriori actu accipit ab illis, exemplaque et imagines ortu posteriori fingit in se ipsa respiciens ad essentias viliores et abstrahens a materia, quae secundum ipsius naturam sunt inseparabilia, nonne animam ipsa materia obscuriorem indigentioremque faciunt? Nam et materia locus est materiatus rationibus, sicut anima speciebus (immateriatis). *Esset vero illa locus prioribus, ista secundariis, et illa ducatum obtinentibus in essendo, ista subsistentiam habentibus ab illis. Denique, illa factis secundum essentiam, ista nuncupatis secundum intentionem. Quomodo igitur anima, mentis et intellectualis primae essentiae particeps, et quae cognitionem inde consummatam habet, quomodo, inquam, etiam obscurissimarum totius vitae specierum erit receptaculum,*

[288] evidentia] Mathematicas species non esse collectas in unum a singularibus quantis *add. in marg. ed.*

[289] utque] Allegationes ex Aristotele *add. in marg. ed.*

infimi scilicet in rebus gradus omniumque imperfectissimi ad essendum? Verum hanc opinionem oppugnare pluribus, quae dudum a multis est flagellata ex merito, supervacaneum existimo.

Quod si non existunt mathematicae species per abstractionem materiatorum neque per collectionem eorum, quae insunt in singularibus communia, neque omnino sunt ortu posteriores aut ex sensilibus, necesse utique fuerit ut anima[290] *vel a se ipsa, vel a mente transsumat illas, vel denique et a se et a mente simul. Verum si a se sola, quomodo erunt imagines intellectualium specierum, quomodo erunt mediae inter partibilem et impartibilem naturam, cum nullam a primis consummationem ad essendum sortiantur? Quomodo denique prima exempla, paradigmata vel ideae, quae sunt in mente, sunt universorum principes? Sin vero a mente sola transsumuntur in animam, quomodo anima manebit ipsa seipsa operans, seque ipsam movens, si rationes quae sunt in ipsa ad normam subsistentiae illarum rerum, quae ab alio cientur, influxerunt in ipsam aliunde? Et quid differet anima a materia, quae solum potestate est omnia, nihil vero de materiatis speciebus generat? Restat igitur ut anima et a se ipsa, et a mente transsumat ista, et ut ipsa sit absoluta consummatio specierum, quae ab intellectualibus primis exemplis seu paradigmatibus a seipsis genitis subsistentiam habent, et accessum ad ipsum ESSE*[291] *sortiuntur. Itaque nequaquam est anima tabula rasa ab omnibus rationibus vacua, sed est scripta semper tabula, scribitque et ipsa in seipsam et scripturis impletur a mente. Est enim anima mens seu intellectus quidam, qui seipsum secundum priorem se intellectum revolvit, imago illius et figura seu typus foris factus. Si igitur ille omnia intellectualit*/fol. 117/*er est, erit et anima omnia animaliter ; et si ille exemplariter*[292]*, anima imaginis in modum; et si ille contractim et unitim, anima divisim.*

Quod cum et Plato intellexisset[293]*, animam ex omnibus constituit divisitque secundum numeros, revinxitque analogiis et rationibus harmonicis, inque ipsa contulit principia prima figurarum effectricia, puta rectum, et curvum, movetque circulos, qui sunt in ipsa intellectualiter. Omnia ergo mathematica primum sunt in anima, et ante numeros numeri seipsos moventes, et ante figuras apparentes figuras vitales, et ante concordata vel concinnata ipsae rationes concordantiarum seu harmonicae, et ante corpora, quae moventur in circulum ipsi inconspicui cycli conditi sunt. Estque anima omnium rerum consummatio, et haec est exornatio quaedam alia* (praeter sensilem) *quae et ipsa seipsam adducit* (ad res) *et a familiari et cognato principio adducitur, quaeque vita et seipsam implet et a creatore impletur, incorporeo et indistanti modo* (Non longa abest ab

[290] anima] anima hic intellegit mundo praecipue Deum creatum Platonis, mentem vero quam christiani dicerent ipsum Deum creatorem, cuius imagines sunt omnes animae creatae, corporibus vivificandis praefectae *add. in marg. ed.*

[291] *sic in ed.*

[292] exemplariter] Christianis et sunt animae exemplaria creatoris et sustentantur etiam num ab illo per quandam velut irradiationem vultus divini ipsas *add. in marg. ed.*

[293] intellexisset] in Timaeo, qui est citra omnem dubitationis aleam commentarius quidam in primum caput Geneseos, seu Libri I Mosis, transformans illum in philosophiam Pythagoricam, ut facile patet attento legenti et verba ipsa Mosis identidem conferenti *add. in marg. ed.*

unoquoque; in ipso vivimus, movemur, sumus). *Et cum profert explicatque rationes eius, tunc et scientias detegit omnes, et virtutes. Essentiatur igitur in hisce speciebus anima, nec existimandum numerum, qui in ipsa inest, esse multitudinem unitatum, neque forma et idea eorum, quae locis sunt dislocabilia, subaudienda est corporea, sed omnia vitaliter et intellectualiter supponenda, et prima exempla apparentium numerorum*[294] *et figurarum et rationum et motuum, et sequendus hic Timaeus, qui omnem ipsius ortum et fabricam a generibus mathematicis consummat et perficit, inque ipsa rerum omnium causas reponit. Septem enim illi termini numerorum omnium secundum causam in ipso praeextiterunt, rursum figurarum principia architectonico seu fabrili modo in ipsa reposita sunt, et motuum omnium primus et principalissimus, qui motus reliquos omnes circumdat incitatque, una cum ipsa extitit. Omnium enim quae moventur principium est circulus et motus circularis. Sunt igitur essentiales et se ipsas moventes mathematicarum rerum rationes quae consummant animas, quas anima proferens et propagans evolvensque omnem varietatem scientiarum mathematicarum facit subsistere. Neque erit unquam ut desinat illa progignere et eruere, semper alia post alia dum rationes suas impartibiles simplicitate exuit. Omnia enim prius accepit, specierum principum et primaevarum in modum, et secundum infinitam suam facultatem, ex principiis anticipatis omnis generis speculationum propagationes architectatur.* Hactenus Proclus.

[294] numerorum] animam mundi Timaeus intellegit, puto, 1, 2, 4, 8, 3, 9, 27 *add. in marg. ed.*

Appendix 3

FEDERICO BORROMEO, *DE FLORIBUS MATHEMATICIS*
MS AMBROSIANA R 181 (16) INF.; 1, FOL. 1R–2V

/fol. 1r/Divisio mathematicarum ex Proclo *In Euclidis Elementa*

Aliae circa intellegibilia, aliae circa sensibilia versantes. Circa intellegibilia duae sunt: geometria et arit<h>metica, quae et purae ac praecipuae mathematicae appellantur. Nam prior magnitudines, secundum se et partes suasque passiones speculatur, altera numeros pariter secundum se, hoc est, in abstracto, ut aiunt, et species et accidentia considerat.
Circa sensibilia sex: astronomia quae et astrologia, perspectiva quae et optica, geodesia, musica quae et canonica, supputatrix quae et arithmetica, practica et mechanica.

Astrologiae duae sunt partes: speculativa, quae caelos, sydera et syderum motus, magnitudines et alia id genus speculatur, scientia mixta appellata, quoniam illius res subiecta naturalis est, rationes vero mathematicae a geometria desumptae. Altera practica in tres divisa: gnomonicam, meteoroscopicam et dioptricam. Prior rationes umbrarum gnomonum, videlicet solaria horologia docet conficere, pervestigat. Altera considerat <e>levationes et distantias syderum, signorum ortus et occasus, aliaque docet theoremata ac problemata. Tertia planetarum praecipue apparitiones distantiasque dioptris instrumentis exquirit, rationes reddens ex ipsa geometrica sicut et duae priores. Est et astrologia divinatrix, sed haec est ars coniecturalis, non scientia.

Optica a geometria ortum habet, ut enim geometria utitur lineis intelligibilibus, tamen uti dictum est, sic haec radiis seu lineis visualibus ac in duas distribuitur: in specularіam, causas reddens apparentiarum eorum quae nobis aliter apparere solent ob eorum quae sub visum cadunt alios situs et distantias, uti parallelarum coincidentiae et quadratorum, velut circulorum aspectus, et sciographia, id est umbrarum designatricem, picturae maxime conferrintes, quae ostendit qua ratione fieri possit, ut ea quae in imaginibus apparent, haud inconcinna vel deformia ob designa<ta>rum rerum distantias altitudinesque videantur.

/fol. 1v/ Geodesia dicitur quae res quantas praesertim terrestres, unde nomen habet, metitur, ut sunt rerum materialium acervi tamquam coni, ut putei tamquam cylindri quod non lineis intellectibilibus, ut geometria, assequitur, sed sensibilibus, aliquando quidem certioribus quodammodo, veluti cum radiis solaribus utitur, interdum vero crassioribus, ut cum spartos adhibet et perpendiculo. Quae quidem non secus ac geometria in duas secatur, hoc est, in eam quae planas figuras, et in eam quae solidas dimetitur.

Musica apparentes so<ni>tus concentus et rationes considerat, seu circa numeros sonoros versatur sensus egens adminiculo, hoc est auditus talisque existit, teste Platone in Timaeo, ut natura menti ipsas aures praeposuisse videatur. Est quae Geminus et preterea qualem musici huius temporis habeant rationum expertes. Huius tria sunt genera apud Boet<h>ium harmonicum, chromaticum et diatonicum.

Supputatrix numeros considerat non intellectiles, ut arithmetica, sed sensiles, quales in ipsis rebus sunt. Huius duae sunt partes: vulgaris[295] et algebra magis recondita. Vulgaris[296] quattor operationibus, ut aiunt, constat: compositione seu collectione, ductione, subductione, divisione.

Mechanica, quae in rerum sensibilium cognitione occupatur, rationes eorum quae fiunt exquirens multiplex est. Alia est instrumentorum ef<f>ectrix a Graecis ὀργάνω<ν> ποι<η>τικὴ dicta maxime instrumentor<orum> quae bello gerendo sunt apta tam ad oppugnationem quam ad defensionem, qualia Archimedes Marcello Syracusas obsidente fecit, quibus patriam sui tutatus est, qualia quoque sunt organa architectonica, veluti tractoria quibus onera trahuntur et elevantur, de quibus Vitru<vius> in architecturae suae libris; alia ταυματοποιετικὴ Graecis, hoc est mirabilium rerum effectrix, et haec multiplex est, sive spiritibus, id est[297] aeris impulsibus maximo artificio utens, uti in organis musicis de quibus apud Ctesibium et Heronem, sive ponderibus quorum /fol. 2r/ motus et status ad aequilibrium pertinent, sive nervi aut spartis animatas convolutiones et motus imitata aequilibrantium omnino, et eorum quae centro ponderantia vocantur cognitio.

Alia sph<a>eropoea Graecis, id est sph<a>eram effectrix ad celestium circunvolutionum imitationum, qua usus Archimedes sph<a>eram illam admirandam fabricatus est, atque haec inter mechanica praecipua esse videtur, et ut uno verbo dicamus, omnis ars quae materiam movendi vim habet, mechanica dici potest, de qua plura apud Vitruvium.

Denique et militaris ars, quae circa instruendas et coordinandas acies versatur, castraque metitur, et ipsa mechanica nominatur, Graece τακτική, nam et haec, quemadmodum egena, omnes mechanicas rationes ab ipsa geometria petit.

/fol. 2v/Per il libro *De floribus mathematicis.*

[295] vulgaris *post corr. ex* communis
[296] vulgaris *post corr. ex* communis
[297] id est *p. corr. ex* sive

Bibliography

Manuscripts

Bologna, Biblioteca Universitaria, 2293, fol. 1–149v.
Milan, Biblioteca Ambrosiana P. 51 sup.
Milan, Biblioteca Ambrosiana R 181 (16) inf. 1.
Modena, Biblioteca Estense, α. T. 9. 11, fol. 67–241v.
Munich, Bayerische Staatsbibliothek, A. gr. b. 954.
Munich, Bayerische Staatsbibliothek, Clm 10674, fol. 337r–340v.
Munich, Bayerische Staatsbibliothek, Clm 6, fol. 1r–154v.
Munich, Bayerische Staatsbibliothek, Cod. Graec. 427, fol. 1–234v.
Naples, Biblioteca Nazionale di Napoli, VIII, 33, fol. 21r–300r.
Oxford, Bodleian Library, Barocci 161.
Oxford, Corpus Christi College Library, 97.
Paris, Bibliothèque nationale de France, Baluze 281.
Paris, Bibliothèque nationale de Paris, Latin 7218.
Salamanca, Biblioteca de la Universidad de Salamanca, 2320, fol. 3v–133v.
Vatican City, Biblioteca Apostolica Vaticana, Vat. Barb. Gr. 101.
Vatican City, Biblioteca Apostolica Vaticana, Urb. Gr. 71.
Vatican City, Biblioteca Apostolica Vaticana, Vat. Lat. 4575, fol. 1r–19r.
Vatican City, Biblioteca Apostolica Vaticana, Vat. Lat. 6996, fol. 2r–159r.
Vatican City, Biblioteca Apostolica Vaticana, Vat. Lat. 6998, fol. 3r–116r.
Vatican City, Biblioteca Apostolica Vaticana, Vat., Barb. Gr. 145.
Venice, Biblioteca Marciana VI. 11.
Venice, Biblioteca Marciana, Gr. Z. 306.

Primary Sources

Alessandro Piccolomini, *De certitudine mathematicarum scientiarum*, Antonio Blado, Roma 1547.

Conrad Dasypodius, *Mathematicum, complectens praecepta mathematica, astronomica, logistica*, Volumen II, J. Rihel, Strasbourg 1570.

— *Protheoria Mathematica in qua non solum disciplinae mathematicae omnes, ordine convenienti enumerantur, verum etiam universalia Mathematica praecepta, explicantur*, Iodocus Martinus, Strabourg 1593.

Euclid, Proclus, *Εὐκλείδου Στοιχείων βιβλία 15 ἑκ τοῦ Θέωνος συνουσίων. Εἰς τοῦ ἑαυτοῦ τὸ πρῶτον ἐξηγημάτων Πρόκλου βιβλία 4*, I, ed. Simon Grynaeus, Hervagium, Basel 1533.

Federico Borromeo, *De Pythagoricis numeris libri tres*, ed. Manuel Bertolini, Bulzoni, Roma 2017 (Biblioteca Ambrosiana/Fonti e Studi 26).

Francesco Barozzi, *Procli Diadochi Lycii philosophi platonici ac matematici probatissimi in primum Euclidis Elementorum librum commentariorum ad uniuersam mathematicam disciplinam principium eruditionis tradentium libri IV*, Gratiosus Perchacinus, Padova 1560.

— *Opusculum in quo una oratio et duae quaestiones, altera de certitudine, et altera de medietate mathematicarum, continentur*, E.G.P., Padova 1560.

Giorgio Valla, *De expetendis et fugiendis rebus opus*, Manutius, Venezia 1501.

Giovanni Aurispa, *Carteggio di Giovanni Aurispa*, ed. Remigio Sabbadini, Istituto storico italiano, Roma 1931 (Fonti per la storia d'Italia).

Johannes Kepler, *Harmonices mundi libri V*, Ioannes Plancus, Linz 1619.

— *Opera omnia*, ed. Christian Frisch, vol. II, Heyder & Zimmer, Frankfurt and Erlangen 1859.

Proclus, *Procli Diadochi in Primum Euclidis Elementorum Librum Commentarii*, ed. Godfried Friedlein, B. G. Teubneri, Leipzig 1874.

Secondary Sources

Adamson, Peter, Filip Karfík, « Proclus' Legacy », in Pieter D'Hoine, Marije Martijn (eds.), *All From One: A Guide to Proclus*, Oxford University Press, Oxford 2016, p. 290–332.

Aurivillius, Pehr Fabian, *Catalogus librorum impressorum Bibliothecae Regiae Academiae Upsaliensis,* Stenhammar et Palmblad, Upsala 1814.

Boncompagni, Baldassarre, *Intorno al comento di Proclo sul primo libro degli Elementi di Euclide*, Tipografia delle Scienze Matematiche e Fisiche, Roma 1784.

Brach, Jean Pierre, « Pythagorean Number Mysticism in the Renaissance. An Overview », in Irene Caiazzo, Constantonos Macris, Aurélien Robert (eds.), *Brill's Companion to the Reception of Pythagoras and Pythagoreanism in the Middle Ages and the Renaissance*, Brill, Boston 2021 (Brill's Companions to Classical Reception, 24), p. 457–488.

Valentino Capocci, *Bibliothecae Apostolicae Vaticanae, Codices Barberiniani graeci*, Tomus I. Codices 1–163, Bibliotheca Vaticana, Città del Vaticano 1958.

Luis Miguel Carolino, « Mathematics and the Late Aristotelian Theory of Science: The *Quaestio de Certitudine Mathematicarum* in Seventeenth–century Portuguese Universities », in Victor Navarro-Brotons, William Eamon (eds.), *Beyond the Black Legend: Spain and the Scientific Revolution*, Publicacions de la Universitat de València, Valencia 2007, p. 399–412

Guy Claessens, « Clavius, Proclus, and the Limits of Interpretation: Snapshot–idealization versus Projectionism», *History of Science,* 47/3 (2009), p. 317–336.

— « Het denken verbeeld. De vroegmoderne receptie (1533–1650) van Proclus' Commentaar op het eerste boek van Euclides' Elementen », Ph.D. Diss. KU Leuven 2011.

— Imagination as Self–knowledge: Kepler on Proclus' Commentary on the First Book of Euclid's Elements », *Early Science and Medicine*, 16/3 (2011), p. 179–199.

— « Proclus in the Renaissance », in Marco Sgarbi (ed.), *Encyclopedia of Renaissance Philosophy*, Springer, Cham 2018. <https://link.springer.com/referenceworkentry/10.1007/978-3-319-02848-4_412-1>.

— « Reception and the Textuality of History: Ramus and Kepler on Proclus' History and Philosophy of Geometry », in André Lardinois, Sophie Levie, Hans Hoeken, Christoph Lüthy (eds.), *Texts, Transmissions, Receptions: Modern Approaches to Narratives*, Brill, Boston 2015 (Radboud Studies in Humanities, 1), p. 281–294.

— « The Drawing Board of Imagination: Federico Commandino and John Philoponus », *Journal of the History of Ideas*, 76/4 (2015), p. 499–515.

Cozzoli, Daniele, « Alessandro Piccolomini and the Certitude of Mathematics », *History and Philosophy of Logic*, 28/2 (2007) p. 151–171.

Crapulli, Giovanni, *Mathesis universalis: Genesi di una idea nel XVI secolo*, Edizioni dell'Ateneo, Roma 1969 (Lessico intellettuale europeo, 3).

De Caro, Mario, « Galileo e il platonismo fisico–matematico », in Riccardo Chiaradonna (ed), *Il platonismo e le scienze*, Carocci, Roma 2012 (Colloquium philosophicum, Nuova serie, 3), p. 119–138.

— « Galileo's Mathematical Platonism », in Johannes Czermak (ed.), *Philosophy of Mathematics*, Holder – Pichler – Tempsky, Wien 1993, p. 1–9.

— « Sul platonismo di Galileo », *Rivista di filosofía*, 82 (1996), p. 25–40.

De Pace, Anna, *Le matematiche e il mondo. Ricerche su un dibattito in Italia nella seconda metà del Cinquecento*, Francoangeli, Milano 1993 (Filosofia e scienza – Studi).

De Risi, Vincenzo, *Leibniz on the Parallel Postulate and the Foundations of Geometry, The Unpublished Manuscripts*, Birkhäuser, Berlin 2018 (Sciences Networks Historical Studies 51).

Del Gallo, Elena, « Gabia, Giovan Battista », in *Dizionario Biografico degli Italiani*, Volume 51, 1998. <www.treccani.it>.

Dubouclez, Olivier, *Descartes et la voie de l'analyse*, Presses Universitaires de France, Paris 2013 (Épiméthé essais philosophique).

Escobar, Jorge M., « Kepler's theory of the soul: a study on epistemology », *Studies in History and Philosophy of Science Part A*, 39/1 (2008) p. 15–41.

Ferraro, Giovanni, « Dimostrazioni Matematiche e Conoscenza Scientifica in Alessandro Piccolomini », in Howard Burns, Francesco Paolo Di Teodoro, Giorgio Bacci (eds.), *Saggi di Letteratura architettonica da Vitruvio a Winckelmann III*, Olschki, Firenze 2010, p. 215–233.

Field, Judith Veronica, *Kepler's Geometrical Cosmology*, Athalone, London 1988 (Bloomsbury Academic Collections: Philosophy).

Giacobbe, Giulio Cesare, « Il *Commentarium De Certitudine Mathematicarum Disciplinarum* di Alessandro Piccolomini », *Physis,* 14/2 (1972), p. 162–193.

— « Francesco Barozzi e la *quaestio de certitudine mathematicarum* », *Physis,* 14/4 (1972), p. 162–193.

Giacomelli, Ciro, « I libri greci di Matteo Macigni. Contributo allo studio di una biblioteca umanistica », *La parola del passato*, 407/2 (2019), p. 361–420.

Goulding, Robert, « Pythagoras in Paris: Petrus Ramus Imagines the Prehistory of Mathematics », *Configurations,* 17/1 (2009), p. 51–86.

— *Defending Hypatia: Ramus, Savile, and the Renaissance Rediscovery of Mathematical History*, Springer, Dordrecht 2010 (Archimedes, New Studies in the History and Philosophy of Science and Technology, 25).

Guarna, Valeria, *L'Accademia veneziana della Fama (1557–1561). Storia, cultura e editoria, Con l'edizione della Summa delle Opere (1558) e altri documenti inediti*, Vecchiarelli, Roma 2018 (Cinquecento, testi e studi letter. Ital).

Guicciardini, Niccolò, *Isaac Newton on Mathematical Certainty and Method*, MIT Press Ltd, Cambridge, MA 2011.

Helbing, Mario Otto, « La fortune des commentaires de Proclus sur le premier livre des Eléments d'Euclide à l'époque Galilée », in Gerald Bechtle, Dominic J. O'Meara (eds.), *La philosophie des mathématiques de l'Antiquité tardive*, Actes du colloque international Fribourg, Suisse 1998, p. 173–193.

Higashi, Shin, *Penser les mathématiques au XVIe siècle*, Editions Classiques Garnier, Paris 2018 (Histoire et philosophie des sciences, 17).

Hintikka, Jaakko, « Method of Analysis: A Paradigm of Mathematical Reasoning? », *History and Philosophy of Logic*, 33/1 (2012), p. 49–67.

Hintikka, Jaakko & Unto Remes, *The Method of Analysis: Its Geometrical Origin and Its General Significance*, Springer, Dordrecht 1974 (Boston Studies in the Philosophy of Science, 25).

Jardine, Nicholas, « Problems of Knowledge and Action: Epistemology of the Sciences », in Charles B. Schmitt, Quentin Skinner, Eckhard Kessler, Jill Kraye (eds.), *The Cambridge History of Renaissance Philosophy*, Cambridge University Press, Cambridge 1988, p. 685–711.

— « The Forging of Modern Realism: Clavius and Kepler against the Skeptics », *Studies in History and Philosophy of Science*, 10/2 (1979), p. 141–173.

Kessler, Eckhard, « Clavius entre Proclus et Descartes », in Luce Giard (ed.), *Les jésuites à la Renaissance, Système éducatif et production du savoir*, PUF, Paris 1988 (Bibliothèque d'histoire des sciences).

Kibre, Pearl, *The Library of Pico della Mirandola*, Columbia University Press, New York 1936.

Koyré, Alexandre, *Metaphysics and Measurement: Essays in Scientific Revolution*, Harvard University Press, Cambridge, MA 1968.

— « Galileo and Plato », *Journal of the History of Ideas*, 5 (1943), p. 400–428.

Levitin, Dimitri, *Ancient Wisdom in the Age of the New Science*, Cambridge University Press, Cambridge 2015 (Ideas in Context, 113).

— *The Kingdom of Darkness: Bayle, Newton, and the Emancipation of the European Mind from Philosophy*, Cambridge University Press, Cambridge – New York 2022.

Longeway, John, « The Place of *Demonstratio Potissima* in some 16th–Century Accounts of Mathematics », in Joël Biard (ed.), *Raison et démonstration*, Brepols, Turnhout 2015 (Studia Artistarum, 40), p. 223–251.

Mahoney, Michael S. « Barrow's Mathematics: between Ancients and Moderns », in Mordechai Feingold (ed.), *Before Newton: The Life and Times of Isaac Barrow*, Cambridge University Press, Cambridge – New York 2021 (Transformations: Studies in the History of Science and Technology), p. 179–249.

Maierù, Luigi, « La diffusione di Proclo, commentatore di Euclide, nel Cinquecento », in *11° Annuario del Liceo Scientifico 'B.G. Scorza'*, Calabria Letteraria Editrice, Soveria Mannelli 1999, p. 49–68.

— « Metafìsica ed enti geometrici: Benito Pereyra, Pedro Fonseca, Francisco Suárez », in *Sciences et religions. De Copernic à Galilée* (1540–1610) Actes du colloque international, Rome 1996. Rome (Publications de l'École française de Rome, 260), p. 47–67.

Mancosu, Paolo, *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, Oxford University Press, Oxford 1999.

Martens, Rhonda, *Kepler's Philosophy and the New Astronomy*, Princeton University Press, Princeton, NJ 2000.

Martini, Emidio, Domenico Bassi, *Catalogus codicum Graecorum Bibliothecae Ambrosianae*, t. I–II, Hildesheim–New–York, Milano 1906.

Mioni, Elpidio, *Bibliothecae Diui Marci Venetiarum codices graeci manuscripti. Volumen II: Thesaurus Antiquus. Codices* 300–625, Ministero per i beni culturali e ambientali, Roma 1985 (Indici e Cataloghi, Nuova Serie VI).

Morrow, Glenn R. *Proclus: A Commentary on the First Book of Euclid's Elements,* Princeton University Press, Princeton, NJ 1970.

Navarro-Brotons, Victor, Enrique Rodríguez Galdeano, *Matemáticas, cosmología y humanismo en la España del siglo XVI: Los Comentarios al segundo libro de la Historia natural de Plinio de Jerónimo Muñoz*, Instituto de Estudios Documentales e Históricos sobre la Ciencia, Valencia 1998.

— « The Cultivation of Astronomy in Spanish Universities in the Latter Half of the 16th Century », in *Universities and Science in the Early Modern Period*, Springer, Dordrecht 2006 (Archimedes, New Studies in the History and Philosophy of Science and Technology, 12), p. 83–98.

— *Disciplinas, saberes y prácticas: Filosofía natural, matemáticas y astronomía en la sociedad española de la época moderna*, Publicacions de la Universitat de València, Valencia 2014.

— *Jerónimo Muñoz: Matemáticas, cosmología y humanismo en la época del Renacimiento*, Publicacions de la Universitat de València, Valencia 2019.

Oosterhoff, Richard J. *Making Mathematical Culture: University and Print in the Circle of Lefèvre d'Étaples*, Oxford University Press, Oxford 2018 (Oxford – Warburg Studies).

Osterhage, Wolfgang, *Johannes Kepler: The Order of Things*, Springer, Berlin 2020 (Springer Biographies).

Palmieri, Paolo, « On *scientia* and *regressus* », in Henrik Legarlund, Benjamin Hill (eds.), *Routledge Companion to Sixteenth Century Philosophy*, Routledge, New York 2017 (Routledge Philosophy Companions), p. 319–349.

Puntoni, Vittorio, *Indice dei codici greci della Biblioteca Estense di Modena,* Firenze 1986.

Rabouin, David, « Le rôle de Proclus dans les débats sur la 'mathématique universelle' à la Renaissance », in Alain Lernould (ed.), *Etudes sur le Commentaire de Proclus au premier livre des Eléments d'Euclide*, Presses Universitaires du Septentrion, Villeneuve D'Ascq 2007, p. 217–235.

— *Mathesis universalis: L'idée de « mathématique universelle » d'Aristote à Descartes*, Presses Universitaires de France (PUF), Paris 2009 (Épiméthé essais philosophique).

Randall, John H., *The School of Padua and the Emergence of the Modern Science*, Editrice Antenore, Padova 1961.

Raschieri, Amedeo A. « Giorgio Valla Editor and Translator of Ancient Scientific Texts », in Paula Olmos (ed.), *Greek Science in the Long Run: Essays on the Greek Scientific Tradition (4th c. BCE–16th c. CE)*, Cambridge Scholars Publishing, Cambridge 2012, p. 127–149.

Rigaud, Stephen P. *On the Arenarius of Archimedes*, S. Collingwood, Oxford 1837.

Rommevaux, Sabine, *Clavius, une clé pour Euclide au XVIème siècle*, Vrin, Paris 2006 (Collection Mathesis).

Rose, Paul L. *A Venetian Patron and Mathematician of the Sixteenth Century: Francesco Barozzi: 1537–1604*, Giardini Editori, Pisa 1977.

— « Bartolomeo Zamberti's Funeral Oration for the Humanist Encyclopaedist Giorgio Valla », in Cecil H. Clough (ed.), *Cultural Aspects of the Italian Renaissance: Essays in Honor of Paul Oskar Kristeller*, Manchester University Press, Manchester 1976, p. 299–310.

— « Certitudo Mathematicarum from Leonardo to Galileo », in *Leonardo nella scienza e nella tecnica: Atti del Simposio Internazionale di storia della Scienza*, Giunti Barberà, Firenze – Vinci 1969, p. 43–49.

— *The Italian Renaissance of Mathematics: Studies on Humanists and Mathematicians from Petrarch to Galileo*, Libraire Droz, Genève 1975 (Travaux d'humanisme et renaissance CXLV).

Schöttler, Tobias, « From Causes to Relations: The Emergence of a Non–Aristotelian Concept of Geometrical Proof out of the *Quaestio De Certitudine Mathematicarum* », *Society and Politics*, 6/2 (2012), p. 29–47.

Stewart, Ian, « Mathematics as Philosophy: Barrow and Proclus », *Dionysius*, 18 (2000), p. 151–181.

Velilla-Jiménez, Helbert E. « El debate sobre la certeza de las matemáticas en la filosofía natural de los siglos XVI y XVII (*De quaestio* [sic] *de certitudine mathematicarum*) », *Estudios de Filosofía*, 3 (2018), p. 59–93.

— « Francisco Sánchez and the *Quaestio de certitudine mathematicarum*: A Sceptical approach », *Endeavour*, 46/4 (2022), p. 1–10.

Wilson, Nigel, *A Descriptive Catalogue of the Greek Manuscripts of Corpus Christi College Oxford*, D. S. Brewer, Oxford 2011.