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EVALUATING MODERN MATHEMATICS CURRICULA

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Abstract

This article intends to understand the degree in which expectations about the modern mathematics reform were fulfilled. Focusing on the Portuguese case, we probed four quantitative studies developed by governmental institutions at the time of the reform, usually intended to evaluate its implementation at several grade levels. Re-appreciated in modern times, these studies provided us with insights about the reform. More specifically, following Gimeno's distinction among several curricular levels, the contemporary inspection of those studies allowed us to have an insight of the curriculum presented to teachers, the curriculum modelled by them, and the attained curriculum at the time of the reform.

Keywords: *history of mathematics education, modern mathematics reform, curricular studies, meta studies*

Evaluación del currículo de matemáticas modernas

Resumen

Este artículo pretende comprender el grado en que se cumplieron las expectativas sobre la reforma matemática moderna. Centrándonos en el caso portugués, examinamos cuatro estudios cuantitativos desarrollados por instituciones gubernamentales en el momento de la reforma, generalmente destinados a evaluar su implementación en varios niveles de grado. Reconocidos en los tiempos modernos, estos estudios nos proporcionaron ideas sobre la reforma. Más específicamente, siguiendo la distinción de Gimeno entre varios niveles curriculares, la investigación contemporánea de esos estudios nos permitió tener una idea del currículum presentado a los maestros, el currículo modelado por ellos y el currículo alcanzado en el momento de la reforma.

Palabras clave*: historia de la educación matemática, reforma matemática moderna, estudios curriculares, meta estudios*

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INTRODUCTION[1]

We can date from the 1950s the beginning of the reform of Modern Mathematics. Born out of the need to recompose programs, adapting them to new content and methods that economic development and the political situation demanded from the post-war period, the reform took place in all levels of education from primary to higher education in most countries of the world. In Portugal, new ideas circulated from the end of the 1950s until the end of the 1980s. From the mid-1970s onwards, other curriculum options were developed internationally and reform was declining (Furinghetti, Matos, & Menghini, 2013).

The reform created high expectations for the improvement of mathematics learning. According to the reformers, it was well adjusted to recent psychological findings, guarantying shorter learning times and, as it was closer to up-to-date mathematical research, it would facilitate de formation of highly skilled technicians (mathematicians, physicists, engineers, etc.) needed, either for an improved development of the society, or to ensure advantages in economical or military competition.

It is the purpose of this article to understand the degree in which these expectations were fulfilled. Focusing on the Portuguese case, we probed four quantitative studies developed by governmental institutions at the time of the reform, usually intended to evaluate its implementation at several grade levels. Re-appreciated in modern times, these studies provided us with insights about the reform. More specifically, following Gimeno's distinction among several curricular levels (2000), the contemporary inspection of those studies allowed us to have an insight of the curriculum presented to teachers, the curriculum modelled by them, and the attained curriculum at the time of the reform.

From the 1960s until the 1980s decades, the Portuguese school system begun with the mandatory primary school (grades 1-4). Those that wanted to continue school should choose between the technical schools or the l*iceus*. Technical schools started with a preparatory cycle (grades 5-6) followed by courses for specific professions. The course of *liceus* was divided into three cycles (grades 5-6, 7-9, 10-11). From 1968, the mandatory Preparatory Cycle for Secondary Education (CPES) unified the first two cycles of technical schools and *liceus*. From 1975, the courses of *liceus* and technical schools were gradually unified into a "secondary" course and, from 1977, this course was extended and a 12th grade was created.

This text addresses three sub-systems where the new curriculum was evaluated. Firstly, we used a report elaborated in 1969 about an experiment on the last two years of secondary schools. Despite its limitations, this document, which was never published and became available to us in 2014, allowed us to examine the curriculum presented to teachers and the curriculum modelled by them. Secondly, we used two studies focused on CPES, an unpublished report about the results of 6th graders in a national exam of 1972 and a published inquiry about curricular effectiveness in 1986, that gave us insights about the curriculum modelled by teachers and the accomplished curriculum. Thirdly, we compiled the results of several published studies intending to evaluate mathematical teaching and learning in the late 1970s of grades 7 through 9 and discuss both the curriculum presented to teachers and the accomplished curriculum.

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THE SEBASTIÃO E SILVA EXPERIMENT

In Portugal, since 1963, a well-known pedagogical experience of modern mathematics took place in the last cycle of *liceus*, under the leadership of a Commission chaired by José Sebastião e Silva (Almeida, 2013). With the support of the Organization for Economic Cooperation and Development, the project began with three pilot classes composed of students with high performance in mathematics. The number of classes involved was gradually increased.

In 1968 Manuel Sousa Ventura, a mathematics teacher of *liceus*, then working at the Ministry of Education, was assigned to write an evaluation of the project (Ventura, 1968). Details about the report and the context in which it was produced were studied in Almeida and Matos (2021). His research was very superficial and did not address key questions about the experience, however, we studied it because it contains information concerning some aspects of the experience that are relevant for our purpose.

Focussing on the school years 1965/66 and 1966/67, the Ventura report (1968) evaluated the experience considering quantitative and qualitative "aspects". To study the first ones, pilot classes in seven liceu were paired with control classes (called "testimony classes") with traditional mathematics programs. Quantitative data collected included: number of students at the beginning and at the end of each academic year studied; frequency of classifications in each discipline; likewise for the final exam classifications. The Report includes 23 graphs with the percentages of the classifications during the school year (11 graphs) and in the final exams in Mathematics (12 graphs) of the seven *liceus*.

Ventura did not perform an analysis of the quantitative results and actually devalued any attempt to compare the classes of experience with control classes, arguing that data lack statistical significance (…) [because] the pilot class was organized and operated under different conditions than the testimony group (number of students, number of weekly hours in the subject of Mathematics, teachers in a given class, as a rule, would not be the same as in the other class, etc., etc.). (Ventura, 1968, p. 10)

In addition, students in the experiment took special final national mathematics exams and, as several accounts indicate (Almeida & Matos, 2021), students' motivation was different as the enthusiasm surrounding the experiment, namely, parents' intervention to include their siblings in the pilot classes.

Although Ventura's presentation of the quantitative results is just a set of graphs about students' performance in examinations and during the school year, his graphs provided us with valuable data concerning the experience. These 23 line graphs have an identical structure: the abscissa axis indicates classifications from 0 to 20, called "Number - frequency of classifications", and the ordinate axis is called "Percentages". For each year and each liceu, the same graph shows data from the classifications in mathematics of the experience and the control classes.

We looked specifically for differences between pilot and control classes in students' performance either during the school year or the examinations, especially concerning the curriculum modelled by teachers and the accomplished curriculum of the experiment.

We began by analysing the graphs with grades in the discipline of Mathematics during the school years 1965/66, 1966/67. Three times a year, at the end of a term ("*período*"), students were given a grade from 0 to 20 points and for students to pass the 6th year of the course of *liceus* or having access to the final exam at the end of the 7th year, they should add at least 29 points each year. If, in the course of the school year, students thought they had no chance of obtaining the 29 points at a given discipline, they could cancel their enrolment and take the exam as "external" students. In the graphs with the term grades in Mathematics, scores for the three school terms, so the cumulative "frequencies" resulted in values close to 300%.

In a first step, we built a table of percentages of frequencies for each graph of the term grades, in a total of 11 tables. As the Report provided no numerical data, we performed a conversion of the line segment lengths in the graphs into numerical data, necessarily incorporating some inaccuracy, as the percentages did not always add up to exactly 300%. We admitted, however, that a 10% error in the total frequency percentages would not fundamentally alter our analysis. Even so, there were still inconsistencies in the data that should have originated from errors in the graphs' original data or in their elaboration, so we excluded three cases, all from the control classes, in which the total frequencies (considerably) exceeded this margin. In a second phase, we calculated the means (necessarily approximate), the medians, and the modes of the term grades in Mathematics of the remaining 19 classes, which are shown in Table 1.

Table 1. *Modes, approximate means, and medians of term grades in Mathematics of the pilot and the control classes in 6 liceus, 1965/66, 1966/67*

	Pilot classes			*Control classes*		
Year/Liceu	*Mode*	*Mean*	*Median*	*Mode*	*Mean*	*Median*
1965/66						
Alex. Herculano	9	11,1	10	10	11,2	11
D. João de Castro	10	10,9	10	10	11,6	11
D. João III	10 & 11	11,4	11	9, 10 & 11	10,8	10
D. Manuel II	12	11,5	12	10	10,6	10
Oeiras	13	12,9	13	(a)	(a)	(a)
Pedro Nunes	10	11,1	11	(a)	(a)	(a)
Santa Isabel	(b)	(b)	(b)	(b)	(b)	(b)
1966/67						
Alex. Herculano	(b)	(b)	(b)	(b)	(b)	(b)
D. João de Castro	11	11,8	12	10	10,1	10
D. João III	(b)	(b)	(b)	(b)	(b)	(b)
D. Manuel II	9 & 15	13,8	14	11	10,4	10
Oeiras	13	11,6	12	(a)	(a)	(a)
Pedro Nunes	10	11,8	11	9 & 10	9,5	9
Santa Isabel	10	11,2	11	9	9,8	9

Note: (a) excluded data; (b) non-existent data.

We then performed eight comparisons between the pilot and the control classes. In six of them, the pilot classes show higher modes, means, and medians. Higher modes denote that the most frequent grade given in during the school year is higher in pilot classes. Table 1 shows that modes in these classes may attain 15, 13, 12 points, which differentiates them from the control classes. Medians also show that in pilot classes the lower half of students has higher grades than in control classes and this difference is often greater than 1 point. Means just confirm these differences. Although our analysis is based on data that must be handled with a grain of salt, we believe it is possible to, at least, state that, from the teachers' perspective, learning was more successful in pilot classes.

It is almost impossible to draw definitive conclusions, apart from the fact that those statistical indicators are higher in pilot than in control classes[2]. Were students in pilot classes better achievers? Were they more motivated as they were participating in a "modern" experiment involving top methods and contents? Was teaching more

[2] We refrained from applying inferential statistics, as the number of cases is low.

successful? Or were teachers just fulfilling high expectancies about the experience and tended to give better grades? As Ventura himself says, in the absence of a more sophisticated methodology, it is impossible to go any further.

To extend our study, we proceeded by analysing the graphs concerning the results from the final exams as the effect of teachers' expectancies would be minimized. At the time, the score of the exams was given in a scale from 0 to 20, and students had to have a final grade equal or above 10 to pass. As the graphs did not include grades smaller than 10, we assumed that they showed the frequency of the grades of the students who attended classes and passed the exam, i. e., those that failed the exam, or that, during the year, cancelled their enrolment, were not included. So, the population from which these new data was collected was not the same as the previous population and weaker students have been excluded.

For each graph of the results of the exams we built a table of percentages of frequencies, totalling 12 tables. As before, we followed a procedure for converting lengths of line segments into numbers and we considered, that a 10% error in the total frequency percentages would not alter our analysis and so we excluded three cases from the pilot classes and four from the control groups, in which the total frequencies (considerably) exceeded this margin. In a second phase, we calculated the means, necessarily approximate, the modes, and the medians of the classifications in the Mathematics exams of the remaining 17 classes, which are shown in Table 2.

Table 2. *Modes, approximate means, and medians of grades of Mathematics exams of the pilot and the control classes in 7 liceus, 1965/66, 1966/67*

	Pilot classes			*Control classes*		
Year/Liceu	*Mode*	*Mean*	*Median*	*Mode*	*Mean*	*Median*
1965/66						
Alex. Herculano	(a)	(a)	(a)	10	12,3	11
D. João de Castro	10	11,1	10	10	12,0	11
D. João III	10	11,7	11	(a)	(a)	(a)
D. Manuel II	(b)	(b)	(b)	(b)	(b)	(b)
Oeiras	13	12,9	13	(a)	(a)	(a)
Pedro Nunes	10	12,1	11	10	10,2	10
Santa Isabel	10	11,9	11	10	10,9	10
1966/67						
Alex. Herculano	(b)	(b)	(b)	(b)	(b)	(b)
D. João de Castro	11	12,9	11	11	11,2	11
D. João III	10	11,6	10	10	11,1	10
D. Manuel II	(a)	(a)	(a)	(a)	(a)	(a)
Oeiras	(a)	(a)	(a)	10 & 12	11,7	12
Pedro Nunes	15	12,9	13	10	10,2	10
Santa Isabel	10	11,4	10	(a)	(a)	(a)

Note: (a) excluded data; (b) non-existent data.

In six cases, the approximate means of the pilot classes are higher than the control classes, except for one case (*D. João de Castro*, 1965/66), but either the modes, or the medians do not provide such a clear distinction. Again, we find better means for the pilot classes. Conclusions, however, were difficult to formulate: did modern mathematics produce better quality of learning? Were the results naturally due to the criteria used for the selection of students in the experimental classes? Did the number of students per class (considerably smaller in the pilot classes) have an influence? Did the greater number of hours of mathematics (6, compared to 4) make a difference? Did students' motivation about modern mathematics explain the results? Were pilot class exams easier?

There was a detail, however, that allowed us to go further. Both in pilot and in testimonial classes, the modes for classifications in Mathematics exams is mostly 10 points, except for the *Liceu D. João de Castro*, which, in 1966/67, presented a mode of 11 points in both classes, the *Liceu Pedro Nunes* with a mode of 15 points in the pilot class of 1966/67 and the *Liceu de Oeiras* with a mode of 13 points in the pilot class in 1965/66 and bimodal (10 and 12 points) in the testimony class of 1966/67. So, in both types of classes, most students' grades laid on the border between passing and failing and a visual inspection of the graphs showed that many resembled L curves. However, the fact that, roughly, modes were similar but means were higher in pilot classes indicated that in the experimental group higher mathematics achievers were obtaining higher examination grades. In fact, when we observed the distribution of the grades, all the comparisons except one (*D. João de Castro*, 1965/66), showed that the examination grades range was more extended in experimental classes. An extreme example was *Liceu Pedro Nunes* where the range of grades for the control classes was 10-11 points in both years, whereas for the experimental classes was 10-16 and 10-15 points.

We already knew that pilot classes were different because they were exposed to a special program, but, from our analysis of the "quantitative" aspects of the Report, we argued that the curriculum modelled by teachers in these classes produced a distinct class environment as can be seen by the grades attributed by teachers during the school year. Newspaper articles confirm this opinion (Matos, 2019). Moreover, as exam results show, this environment had the effect of enhancing performance of some students, most likely those that had already a preference for mathematics. This conclusion is consistent with the common current opinion about the experiment. Many contemporary mathematicians and physicists attribute their enthusiasm about science to these classes. However, we must also note that there are also successful persons in areas other than "hard" sciences that express a rejection of those classes. Our analysis also suggests that this may be the case, as results show that the examination curves tend to show a concentration of students are at the borderline between pass and failure and this is not different from what occurs in the control classes.

The qualitative aspects that Ventura's report (1968) allowed us to picture teachers' opinions about the curriculum presented to teachers, namely, the program and the overall experiment. Data was obtained through opinion surveys ("Information Sheets") of two types. Type I Sheets were sent to mathematics teachers who taught experimental classes and asked, "What is your opinion on the results achieved by the experimental groups in which you participated in the course of this experiment?" Type II sheets were sent to all the Rectors, and all the "qualified" teachers of Mathematics, Physics and Philosophy of the *liceus* and asked, "What impressions and suggestions could you give us about these experiment?"

In this case, Sousa Ventura actually conducted a data analysis. He received 130 responses from 30 high schools, of which 25 said they had no opinion. He then emphasized 13 "themes", that is, categories of opinions, indicating the number of responses that he included in each one, rarely distinguishing between responses from teachers of mathematics or other disciplines. About the curriculum presented to teachers, they thought that the programs of the experimental classes were excessively long, the number of hours (6h) per week of these classes was excessive (33 responses) and that these programs must have been coordinated with Physics and Philosophy programs (26 responses). As for the curriculum modelled by teachers, some believed that modern mathematics developed a taste for scientific research, provides greater capacity for analysis and rigor in the type of hypothetical-deductive reasoning (25 responses). This may have been the case for some students, as we have seen previously. Finally, significant numbers of teachers think that pilot classes should be started at all levels of primary and secondary education (37 responses) and that experimental classes should consist of non-selected students with a number of students of not less than 30 (24 responses). These opinions signal a desire to enlarge modern mathematics to all students.

TEACHING AND LEARNING IN CPES

The new CPES for grades 5th and 6th began in 1968, unifying similar courses for *liceus* and technical schools. It was a cycle with a new philosophy fostering the use of innovative teaching methodologies, with specific schools and a teaching staff organized according to new interdisciplinary areas. The new programs advocated active and practical teaching, seeking to awaken the spirit of observation, creative imagination, aesthetic sense, the taste of inventiveness and personal effort, as well as the recognition of the value of work, although the programs of some disciplines reflected the conservative nature of the regime.

The discipline of Mathematics took 3 hours per week for the two years and its program was designed by Sebastião e Silva. Strongly recommending the use of new methodologies, he made two major changes: 1) the older program was organized around geometrical ideas and the new one would give prominence to arithmetic including an initiation to algebra, 2) the language of sets would become a standard way to address mathematical content (Matos, 2014).

The program, which without major changes prevailed until the end of the 1980s, started in the 1st year with sets and their operations, followed by the study of arithmetic, rational numbers, calculation with decimals, measurement of lengths, times and speeds, and finally solid and plane geometry. The 2nd year, in addition to deepening these notions, included the study of proportionalities (direct and inverse). Simple equations were also taught.

Change was not simple. Three months after the start of the new program, the Inspector Joaquim Redinha issued a note to teachers (1969) advising them against spending too much time on sets. A second note in the beginning of 1969/70 ("A programação de Matemática do 1º ano do Ciclo Preparatório", 1969) written also by Redinha and supported by Sebastião e Silva, used stronger terms to caution teachers against the excessive development of the topic and discussed adequate terminology (Matos, 2009). The Inspectorate and Sebastião e Silva believed that the source of the problem laid partially on the available textbooks. However, we may conjecture that the main issue could be essentially placed in the excessive linguistic precision of the program, forcing either textbooks or teachers to spend too much time to teach 10 and 11 years old the details of the definitions of set terminology.

Two reports allowed us to characterize the curriculum modelled by teachers and the accomplished curriculum. The first, conducted by the Inspector Paulo Crato (1972), analysed the responses of the 31217 students of the 2nd year of CPES who took the first edition of the national exam in 21 June 1972 and we already performed a preliminary study (Matos, 2005). The exam was a mandatory written test lasting 90 minutes, which conditioned the progression to the following cycles in *liceus* or technical schools. It had ten questions with several items. The report values a detailed analysis of answers and, for each question, discriminates the percentage of totally correct answers, of totally incorrect answers, the absence of answers, the percentages of different types of answers, and the frequency of some common errors. Crato does not provide an indication of students' total scores but, given the partial results, we may suspect the pervasiveness of low total grades.

Geometry was represented by three of the ten test questions. The first (question 1, with four items) focused on the classification of geometric solids. The responses revealed that more than 80% of students correctly identified cylinders and cones. The identification of prisms (40% correct answers) showed some problems related to the exclusion of cubes or parallelepipeds. The other two geometry questions (questions 9 and 10) involved the calculation of areas and volumes. Both required two phases of calculation, the area of a semi-circle and a rectangle in one question, and of fitting cubes in a parallelepiped in the other. These questions required the use of complex solution strategies and the percentages of correct answers (8% and 13%) reflected that. Even so, the percentages of students who correctly calculated some of the required areas or volumes were very low (rectangle area, 60%; circle area, 20%; parallelepiped volume, 42% and 25%). Paulo Crato attributed these low percentages to "deficiencies in simple calculation" among other reasons. However, the next study suggests, these topics may not have been even taught.

Five questions dealt with arithmetic. Question 3 involved the determination of divisors and obtained success rates of 60% and 49% in its two items, with 81% of students having the concept of divisor and 68% that of common divisor. The main mistake made was not including 1 or the number itself in the set of dividers. Only 15% and 20% of students included strange elements. Question 4 asked for the comparison between two numerals (fractions or decimal numbers). About 60% of the students answered appropriately to each of the four items. Questions 5 and 6 (with two items) required writing and calculating of numerical expressions involving fractions and decimal numbers. These were difficult questions and had very low percentages of correct answers, around 20%, with high rates of completely incorrect or even unanswered questions. Errors in intermediate arithmetic operations played an important role in these results. Question 2, although using numbers, essentially involved operations on sets and will be discussed later.

Question 7 could be solved using an equation. The problem "which number multiplied by 15 gives 240" was correctly solved by 53% of students. A significant percentage of students indicated the correct number, but included an equation unrelated to the problem (15%), or did not write any equation (28%), thus suggesting the use of alternative non-algebraic methods.

Finally, question 8 tested knowledge in proportionality and percentage. Correct answers were scarce (24% and 18%, respectively), with many answers completely incorrect (about 40%), with many students choosing not to answer (22% and 33%).

Through out his analysis, Paulo Crato repeatedly mentioned the deficient performance of arithmetic operations. This happened in all questions that require the performance of arithmetic algorithms (questions 5, 6, 8 and 9) where the percentage of errors was substantial. For example, in question 6, 28% of students made a subtraction mistake, 20% a multiplication and 34% a division. It is unclear whether this is a by-product of modern mathematics, but a small group of teachers from Ventura's research (1968) thinks so.

We looked specifically at questions in which modern mathematics topics were clearly present. The use of mathematical language was strongly present in questions 1, 2, and 3. Paulo Crato studied the understanding of concepts of set theory and its symbolism in questions 1 and 3. The success percentages on the knowledge of the subset concept, the use of braces, and the separation of elements by commas were over 80%. In question 2, the percentages for understanding intersection, union and their symbols were over 70%.

However, despite the correct use of this terminology by large percentages of students, the comprehension of mathematical concepts in these questions was low.

Question 2 in particular focused on the proper use of the language of sets:

> 2. Given the sets
>
> A = {whole numbers less than 6}
>
> B = {integer numbers greater than 3 and less than 8},
>
> represent, using curly brackets and indicating its elements:
>
> a) $A \cap B$;
>
> b) $A \cup B$. (Crato, 1972, p. 6, our translation)

The percentage of correct answers was discouraging (42% and 23%, for items a) and b) respectively). As the question did not require any arithmetic calculation, and students understood the notions of union and intersection of sets, the issue may lie essentially in the domain of language, either due to difficulties in interpreting the question or due to the lack of mathematical writing skills. Crato did not propose an explanation.

In summary, Paulo Crato's study proved us with a glimpse of the accomplished curriculum and globally mathematics learning was low. Looking in particular to the use of modern mathematics language, although students seemed well acquainted with most of its symbols, framing problems in the new language produced very low results.

We can have a perspective of the modern mathematics accomplished and modelled curriculum by teachers in CPES by looking at a large quantitative study performed by the Inspectorate (Monteiro, Sá, & Loureiro, 1986) focusing on the degree at which the several topics of the curriculum were actually taught during the school year 1985/86. A teachers' questionnaire was sent to a sample of Preparatory Schools covering the continental part of the country asking for the degree of coverage of 20 topics in the programs of each of the two years course. A total of 48 schools comprising 478 first year classes and 391 second year classes was studied. The authors concluded that there was a considerable imbalance of curriculum coverage as geometry was barely taught in the two years and this fact explained the high rate of failure in the national exams. In the end of the study the Inspectorate recommended that it was time to change all the programs at non-university levels, which occurred by the end of the 1980s decade.

We looked particularly at the accomplished curriculum in the two years by looking both at the percentages of classes that taught the complete topic and the average number of class times devoted to the topic. To do that, we aggregated the topics of the original study (numbered from 1 to 20) into 7 topics for the first year (Table 3) and 6 for the second year (Table 4).

Table 3. *Accomplished curriculum in the first year, per topic*

Topics	*Sets*	*Complementary set*	*Arithmetic operations*	*Equations*	*Multiples, divisors*	*Numerical expressions*	*Geometry*
Percentage of classes that taught the topic entirely (*n*=478)	90,2	7	81,4	73	44	87	20,2
Average of total number of class time devoted to the topic	26,9	0,2	28,9	3,3	2,5	5,4	8,5

Source. Table compiled from Monteiro, Sá, and Loureiro (1986, pp. 30, 40). Aggregation of topics: sets: 1-5; complementary set: 7; arithmetic operations: 6-11; equations: 12; multiples, divisors: 13; numerical expressions: 14; geometry: 15-20.

We also looked at data from topics clearly associated with modern mathematics, namely, sets and complementary sets in the first year and partitive operators in the second. In the first year (Table 3) we notice that sets occupy one third of the whole school year. It seems that the recommendations to limit the topic to two weeks given in 1969 was still largely ignored by the teachers 14 years latter.

Table 4. *Accomplished curriculum in the second year, per topic*

Topics	*Multiples, divisors*	*Partitive operators*	*Fractions*	*Equations*	*Proportionality*	*Geometry*
Percentage of classes that taught the topic entirely (*n*=391)	98,5	20	89,8	75	52	8,9
Average of total number of class times devoted to the topic	14,9	1,7	41,4	5,8	5,8	6

Source. Table compiled from Monteiro, Sá, and Loureiro (1986, pp. 35, 45). Aggregation of topics: multiples, divisors: 1-2; partitive operators: 3; fractions: 3-8; equations: 9; proportionality: 10; geometry: 11-20.

We also noted that the two remaining topics that relate to modern mathematics, and complementary sets in the first year (Table 3) and partitive operators in the second (Table 4), were barely taught. In the minds of the reformers of 1968, the first, complementary sets, was necessary in order to understand subtraction as the cardinal of complementary sets. Apparently, teachers were bypassing this approach (Table 3), most probably teaching subtraction the “classic” way. The same occurs with partitive operators (Table 4) which were not explicitly included in the program of 1968 but took a major role in the program of 1974 where fractions were taught as operators either partitive, multiplicative, or both. This study provides a glimpse of the curriculum modelled by teachers as they devoted residual class time to this notion, which may imply that they also taught fractions “the classic way”, eventually merely retaining the terminology.

Both studies, Crato’s research on the 1972 exam and the Inspectorate report of 1986, confirmed that sets, their basic operations, and associated terminology remained the only successful topic of the modern mathematic reform in CPES. They also showed that most of class time of the first year was devoted to this topic, which naturally diminished the amount of time available to other topics, namely geometry or sharpening arithmetic and algebraic calculations. Both studies also showed that students and teachers alike did not relate well with other notions of modern mathematics. In the 1972's exam we saw students' performance in a very simple arithmetic task hindered by the use of modern language and in the 1986's survey we noticed how teachers avoided the most complex modern approaches to subtraction and fractions.

MODERN MATHEMATICS IN GRADES 7TH THROUGH 9TH

From 1977 until 1979, a major project involving the collaboration of Swedish educators evaluated teaching and learning of grades 7th through 9th and published a total of 16 reports, 15 of which of empirical research by collecting data from students, teachers, parents, members of Executive Councils, and businesses (Matos, 2010). Five of these studies refer specifically to teaching and learning mathematics between 1975 and 1979. These studies assume great importance today, as they provide an opportunity of studying the conditions in which the reform of modern mathematics was applied, in particular, an appreciation of the curriculum presented to teachers and the accomplished curriculum of mathematics. They are particularly helpful as they provide a foreign perspective, as was the case of Wiggo Kilborn, a critique of modern mathematics, often at odds with the perception of the Portuguese educators supervising mathematics programs at the time. Sticking with the purpose of this article, we will look into the several levels of curriculum development we have been discussing.

The 1977 program for grades 7th through 9th was the third version of the original modern mathematics program implemented in 1970. For the purposes of our research, there were no major differences among these versions. It involved the study of rational and real numbers, first and second degree equations and inequalities, applications (functions). Geometry included transformational geometry (isometries, homotheties), Pythagorean theorem, space geometry and trigonometry. The new language of mathematics was incorporated in all these topics: the program began with a propaedeutic to logic, binary relationships precede the study of functions, and vector algebra predating the study of geometry.

Reports were very critical of the curriculum presented to teachers. For example, one of them states, by analyzing the curricula for the 7th, 8th and 9th grades it appears that they are curricula typical of the first generation of modern mathematics; they show, moreover, a keener interest in teaching pure mathematics that will make the students good mathematicians. (Catela & Kilborn, 1979, p. 41)

The reports mentioned several times the bad results the new approaches produced in foreign countries. Adopting simultaneously a pedagogical and a critical tone the reports explained that in other countries the initial enthusiasm was replaced by a disenchantment caused by learning difficulties, motivation, and major limitations of the students in solving even simple problems. These opinions were supported by references to the three ICME held recently (in 1976) and to international studies, remarking what extent is modern mathematics necessary, both in Portugal and elsewhere, to the child's education or profession in the future. The answer to this question is also one of the main reasons many countries have changed their [modern] mathematics curricula, using other alternatives according to the capabilities and needs of children. (Catela & Kilborn, 1979, p. 42).

Textbooks, another dimension of the curriculum presented to teachers, were also assessed by the Project. At the time, there was a dominant collection of textbooks in use in schools and alternative manuals were residual. Shortages in the use and access to textbooks were reported (Catela & Kilborn, 1979). For example, although in the year 1975/76 new programs had been published, but new manuals did not immediately follow. Even in 1977/78 the project documented some problems accessing books. In general, students could only acquire the book in December, at the end of the 1st school term.

The content of these books drew strong criticism from the evaluators (Catela & Kilborn, 1979). Firstly, they argued that books were almost replicas of the programs, which could pose problems for students and teachers. They indicated that programs used concepts and sequences very different from regular teaching, so that the structure of "pure" logic of the books did not facilitate their understanding by teachers who were not confortable with meanings and assumptions on which these concepts and sequences were based. The structure of the books was also questioned. Students were often required to go through five or more pages of text before having the opportunity to solve tasks.

For the authors of the reports, the Portuguese books, unlike those of other countries, did not accommodate an individualized learning, because they did not contain diagnostic tests allowing the verification of knowledge by the students. On the other hand, there were few exercises at the end of each section that allowed students to assess understanding before moving on. They also pointed out the linear structure of the books. Students only had a single contact with each topic, which implied that there were no ways to compensate later for a more hasty study. However, born out of foreign tradition about textbooks, the reports may be underestimating the pervasive use of "exercise books", which sometimes were even replacing the "regular" ones.

The accomplished curriculum was also studied by the Project. They used longitudinal testing (three comparable tests, five questions each) through the years 1977/78 and 1978/79. Each question of the tests should encompass three different levels of difficulty and so each had three items and it was expected that 75% of students responded correctly to the first, 50% to the second and 25% the third. It was therefore expected that students obtain an average score of 50% in each test question. Almost every question privileged knowledge of algorithmic or algebraic nature, in which the greater complexity meant replacing integers by fractions, additions and subtractions by multiplications and divisions, or by incorporating powers.

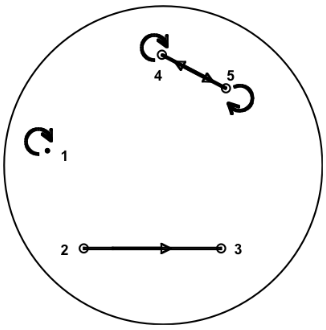
The first two tests, M-I with topics from 7th grade and M-II with 8th grade contents, were prepared by Maria Emília Catela, Wiggo Kilborn and the authors of the new programs (Catela & Kilborn, 1979). The tests were previously verified but the results were very weak. However it was decided to keep M-I and change some details of M-II. The test for grade 7, M-I, was also applied to grade 8. The results were comprehensively examined elsewhere (Ponte, Matos, & Abrantes, 1998).

Overall results were extremely low. Performances were much lower than expected, even for 7th grade contents tested on the 8th. On average, students in the 7th grade got a rating of 13%, and the 8th achieved 24% in the contents of 7th grade and 25% in the 8th. On average, students in 9th grade scored 29% (Catela, 1980). Another report (Catela, 1980) blamed these results on the excessive difficulty of the tests. Referring to the initial tests that were tried, she stated these trial tests were constructed by the authors of the programs which are also teachers. The level of the tests M-I and M-II (1st trial version) shows, therefore, the notion that these teachers have of the level of their own classes, and it was ultimately proved that this notion contains higher expectations than the actual knowledge of students. (Catela, 1980, p. 24)

Searching for explanations for this mismatch, the author concluded that one reason for the high expectation on the part of authors of the programs (and perhaps teachers in general) may be the fact that modern mathematics, whose concepts have guided the current programs, is considered easier for students than conventional mathematics. In fact, when mathematics was introduced in secondary programs in several countries there has been no investigation of how students would accept it in terms of learning, and it was immediately assumed that Modern Mathematics was actually simpler. However, experience has shown otherwise. (Catela, 1980, pp. 24-5)

The answers to the questions of the tests involving modern mathematics were of particular importance for us. Unfortunately, there was only one such question, question 5 of M-I that dealt with binary relationships:

> 5.1. Given the sets
>
> > A = {0, 1, 3} and B = {-1, 1, 2} and the condition $x + y < 1$, indicate the ordered pairs of the relationship defined from A to B.
>
> 5.2. Consider the binary relationship defined in the set {1, 2, 3, 4, 5} and represented by the diagram. Indicate the missing pairs for the relation to be reflexive.



> 5.3. Represent in extension the classes in which the set {11, 18, 21, 28, 37, 31} is divided by the relationship defined by the condition "if it has the same unit figure of y".[3]

From the available copies of the report it is difficult to read the number of correct answers per class for each item. But in one 7th grade class, which has all numerals legible, the following percentages of correct answers were obtained: 5.1 - 0%, 5.2 - 0%, 5.3 – 6,7 %. It is also possible to know the average percentage of correct answers per class, as computed by the Project. For example, the mean percentage of correct answers for the previous class was 2,2% which corresponds to the average of three percentages (Catela & Kilborn, 1979, p. 21). For purposes of this article, these percentages means were grouped at 10% intervals and the data obtained are presented in Table 5.

[3] The answers were: 5.1: (0, -1), (1, -1); 5.2: (2, 2), 3,3); 5.3: $\{11, 21, 31\}$, $\{18, 28\}$, $\{37\}$.

Table 5. *Number of classes with correct answers to question 5 of test M-I per grade*

	7th grade		8th grade	
Average percentage of correct answers	Number of classes	%	Number of classes	%
0 a 10 %	16	64	12	55
10 a 20 %	4	16	8	36
20 a 30 %	3	12	1	5
30 a 40 %	2	8	1	5
40 a 50 %	0	0	0	0
Total	25	100	22	101

Source. Compiled from Catela and Kilborn (1979, p. 21).

Performance in this question about binary relationships was very weak. For example, sixteen 7th grade classes (64% of the total 7th grade classes) and twelve 8th grade classes (55%) had an average percentage of correct answers less that 10%. According to the expectations of the authors of the test, the average percentages should be 50%. It is, however, 8,8% in 7th grade and 9,7% in the 8th, which, at the same time correspond to the lowest percentage of correct responses of the test.

The aggregation of the correct answers to the three items just gives us a partial view. Looking at the disaggregated maps (Catela & Kilborn, 1979, pp. 21, 22), we detected many classes in which few or no students (0 or 1) answered correctly, and some (few) classes with slightly better results but which were still very far from the expected results. In the Northeast region, in particular, the classes showed a high number of null responses (33 in 57, compared to 28 in 84 in the region of Lisbon). The best results (over 30% average of correct answers) are obtained in three groups of schools in the city of Lisbon which had teachers well acquainted with the reform.

What are the reasons behind such poor results? The M-I test was developed in cooperation with the authors of the program and that there was a pre-testing. We should also reject the hypothesis that students had difficulties computing these items. These difficulties were clearly present in the items related to algebra or fractional numbers, but the answer question 5 only requires a correct linguistic interpretation.

The poor performance of students on the issue of binary relationships seemed to be due to two factors: shortcomings in the teaching process and intrinsic difficulty. In other words, it is very likely that, not having learned binary relations during their initial scientific formation, teachers tended either to teach it inappropriately, or not to teach it at all, as we have seen happening in CPES. But even classes taught by teachers conversant with the new ideas, as those from Lisbon, performed poorly. And even these teachers seemed not being able to teach in such a way that learning would remain stable as students moved from the 7th to the 8th grade, contrary to what happens in other "classical" items. Binary relations seem indeed to have an intrinsic difficulty of their own. This is stated in the report that concluded that this topic was not appropriate for these students.

These results are coherent with what we found in Paulo Crato's study (1972). The introduction of terminology characteristic of modern mathematics, although superficially seized by students, kept them from answering some mathematically simple questions. This study also suggests problems with teacher preparation and the inadequacy of certain topics to the mental age of the students.

CONCLUSIONS

With the intention to understand how the modern mathematics curricular reorganisation fulfilled the expectations of an improvement in mathematics teaching and learning, we probed four quantitative studies developed by Portuguese governmental institutions intended to evaluate this reform at several grade levels. Framing our analysis on Gimeno's curricular levels (2000), we analysed the reform as it was implemented in three sub-systems re-appreciating investigations performed in the past. Firstly, analysing the Sebastião e Silva experiment, we concurred with common wisdom that agrees that curriculum modelled by teachers in experimental classes produced a distinct class environment especially favouring students with a penchant for mathematics and sciences, as was the intention of the experiment, although effects are not clear for the rest of the students. We also found that, by 1966, there was a significant number of teachers believing that the new program was taking an excessive amount of class time and that the new ideas should not be limited to a small number of students.

We then focused our attention to the implementation of the reform in CPES. The new curriculum now was encompassing all the students and the results were not very good. Integrating two large scale studies, we found that there are reasons to believe that the curriculum presented to teachers was too large and paid excessive attention to the details of mathematical language. Curriculum modelled by teachers reflected these shortcomings and from the beginning of the new course in 1968 until the curricular study of 1986 teachers were spending to much time with the elementary aspects of sets and their operations neglecting, or even bypassing, other topics the reformers thought should frame the arithmetic operations (complementary sets and partitive and multiplicative operators) and geometry. The accomplished curriculum evaluated in 1972 confirms in broader terms these findings.

Lastly, we addressed curriculum in grades 7th through 9th of the new unified course. A large project evaluating several dimensions of the Portuguese school systems conducted by Portuguese and foreign researchers provided a bleak perspective of mathematics teaching and learning at these grades in the second half of the 1970's decade. Curriculum presented to teachers, either the program or the textbooks, was considered out-dated and not adjusted to students' needs and the accomplished curriculum and mathematics learning was thought to be well below the expectations of the educators in the Ministry of Education overseeing mathematics education.

When these reports were conducted, the debate in Portugal about the teaching and learning of mathematics was very limited (Matos, 2008). The regime that ended in 1974 limited for nearly half a century the democratic functioning of organizations in which social forces could meet. As a consequence, these reports had no effect on actual teaching of mathematics. Apparently, people in charge had a sense that things were falling apart but had run out of ideas about how to mend the situation. The reformers of the 1960s understood that the proper use of mathematical language (almost exclusively understood as the apparatus related to sets, their operations and Boolean logic) would guarantee a good understanding, that is, the use of an appropriate language would mirror the clarity of reasoning. We know today that this is not the case, that is, the relationship between comprehension and language is much more complex than previously thought. It is widely accepted today that one of the problems of the reform of modern mathematics in Portugal, and which led to its discredit during the 1980s, was the excessive emphasis on formalism and language. In 1981 the Portuguese Society of Mathematics conducted a series of meetings attended by mathematics teachers and mathematicians to debate the mathematics programs, which led to a document strongly criticizing the situation and proposing an urgent changes (Matos, 2008). This document judged the discipline of mathematics as hermetic, more formalized, with a stronger emphasis on symbolism, with a more cumbersome language and detached from reality and applications. (Os programas em debate, 1982, p. 20)

However, only in 1989 new programs were adopted.

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