"...CUPIENS MATHEMATICAM TRACTARE INFRA RADICES METAPHYSICE..." ROGER BACON ON MATHEMATICAL ABSTRACTION[1]

"...CUPIENS MATHEMATICAM TRACTARE INFRA RADICES METAPHYSICE..." ROGER BACON SOBRE LA ABSTRACCIÓN MATEMÁTICA

Dominique Demange

Université Paris Nanterre

Abstract

In some passages of the *Opus maius* and the *Opus tertium*, Roger Bacon holds that mathematical objects are the immediate and adequate objects of human's intellect: in our sensible life, the intellect develops mostly around quantity itself. We comprehend quantities and bodies by a perception of the intellect, because their forms belong to the intellect, namely, an understanding of mathematical truths is almost innate within us. A natural reaction to these sentences is to deduce a strong Pythagorean or Platonic influence in Roger Bacon's theory of mathematical knowledge. However, Bacon has always followed Aristotle's view according to which numbers and figures have no real existence apart the sensible substances, and universal knowledge comes from sensory experience as well. It appears that Bacon's claim that quantity is the first object of human's intellect comes from an original reading of a passage of Aristotle's *On Memory and Reminiscence*. In this paper, we try to clarify Bacon's views about mathematical abstraction and intellectual perception of mathematical forms in his Parisian questions on *Physics* and *Liber De causis*, the *Perspectiva*, *Opus maius*, *Opus tertium*, the *Communia mathematica* and the *Geometria speculativa*. We conclude that Bacon considered mathematical abstraction as a mode of perception of the internal structure of the physical world: mathematical abstraction does not mean for Bacon an act of separation of ideal forms from the sensible matter, but a possibility of intuition of the internal structure of the sensible world itself, a faculty which is necessary for human's perception of space and time.

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Keywords

Abstraction; Quantity; Euclidian Geometry; Medieval Visual Theory; Medieval Mathematics; Medieval Physics; Mathematical Physics

Resumen

En algunos pasajes del *Opus maius* y del *Opus tertium*, Roger Bacon sostiene que los objetos matemáticos son los objetos inmediatos y adecuados del intelecto humano: en nuestra vida sensible, el intelecto se desarrolla fundamentalmente en torno a la cantidad. Comprendemos las cantidades y los cuerpos mediante una percepción del intelecto, porque sus formas pertenecen al intelecto, es decir, para nosotros la comprensión de las verdades matemáticas es prácticamente innata. Una reacción natural a estas afirmaciones consistirá en deducir una fuerte influencia pitagórica o platónica en la teoría del conocimiento matemático de Roger Bacon. Sin embargo, Bacon siempre ha seguido el punto de vista de Aristóteles, según el cual los números y las figuras no tienen una existencia real aparte de las sustancias sensibles – y el conocimiento universal proviene también de la experiencia sensorial. Parece que la afirmación de Bacon de que la cantidad es el primer objeto del intelecto humano tiene su origen en una lectura original de un pasaje de *Sobre la memoria y la reminiscencia* de Aristóteles. En este trabajo se intentan aclarar las opiniones de Bacon sobre la abstracción matemática y la percepción intelectual de las formas matemáticas en sus cuestiones parisinas sobre la *Física* y el *Liber de causis*, la *Perspectiva*, el *Opus maius*, el *Opus tertium*, la *Communia mathematica* y la *Geometria speculativa*. Concluimos que Bacon consideraba la abstracción matemática como un modo de percepción de la estructura interna del mundo físico: la abstracción matemática no significa para Bacon un acto de separación de las formas ideales de la materia sensible, sino una posibilidad de intuición de la estructura interna del mundo sensible, facultad que es necesaria para la percepción humana del espacio y del tiempo.

Palabras clave

Abstracción; Cantidad; Geometría euclidiana; Teoría visual medieval; Matemáticas medievales; Física medieval; Física matemática

Part IV of the *Opus maius* is dedicated to mathematics.[2] To all the sciences, mathematics is "the gate and the key".[3] Knowledge of mathematics is required in physics, theology, music, astronomy, logic – in all sciences. The priority of mathematics is proven not only for the logical construction of the sciences, but also in accordance with the order of the acquisition of knowledge. Our understanding of mathematics is immediate, "almost innate" and its knowledge is certain.[4] Certainty is gained in mathematics in two ways: by demonstration through necessary causes and by an

[2] On the topic of this paper, see George Molland, "Roger Bacon's Knowledge of Mathematics", in *Roger Bacon and the Sciences. Commemorative essays*, edited by J. Hackett (Leiden-New York-Köln: Brill, 1997), 151-174; David C. Lindberg, "On the Applicability of Mathematics to Nature: Roger Bacon and his Predecessors", *The British Journal for the History of Science* 15/1 (1982): 3-25; Cecilia Panti, "Natural Continuity and the Mathematical Proofs Against Indivisibilism in Roger Bacon's *De Celestibus* (*Communia Naturalium*, II)" in *Roger Bacon's Communia Naturalium. A 13th Century Philosopher's Workshop*, edited by P. Bernardini and A. Rodolfi (Micrologus Library) (Firenze: SISMEL-Edizioni di Galluzzo, 2014), 159-190. For the works of Roger Bacon, I will refer to the following editions: *Letter to Pope Clement IV*, edited and translated by N. Egel, *Revista Española de Filosofía Medieval,* 27/2 (2020): 143-174; *De viciis contractis in theologia*, edited by R. Steele, *Opera hactenus inedita Rogeri Baconi,* fasc. 1 (Oxford: Clarendon Press, 1905); *Summulae Dialectices*, edited by A. De Libera. "Les *Summulae dialectices* de Roger Bacon", *Archives d'Histoire Doctrinale et Littéraire du Moyen Âge* 53 (1986): 139-289 and 54 (1987): 171-278; *Questiones supra libros prime philosophie Aristotelis*, edited by R. Steele and F. M. Delorme, *Opera hactenus inedita Rogeri Baconi,* fasc. 10. (Oxford: Clarendon Press, 1930); *Questiones supra libros octo physicorum aristotelis*, edited by F. M. Delorme and R. Steele, *Opera hactenus inedita Rogeri Baconi,* fasc. 13. (Oxford: Clarendon Press, 1935); *Questiones supra librum de causis*, edited by R. Steele and F. M. Delorme, *Opera hactenus inedita Rogeri Baconi,* fasc. 12. (Oxford: Clarendon Press, 1935); *Opus maius*, edited by J. H. Bridges, 3 vols. (Oxford and Edinburgh: Clarendon Press, 1897-1900); *Opus maius*, Part IV, English translation by P. W. Dennis, *Roger Bacon's Mathematical Thought: A translation of Part IV of the Opus maius with Introduction and Commentary*, by P. Willard Dennis, Ph Dissertation (Dallas: University of Texas, 2011); *Opus tertium*, edited and translated into German, with notes, by N. Egel (Hamburg: Felix Meiner Verlag, 2019); *De multiplicatione specierum*, edited and translated by D.C. Lindberg, *Roger Bacon's Philosophy* of *Nature* (Oxford: Clarendon Press, 1983); *Communia Naturalium*, edited by R. Steele, *Opera hactenus inedita Rogeri Baconi*, fasc. 2-4 (Oxford: Clarendon Press, 1910-1913); *Communia Mathematica*, edited by R. Steele, *Opera hactenus inedita Rogeri Baconi*, fasc.16. (Oxford: Clarendon Press, 1940); *Perspectiva*, edited and translated into English by D. C. Lindberg, *Roger Bacon and the Origins of Perspectiva in the Middle Ages* (Oxford: Clarendon Press, 1996); *Geometria Speculativa*, edited and translated into English by G. Molland, "Roger Bacon's *Geometria Speculativa*", in *Vestigia Mathematica. Studies in medieval and early modern mathematics in honour of H.L.L. Busard,* edited by M. Folkerts and J. P. Hogendijk (Amsterdam: Rodopi B.V. Editions, 1993), 265-303. I will also refer to the spurious (pseudo-Bacon) *Questiones super Libros IV Physicorum Aristotelis*, edited by F. M. Delorme, *Opera Hactenus Inedita Rogeri Baconi*, fasc. 8 (Oxford: Clarendon Press, 1928) and *Questiones altere supra libros prime philosophie Aristotelis*, edited by R. Steele and F. M. Delorme, *Opera hactenus inedita Rogeri Baconi, fasc.* 11 (Oxford: Clarendon Press, 1932).

[3] *Opus maius* IV, dist.1, chap.1, ed. Bridges I, 97; transl. Dennis, 62.

[4] *Opus maius* IV, dist.1 chap.3, ed. Bridges I, 103: "mathematicarum rerum cognitio est quasi nobis innata"; transl. Dennis, 71: "... an understanding of mathematical truths is almost innate within us." I will give an interpretation of this '*almost* innate' ('*quasi* innata') in part II of this paper.

immediate verification of its conclusion by sensory experience.[5] No other science can reach the same degree of evidence and certainty. Mathematical knowledge is the most clear and certain, its evidence is brought from the very nature of its objects, since mathematical objects (figures, numbers, etc.) are the immediate and adequate objects of human's intellect:

> And, seeing that it has already been demonstrated by its particular property that mathematics is prior to the other sciences, and is useful and necessary to them, it is now to be demonstrated by arguments taken up on the part of its subject. Thus, first, it is natural to proceed from the senses to the intellect, since, by abandoning the senses, one also abandons the knowledge which derives from those senses. In the first book of the *Posterior Analytics*, it is said that as the senses advance, the human intellect advances. However, quantity is particular to the senses, because it involves the common sense and is perceived by the other senses. Nothing can be perceived without quantity; for this reason, the intellect develops most as a result of quantity. Secondly, the very act of understanding itself is not completed without continuous quantity, because Aristotle, in his book *On Memory and Reminiscence*, states that our whole intellect is associated with continuity and time. From this, we comprehend quantities and bodies by the perception of the intellect, because their forms belong to the intellect. The forms of incorporeal things, however, are not apprehended by our intellect; or if they were formed in the intellect, as Avicenna indicates in the third book of the *Metaphysics*, we nevertheless do not perceive these forms, because our intellect is more strongly oriented around bodies and quantities. Therefore, by our way of argumentation and attention to the corporeal and the quantifiable, we seek knowledge of incorporeal things, as Aristotle does in eleventh book of the *Metaphysics.* Therefore, the intellect develops mostly around quantity itself, and it is in this way, according to the common condition of understanding, that quantities and bodies are apprehended by the human intellect.[6]

[5] *Opus maius* IV, dist.1, chap.3, ed. Bridges I, 105-106; transl. Dennis, 74: "In mathematics, we can arrive at complete truth without error, and at certainty free from doubt, since mathematics can provide a demonstration through a proper and necessary cause. And this demonstration makes the truth known. Similarly, mathematics can provide an example sensible to all, and experience perceptible to the senses through the drawing of figures and counting, so that all is made clear to the senses; for this reason, there can be no doubt in this science." The idea that mathematics provide the proper and necessary cause of natural phenomena comes from Grosseteste's reading of *Posterior Analytics*: see Lindberg, "On the Applicability", 10-14; Jeremiah Hackett, "Robert Grosseteste and Roger Bacon on the Posterior Analytics", in *Erkenntnis Und Wissenschaft/ Knowledge and Science: Probleme der Epistemologie in der Philosophie des Mittelalters/ Problems of Epistemology in Medieval Philosophy*, edited by P. Antolic-Piper, A. Fidora and M. Lutz-Bachmann (Berlin and Boston: De Gruyter, 2004), 161-212.

[6] *Opus maius*, IV, dist.1, chap.3, ed. Bridges I, 107-108; transl. Dennis, 77-78: "Et quoniam jam per proprietatem ipsius scientiae ostensum est, quod mathematica est prior aliis, et eis utilis et necessaria, nunc ostenditur hoc per rationes sumptas a parte sui subiecti. Et primo sic, quia nobis est via nata a sensu ad intellectum, quoniam deficiente sensu deficit scientia quae est secundum illum sensum, ut dicitur Primo Posteriorum, quoniam secundum quod proficit sensus, proficit

It would be interesting to know where Roger Bacon found in Aristotle this Cartesian formula: "*quanta et corpora intelligimus intuitu intellectus*" – we understand quantity and extended bodies by direct intuition of the intellect. In addition, one cannot but be intrigued about Bacon's exegesis of Aristotle's *On Memory and Reminiscence*: while Aristotle writes that intellection of the triangle is not without imagination of some concrete triangle, which *accompanies* the intellection, Bacon reads that the act of intellection immediately relates to the extended triangle itself, for "we comprehend quantities and bodies by the perception of the intellect, because their forms belong to the intellect".[7] As we shall see, some passages of the *Opus tertium* clarify Bacon's reading of this source. Obviously, Bacon had good reasons for considering that for Aristotle, the immediate and intuitive object of human's intellect in the present life is quantity – and hence figures, numbers, space and time.

Roger Bacon's exegesis of Aristotle is of course surprising for the modern reader. For us, the question of the intellectual knowledge of mathematical objects in Aristotle's philosophy is an almost insoluble problem.[8] In the scope of Aristotle's psychology,

humanus intellectus. Sed quantitas est maxime sensibilis, quia est sensibile commune, et ab aliis sensibus sensitur, et nihil potest sentiri sine quantitate quapropter maxime potest intellectus proficere circa quantitatem. Secundo, quia ipse actus intelligendi secundum se ipsum non perficitur sine quantitate continua, quia dicit Aristoteles in libro de Memoria and Reminiscentia quod omnis intellectus noster est cum continuo et tempore. Unde quanta et corpora intelligimus intuitu intellectus, quia species eorum apud intellectum sunt. Incorporeum autem species non recipiuntur intellectu nostro; aut si fiant in eo, secundum quod Avicenna dicit tertio Metaphysicorum, non tamen hoc percipimus propter occupationem fortiorem intellectus nostri circa corpora et quanta. Et ideo per viam argumentationis et admirationis corporalium et quantorum investigamus rerum incorporalium notitiam, sicut Aristoteles facit in libro undecimo Metaphysicorum. Quapropter proficiet maxime intellectus circa ipsam quantitatem, eo quod quanta et corpora in quantum huiusmodi appropriantur intellectui humano secundum statum communem intelligendi."

[7] Aristotle, *On Memory and Reminiscence*, 449b30-450a12: "Now, since we have already spoken about imagination in our discussions *On the Soul*, and since it is not possible to think without an image – for the same affection that occurs in drawing a diagram also occurs in thinking: for when drawing a diagram we make no use of the fact that the quantity of the triangle drawn is determinate, but still we draw it as having a determinate quantity; and similarly a person who thinks, even if he does not think about a quantity, he posits a quantity before his eyes, but does not think about it as a quantity; and if the object by nature has quantity, but an indeterminate quantity, he posits a determinate quantity, but thinks about it as quantity only. Now, the reason why it is impossible to think anything without continuity, and impossible to think about things that are timeless without time, belongs to another discussion." Transl. David Bloch, *Aristotle on Memory and Recollection. Text, Translation, Interpretation, and Reception in Western Scholasticism* (Leiden, Boston: Brill, 2007), 27-28.

[8] Emily Katz, "Geometrical Objects as Properties of Sensibles: Aristotle's Philosophy of Geometry", *Phronesis* 64 (2019): 465-513: "According to many, geometry for Aristotle cannot be about sensible things, so that geometrical objects have actual existence only in the mind. For others, some simple geometrical properties are in sensibles, but geometrical objects themselves are fictions. For still others, geometrical objects underlie physical reality, either as potential or actual parts of sensibles, or as entities distinct yet somehow derived from sensibles. There are also many sources of controversy in the details."

quantity (discrete or continuous) is not perceived at the first level of the five exterior senses, but at the secondary level of the internal senses (common sense and imagination), which are parts of the sensitive soul,[9] whereas intellect (*noûs*), dealing with universal concepts, is a 'separate' faculty.[10] If figures and numbers are perceived by the common sense, there is no doubt however that geometry and arithmetic are intellectual sciences, dealing with intelligible (absolutely abstracted) objects. Aristotle himself had recognized that the geometer, while reasoning on a particular triangle of his imagination, was thinking about the triangle in general – the universal concept of triangle.[11] This assumption was necessary for preserving the scientific character of geometry. Alas, this intellectual object could have no place in Aristotle's psychology. If intellectual objects (universals) are completely abstracted from matter, quantity, space and time, the geometer could never demonstrate that the sum of the angles of the triangle in general is equal to two right angles, without integrating in it spatial properties such as continuity and extension. In some places, Aristotle does not hesitate to define the triangle by its integrative parts (lines, angles), thus contradicting his claim that a formal division (which produces the definition) must not be confused with the division of the concrete compound.[12] Adding complexity, Aristotle's appeal to the idea of the intelligible matter of figures was thus inevitable for preserving the consistency of abstractionism: the triangle was abstracted from sensory or individual matter, but not from the intelligible matter which becomes a part of its definition.[13]

The problem of the intelligibility of quantity and its forms (numbers, figures) is therefore anything but simple in the Aristotelian tradition. In the passage of the *Opus maius* quoted above, Roger Bacon does not mention any of these problems; he takes for granted that figures, numbers, and quantity in general are immediate objects of man's intellect. But this was not the common view at that time. In the first half of the

[9] Aristotle, *De anima*, III, c.1, 424a14-21: common properties (movement, rest, figure, magnitude, number, and unity) cannot be perceived by exterior senses alone. Aristotle does not introduce here the concept of abstraction, he says that these properties are perceived by the way of *movement*: "...for all these we perceive by movement, e.g. magnitude by movement, and therefore also figure (for figure is a species of magnitude), what is at rest by the absence of movement: number is perceived by the negation of continuity, and by the special sensibles..."

[10] The intellect cannot function without the products of imagination (and therefore without the sensitive soul) but its operation is not sensitive, and this superior faculty is "not mixed with the body" – *De anima*, III, c.4, 429a25.

[11] Aristotle, *On Memory and Reminiscence*, 449b30-450a12, quoted in footnote 7.

[12] For instance, Aristotle, *Posterior Analytics* I, chap.4, 73a35-36: "Something holds of an item in itself both if it holds of it in what it is – e.g. lines of triangles and points of lines (their essence comes from these items, which inhere in the account which says what they are)", to be compared with *Metaphysics* VII (Z), chap.10, 1034b20-1036a25.

[13] Aristotle, *Metaphysics* VII (Z), chap.10, 1036a9-11: "Some matter is perceptible, e.g. bronze, wood, and all changeable matter, while some is intelligible, namely that which is present in perceptible things but not *qua* perceptible. Such is the matter of the objects of mathematics." On this concept, see Christoph Helmig, "Aristotle's Notion of Intelligible Matter", in *Quaestio* 7 (2007): 53-78.

thirteenth century, it was commonly held that the object of the intellect is the universal, not the singular being – and universal concepts do not integrate the quantitative properties of their objects.[14] Later in the thirteenth century, Peter John Olivi will clarify the question. He will carefully distinguish intellectuality from intelligibility. According to the French Franciscan, the Aristotelian tradition confused intelligibility (the possibility for something to be an object of intellection) and intellectuality (the essences of intellectual beings, which are separated forms). According to Olivi, separation from matter is the condition for intellectuality, not for intelligibility. The object of the intellect is not the intellectual form, but all being in general: everything is intelligible. Therefore, quantity as such, and individual properties of sensible things, are directly intelligible.[15]

This revolution seems to have been prepared by Roger Bacon. Among the various problems involved, we have of course the question of the first object of the intellect (is it the universal or the singular?) and the various theorical aspects of abstraction. Since he tells us that numbers and figures are immediate objects of the intellect, one could imagine that Bacon adopts a Platonic or Pythagorean view: a theory of direct intellection of mathematical ideal essences – but this is not the case. He has always defended the Aristotelian dogma that all our scientific knowledge comes from sensory experience, and that numbers and figures have no real existence apart from the

[14] See the classical study of Camille Bérubé, *La connaissance de l'individuel au Moyen Age* (Paris-Montréal: Presses Universitaires de Montréal, Presses Universitaires de France, 1964); and more recently, Peter King, "Thinking about Things: Singular Thought in the Middle Ages" in *Intentionality, Cognition, and Mental Representation in Medieval Philosophy*, edited by G. Klima (New York: Fordham University Press, 2015), 104-121. About the passage of *On Memory and Reminiscence* (quoted footnote 7): the universal concept of quantity, abstracted from various determinate quantities, has no quantity at all and therefore cannot be the object of any universal geometrical demonstration. If universal propositions of geometry do not posit empirical quantitative properties, they however posit determinate ones (such as the sum of the angles, the ratio of surfaces, etc.). There is therefore no possibility of an intellectual geometrical demonstration without the assumption that determinate quantity can be an object of intellection. This may be one of the reasons for Bacon's original reading of this passage of *On Memory and Reminiscence*. Bacon could have compared this passage with some mathematical texts (such as the famous introduction of Ptolemy's *Almagest*), claiming that mathematical objects are the most intelligible ones. He therefore concluded that Aristotle couldn't have thought differently.

[15] Petrus Iohannis Olivi, O.F.M., *Quaestiones in secundum librum Sententiarum*, edited by B. Jansen, 3 vols., Bibliotheca Franciscana Scholastica Medii Aevi (Quaracchi: Collegium S. Bonaventurae, 1922-1926). Qu.58 (I, 450): "Ad tertiam dicunt quod prima est simpliciter falsa, quod scilicet omne per se obiectum intellectus sit simplex et intellectuale, quia tunc nulla quantitas posset esse per se obiectum intellectus nec conditiones sensibiles et individuales rerum sensibilium, et ita intellectus non haberet totum ens pro obiecto." Qu.72 (III, 50): "non omne intelligibile est intellectuale. Alias nulla quantitativa extensio esset intelligibilis vel intellecta a nobis." On Olivi's theory of quantity and its influence on Ockham, see the classical study of Anneliese Maier, *Metaphysische Hintergründe des Spätscholastischen Philosophie*, Kap. 3 ("Das Problem Der Quantität oder der räumlischen Ausdehnung") (Roma: Edizioni di storia e letteratura, 1955), 143-225.

sensible substances from which they are abstracted. Among various passages which could be presented, we find for instance in the *Geometria speculativa* the definition of figures as entities abstracted from determined matter – and the introduction of the *Communia mathematica*, as we shall see, provides similar definitions.[16]

This is enough therefore, for asking Bacon if he has something more to say about our knowledge of abstract objects. This question is complicated, for several reasons. First, the scope of the concept of abstraction is very wide. 'Abstraction' (*aphairesis*) was Aristotle's response to Plato's theory of ideal figures and numbers, and it was thereafter implicated in the general dispute about the ontological status of universals.[17] In the scholastic tradition, abstraction in general was understood, sometimes confusedly, as the ontological status of immaterial objects, a psychological process, or a logical criterion. While examining the occurrences of the corresponding terms (*abstractio, abstracto, abstractione*, etc.) in Bacon's texts, it appears that we must distinguish:

(1) Logical abstraction: the abstract ('*abstractum*', '*in abstracto*') is understood as the opposite of the concrete ('*concretum*', '*in concreto*'). 'Whiteness' is the abstract name for the concrete term of 'white'. This is a mere logical distinction.[18]

(2) Universal abstraction: the separation of universal properties from the individual determinations – all scientific knowledge is 'abstract' in this sense.[19]

(3) Real abstraction: the real separation (real existence) of a form from its specific matter, of from matter of any kind.[20]

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[16] *Geometria speculativa*, §4, ed. Molland, 270: "geometria speculativa probat conclusiones circa figuras superficiales et lineas absolutas et abstractas a materia determinata, non curans in quo corpore sint nec in qua materia..."

[17] For an extensive study on the subject, see Allan Bäck, *Aristotle's Theory of Abstraction* (Heidelberg New York London: Springer, 2014).

[18] *Summulae dialectices*, I.2 (*De praedicamentis*), §30, ed. A. De Libera, 190: "Dicit igitur Aristoteles quod denominativa sunt quaecumque ab aliquo ut a principali vel abstracto solo casu sunt differentia, id est: in fine, hoc est in extrema parte illorum vocabulorum, ut 'album', 'albedo'." *Q.Primae.phil.*, 278: "...sicut se habet album ad albedinem, sic homo ad humanitatem; set in aliis ita est quod abstracto formatur concretum secundum casum nominis rectum..." See in the same volume the questions pages 130-131: "Quaeritur an differat concretum et abstractum in suspiciendo predicationem generis", pages 169-170: "Quomodo querent sumi species relationis, scilicet in abstractione vel in concretione."

[19] *Q.octo.Phy.*, 94-95: "duplex est abstractio, forme a materia et universalis a particularibus"; "omnis scientia est de universali et de ipso prout est abstractum a singularibus; quare physica, cum sit scientia, erit de universali abstracto a singularibus. " (*ibid.*). *CM*, 59: "Sunt autem quinque modi abstraccionis. Unus est communis omnibus scienciis, scilicet secundum quod universale abstrahitur a singularibus..."

[20] *CM*, 59: "Secunda est abstraccio a motu et materia omnino tam secundum rem quam secundum considerationem, et sic sola causa prima dicitur abstracta et separata. Tertia dicitur abstraccio et

(4) Formal abstraction: the possibility for a property (accidental or essential form) to be considered apart from its subject, or from other properties of the same subject. In this specific sense, the famous *auctoritas aristotelis* "to abstract does not imply to lie" ("astrahentium non est mendacium") was used to justify that geometry or arithmetic could be true sciences thanks to the conceptual separability of spatial figures and numbers from the physical matter.[21]

(5) Psychological abstraction: the psychological mechanism by which the intellect forms universal concepts (such as universal numbers or figures) on the basis of sensory experience.

Bacon's psychology of abstraction has already been studied and discussed.[22] By 'psychology of abstraction', I mean the theory according to which universal objects are extracted from sensory matter by a psychological spontaneous mechanism of separation of the universal characteristics from the individual ones. This idea is derived from an interpretation of Aristotle's *De anima* book III chaps. 4-5, in terms of the abstractive action of the agent intellect, which illuminates the forms of imagination present in the sensitive soul. We find a typical formulation of this theory in the questions on *Physics* attributed to the young Bacon by the editors of the *Opera hactenus inedita,* and another in the *Quaestiones supra librum De causis.*[23] However, in the writings

separatio a materia corporali et motu <...> et sic intelligentie sunt abstracte et separate a materia et motu..."

[21] Aristotle, *Physics* B2, 193b34-35, *Auctoritates Aristotelis* (ed. J. Hamesse) n.57, 145. Roger Bacon, *Q.octo.Phy.*, 95: "Ad argumentum (abstraccio secundum intellectum est impossibile quia sic non respondet rei) respondeo quod illa propositio est duplex: aut quod intellectus intelligat rem ut non se habet, et sic intellectus est falsus, quia non potest hoc facere, sed pro modo solum; vel ut intellectus intelligat rem non prout se habet, et sic est possibile, quia intellectus intelligit lineam que est in materia non prout est in materia, sed aliquo alio modo: ideo dico quod potest esse negatio modi intelligendi vel modus negationis." See Aristotle, *Metaphysics* XIII (M), c.3, 1078a15-22; *De anima* III, c.7, 431b12-16. For a more general view on this subject, see John J. Cleary, *Aristotle in Mathematics. Aporetic Method in Cosmology and Metaphysics* (Leiden-New York-Köln: E. J. Brill 1995) especially Chap.5 ("The Ontological Status of Mathematical Objects"), 268-344.

[22] See Yael Raizman-Kedar, "The Intellect Naturalized: Roger Bacon on the Existence of Corporeal Species within the Intellect", *Early Science and Medecine* 14 (2009): 131-157. For a challenging view, see Jeremiah Hackett, "Roger Bacon on Animal Knowledge in the *Perspectiva*", in *Philosophical Psychology in Arabic Thought and The Latin Aristotelianism of the 13th Century*, edited by L. X. Lòpez-Farjeat and J. A. Tellkamp (Paris: Vrin, 2013), 23-42; and a more general discussion Anselm Oelze, *Animal Rationality*, *Later Medieval Theories 1250-1350* (Leiden-New York-Köln: E. J. Brill, 2018), chap.11, 82-87.

[23] Pseudo-Bacon, *Qu.IV.Phy.*, 31: "Unde sensus particularis primo apprehendit res materiale per suas species, et ulterius depurando ad fantasiam deferuntur, et tunc intellectus agens, cujus creata sunt exemplaria, irradiat supra fantasmata, ipsa a conditionibus materialibus abstrahens in intellectu possibili reponendo." Note that these questions on *Physics* are not Bacon's – see footnote 26. *Q.causis*, 51: "...intelligentia acquirere potest species a rebus corporalibus [...] non tamen antequam denudantur a conditionibus materialibus... species recipiuntur materialiter in organis sentiendi et in inferioribus, et cum sint in una memoria et fantasia tunc illuminatur ab agente irridiante super

of the second period (after 1260), references to psychological abstraction disappear, Bacon adopts a theory of incorporation of species in the soul and its progressive spiritualization by the successive faculties along the path of intromission. The discovery of Alhazen's *Perspectiva* (*De aspectibus*) was one of the major reasons of this change. In the scope of Alhazen's *De aspectibus*, references to an abstractive action of the Intellect on the imaginary forms are not only absent but simply impossible, for the process of perception is intellectualized at the primary level of the perception of individual visible objects.[24] We should therefore conclude that Bacon abandoned the psychology of abstraction in his mature period of work.[25]

This paper will provide confirmation of this view: some passages of the *Opus tertium* and the *Communia naturalium* confirm what we read in the passage of the *Opus maius* quoted at the beginning of this paper. For Bacon (in his mature period), 'abstract' mathematical objects are known by direct intuitive intellection. The absence of the psychology of abstraction is therefore not surprising and even fully consistent with this thesis. Bacon's mature cognitive theory evidently abandons the classical schemes of psychological abstraction and illumination, which are completely absent in the *Perspectiva*. When in the other texts (for instance in the introduction of the *Communia mathematica*), Bacon uses the concept of abstraction (he says for instance that numbers and figures are 'abstracted from the sensory matter'), this is without any implication at the psychological level. He rather reduces abstraction to the three forms of (2) universal abstraction, (3) real abstraction and (4) formal abstraction distinguished above, for the purpose of ontological, logical or epistemological discussions.

However, while examining the texts in greater detail, I wondered if Bacon had *ever* adopted a standard view concerning the psychology of abstraction. I had two major reasons for doubting that. The first comes from the sources. According to a recent study of Silvia Donati, we have to be very cautious with three series of questions on *Physics* and *Metaphysics* from the *Opera hactenus inedita* which do not seem to come from Bacon's teaching.[26] While eliminating these questions from the scope of my study, I also eliminated the most evident passages in favor of the classical psychology of abstraction in the first period of Bacon's career.[27] I also realized that Bacon was already using

huiusmodi species et ita a conditionibus materialibus denudantur..." I will comment on this passage in section I below.

[24] See Abdelhamid I. Sabra, "Sensation and inference in Alhazen's Theory of visual perception", in *Studies in Perception: Interrelations in the History of Philosophy and Science*, edited by P. K. Machamer and R. G. Turnbull, (Columbus: Ohio State University Press, 1978), 160-185 – reprint in Abdelhamid. I. Sabra, *Optics, Astronomy and Logic. Studies in Arabic Science and Philosophy* (Aldershot: Ashgate Variorum, 1994). I will examine this question more in detail in section II.

[25] This is one of Yael Kedar's conclusions, see "The Intellect Naturalized".

[26] Silvia Donati, "Pseudoepigrapha in the *Opera hactenus inedita Rogeri Baconi* ? The Commentaries on the Physics and on the Metaphysics", in *Les débuts de l'enseignement universitaire à Paris (1200-1245 environ)*, edited by O. Weijers and J. Verger (Turnhout: Brepols, 2013), 153-203.

[27] The text of the *Q.IV.Phy.* given in footnote 23.

Alhazen's *Perspectiva* in his discussion of the origin of intellectual knowledge in the 'authentic' Paris questions.[28] Therefore, the idea that Bacon *abandoned* a standard view about psychology of abstraction when he discovered Alhazen's *Perspectiva* had no more solid textual and historical foundations.

The second reason for doubting that Bacon ever adopted a standard view about the psychology of abstraction is his 'extreme realism' about universals.[29] The key aspect of psychological abstractionism (understood in the classical way in the thirteenth century) is that universals are *only* a product of psychological activity. Universal concepts have their origin in the ontological structure of the sensible things (they have a *fundamentum in re*), but they are elaborated and formed by the psychological process *alone*. The universal properties are not received through the senses, but rather psychologically constructed by the soul's powers on the basis of sensory data. It seems, however, that Bacon never adopted this view. From the beginning (from the Paris commentaries), Bacon has always considered that universal properties are given in the exterior thing, and that they act on the senses according to the universal nature of the thing, by means of species multiplying in the medium. How could he therefore imagine the process of 'abstraction' of universal forms from singulars? This is not at all easy to understand. While introducing the classical terminology about abstraction, Bacon could understand it as a mode of reception of the universal properties of exterior things – a reception which can imply some mechanisms of purification or separation, of 'spiritualization', but not of mental construction or reconstruction on the basis of the sensory data.[30]

My first task is therefore to examine the question of abstraction in the Paris questions. In a second part of the paper, I will study the question of the perception and knowledge of visual forms and of spatial relations in the *Perspectiva*. Then, I will comment on a passage of the *Opus tertium*, arguing that quantity is the first object of

[28] *Q.Primae.phil.* (5, 10, 96, 153); *Q.octo.Phy.* (7, 33, 206, 292, 324). In the spurious questions, Alhazen appears in only one place: *Q.altere* (8).

[29] See Theodore Crowley, "Roger Bacon: the Problem of Universals in His Philosophical Commentaries", *Bulletin of the John Ryland's Library* 34 (1951), 264-75; Thomas Maloney, "The Extreme Realism of Roger Bacon", *The Review of Metaphysics* 38 (1985): 807-837; Thomas Maloney, *Three Treatments of Universals by Roger Bacon. A translation with introduction and notes* (State University of New York at Binghamton: Center for Medieval and Early Renaissance Studies, 1989). Notice that Crowley's and Maloney's analysis are unduly complicated by the parasitic presence of the inauthentic *Q. altere* in the field of their studies. See also Jeremiah Hackett, "Roger Bacon (B. CA. 1214/20; D. 1292)", in *Individuation in Scholasticism. The Later Middle Ages and the Counter-Reformation, 1150-1650*, edited by J. E. Gracia (Albany: State University of New York Press, 1994), 117-139; Chiara Crisciani, "Universal and particular in the *Communia Naturalium*: between 'extreme realism' and 'experientia'", in *Roger Bacon's Communia Naturalium. A 13th Century Philosopher's Workshop*, edited by P. Bernardini and A. Rodolfi (Micrologus Library) (Firenze: SISMEL-Edizioni di Galluzzo, 2014), 57-82.

[30] See Raizman-Kedar, "The Intellect Naturalized", 141.

human's intellect. Lastly, I examine the theory of knowledge of abstract objects in the *Communia mathematica* and the *Geometria speculativa*.

I. Abstraction in the Paris questions

In the authentic Paris questions on *Physics* and *Metaphysics*, references to the psychological process of abstraction are rare and vague. We read that universal concepts are immaterial for they are "abstracted from matter" or that mathematical objects are "abstracted from matter" – without any explanation.[31] The discussion concerning the various modes of abstraction in sciences remains at a general level. The metaphysician considers beings in their *esse essentiae*, the mathematician and the physician in their *esse actuale*; therefore, the mathematician abstracts the form of the concrete being from the sensible common matter, whereas the physician considers the forms which depend on sensible matter.[32] According to Bacon's realism, the universal object can be seen from a double point of view: as constitutive part of the thing, or as a mental abstraction. Abstraction is therefore the way the intellect considers the real universal properties apart from the concrete beings: the mental universal concept is a representation, a *similitude* (a species) of the true universal given in the exterior thing.[33]

Bacon's first theory of knowledge is everything but a standard Aristotelian theory. The first pages of the *Questions on Physics* offer a synthetic discussion about the origin of human knowledge which associates innate (concreated) and acquired species, the distinction between distinct and confused knowledge of universals or/and of individuals, and various corresponding species (acquired, innate, confused and distinct).[34] It seems to me that:

(1) This is a theory of the acquisition of knowledge, formulated in terms of change from confused knowledge to a distinct knowledge.

(2) Confusion and distinction are discussed on two levels: (*a*) the natural acquisition of knowledge (with the example of the child who gains more

[31] *Q.Primae.phil.*, 23: "...quod abstrahitur a materia et a partibus per cognitionem, et sic universale est immateriale"; *Qu. prime phil.*, 91: "omnia mathematica sunt abstracta a materia, ideo magis sunt formalia quam alia, et a parte forme est diversitas secundum speciem..."
[32] *Q.octo.Phy.*, 94-98.
[33] *Q.Primae.phil.*, 243: "Et hoc concedo quod universale est quod est vere predicabile de rebus, et est res naturalis et in predicamento, tale est unum in multis et de multis, exclusa omni operatione anime; set aliud est universale quod est similitudo veri universalis, et quod fit per abstractionem intellectus, et tale non est sine operatione anime que est abstractiva, et tale est principium cognoscendi, et de tali procedunt rationes Aristotelis et Algazelis et Avicenne, quia intelligit de universali quod est similitudo expressa veri universalis."
[34] *Q.octo.Phy.*, 6-18.

distinct knowledge by experience and memory),[35] (*b*) the theological difference between the distinct knowledge of man in his perfect state and the confused knowledge he has in the present state (for the union of the soul with the body obscures human's soul).[36] Therefore, due to the weakness of our intellect in the present state, our knowledge begins with the confused universal concept of the individual.[37]

(3) A process of abstraction is never associated to this theory. Bacon is thinking in terms of a direct action of substances and species. The corresponding terms (*abstractio*, *abstractum*, etc.) do not even appear. When talking about the sensory experience, Bacon writes that "our intellection can be obtained by species received from exterior things", without any reference to abstraction.[38] Bacon seems to hold that the intellect *receives* the universal form and matter for he writes that it is more *sensitive* to these elements than the senses themselves.[39]

(4) The concepts of universal/particular natures are widely used in order to discuss the priority of nature concerning the orders of intention (finality) and operation (efficient cause). Bacon is therefore analyzing the psychology of knowledge by taking the point of view of the action of universal and particular natures.[40]

This text of the *Questions in physics* is synthetic and difficult; there are many problems of interpretation concerning the details. However, I don't find in these pages an Aristotelian theory of abstraction.

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[35] *Q.octo.Phy.*, 7: "Nos possumus loqui de anima pueri dupliciter: aut ante exercitium, et sic nihil distinguet, aut post exercitium memorie et experimenti, et sic distinguit aliqua, scilicet illa solum quorum habet memoriam et experimentum. " This point is discussed on the basis of an example taken from Alhazen's *De aspectibus*.

[36] *Q.octo.Phy.*, 11: "...aggeneratur illa confusio per naturam corporis..." See *De viciis contractis in studio theologiae*, 17: "Substancia enim anime, ut dicit, corpus occupat et reddit eam stultam, et facit eam oblivisci sui desiderii proprii, et inquirendi perfeccionem que sibi competit et percipiendi delectacionem perfeccionis sue. Non quod anima sit impressa corpori vel submersa; set quia ligacio est inter illa duo, quod est, desiderium naturale gubernandi corporis, et agitandi affecciones ejus."

[37] *Q.octo.Phy.*, 18: "Ad primam, quia propter debilitatem nostri intellectus est quod non cognoscimus particulare, quia non cognoscit nisi sub confusione, non in propria forma in particulari, quia intellectus noster se habet ad universalia sicut oculus vespertilionis ad lucem diei, ideo non habemus scientiam nec constituimus de particulari, set de universali et confusis, ideo ignata est nobis via a confusis a distincta."

[38] *Q.octo.Phy.*, 10: "intellectus noster potest esse per species acquisitas a rebus extra."

[39] *Q.octo.Phy.*, 6: "Magis sensibile dupliciter: aut quia vehementius et actualius immutat sensum, et sic sensus <est magis sensibilis>, aut quia citius, et sic intellectus est magis sensibilis quia citius immutatur intellectus a materia prima et formis primis quam sensus, et sic dicitur magis sensibilis."

[40] See *Q.octo.Phy.*, 12-18.

The other Paris questions (*Quaestions on Metaphysics* and on the *Librum de causis*) clarify some important points. First, the question of the difference between confused and distinct knowledge at the theological level (see point (2*b*) above) is clarified. The first object of the intellect for separate intelligences (angels) and human's soul in the perfect state is the individual being – not the universal. Separated intelligences (angels) have concreated species of all the corporeal things, and they know all these things as distinct particulars. Their direct cognition of particulars is the basis of their universal knowledge. Admittedly, the first object of human's intellect in the present state is the individual known confusedly, because of the deficiency of man's intellectual soul as linked (*obnubilatus*) to the sensible images in this carnal life; but the normal way of intellection, which will be given in the future life, proceeds from the distinct knowledge of individuals to the knowledge of universals.[41]

Second, the intellect cannot acquire species of corporeal things without separating them from the material conditions. This operation is done by an illumination of the agent intellect. No doubt that we have here a description of the mental process of 'abstraction'; but how does Bacon present it? Abstraction (*abstractio*) is a purification (*depuratio, denudatio*) from specific determinations, such as material and quantitative properties.[42] A purification of the species emitted by the exterior thing is necessary for

[41] *Q.Primae.phil.*, 210: "Set quia non cognoscimus veritates rerum, ideo non diffinimus nec habemus scientiam de hiis, unde scientia non est singularium per defectum nostri intellectus et debilitationem, qui non potest in hac vita cognoscere veritates rerum. Unde sicut unumquodque se habet ad esse, sic ad veritatem et cognitionem aperte. Per hoc patet ad objecta, quia verior est scientia de particulari; set hec erit solum quando complebitur numerus electorum, et cognoscemus tunc universalia per particularia, sicut modo facimus e contrario, quia hoc est secundum possibilitatem nostram, quia modo cognoscimus particularia per universalia. " See also the very interesting discussion about the definition of singulars in *Q.Primae.phil.*, 233-34: "Ideo dicendum quod quantum est a parte rei, [individua] verissime cognoscibilia sunt, a parte nostra non." *Q.causis*, 57-58: "...ideo quia particularia verius habent esse, ideo eorum species sunt apud intellectum intelligentie, unde particulare primo est cognoscibile, universale autem secundo. Set quia nos non intelligimus nisi sub confusione propter obnubilationem intellectus nostri, ideo dicimus quod universale est prius cognoscibile ab intellectu nostro quam particulare [...] omnium causa autem prima in hac vita non <cognoscitur> ab intellectu nostro, et ideo substantiam particularem complete cognoscere non potest in hac vita, set cum erit intellectus in ultima prosperitate, tunc cognoscet particulare primo, universale autem per particulare, modo autem cum est unitus carni econtrario est..." A specific study on the question of confused/distinct knowledge in Bacon would be necessary (and would be quite important for the clarification of some delicate points of Bacon's psychology and epistemology). It seems to me that in his second period, Bacon has not abandoned at all this view about human's knowledge: on the contrary, he has applied it in his *Perspectiva* (and thus completely reinterpreted Alhazen's psychology about our knowledge of individuals).

[42] *Q.causis*, 51: "...intelligentia acquirere potest species a rebus corporalibus [...] non tamen antequam denudantur a conditionibus materialibus... species recipiuntur materialiter in organis sentiendi et in inferioribus, et cum sint in una memoria et fantasia tunc illuminatur ab agente irridiante super huiusmodi species et ita a conditionibus materialibus denudantur..." See also page 59, where Bacon associates abstraction and reception: "...res corporalis immittit speciem sub conditionibus materialibus; set intelligentia sine conditionibus materialibus ita non potest eam sigillare

its *reception* by the spiritual power of the intellect. Nothing therefore contradicts Bacon's realism and the idea of transmission of universal properties through the medium, its final reception by the intellect after some steps of 'purification'. But no doubt that in the *Perspectiva* Bacon will completely abandon this way of presenting the formation of universal concepts (as we shall see in the next section).

Third, sensible properties are immediately intelligible. The same thing which is known and *received* by the senses can be known and *received* by the intellect.[43] Sensible data are the immediate objects of the senses, but since a superior power (*virtus)* can always act on the object of an inferior power, the intelligible power can know the lower objects of the sensation. Therefore, sensory faculties know sensible objects *sub ratione qua sensible*, and the intellect knows the same objects *sub ratione qua intelligible.* The intelligible properties are not abstracted from the sensitive properties. If one wants to use the term of 'abstraction' here, the corresponding concept must be understood as the mode of *reception* of the species of the sensible thing in the intellect. Bacon writes indeed that, after being purified from the corporeal conditions, the "species is received in the intellect" ("*in anima intellectiva recipitur*").[44]

In conclusion: in the *corpus* of texts attributed to the first period of Bacon's career, the classical way of presenting psychological abstraction is abandoned in favour of an original theory based on the mechanism of reception of species, associated with the principle of a direct intellection of the singular being and the sensible data. But this does not mean that Bacon, at that time, takes for granted that quantity and the associated mathematical properties could also be the direct objects of intellection. We don't find in these texts the thesis of the *Opus maius*, according to which we can have direct intellection of quantities and bodies ("*quanta et corpora intelligimus intuitu intellectus*"). On the contrary, Bacon considers that intellection of sensible objects is not possible without removing the concrete dimensions of these objects. A physical being cannot be present in the intellect in a spatial form, but only according to a spiritual mode (*sub esse spirituali*) and without quantitative dimensions. The determinate dimensions of bodies are attached to the quantified matter and cannot multiply their species into the medium nor to the intellect.[45] But if the species do not integrate the

intelligentia [...] Species autem rei corporalis cum sit sub conditionibus materialibus indiget abstractione..."

[43] *Q.causis*, 72: "...idem est sensibile et intelligibile sub alia et alia ratione, ideo potest illud idem quod a sensu cognoscitur et in ipso recipitur ab intellectu intelligi et cognosci sive in ipso recipi, non sub ratione qua sensibile, set sub ratione qua intelligibile."

[44] *Q.causis*, 73 – see the text in the next footnote.

[45] *Q.causis*, 72-73: "Dicendum ergo, quod species rei corporalis habet dimensiones interminatas que sunt sub esse spirituali, et ideo potest recipi in anima. Vel aliter dicendum, quod species rei corporalis potest dici corporalis a corpore quod est substantia, et sic habet conditiones corporales, et hoc sub esse spirituali, vel a corpore quod est quantitas, et sic non habet vel dimensiones sub esse spirituali unde non habet conditiones quantitatis, quia immissio speciei fit via multiplicationis et actionis aliquo modo, et ideo solum debetur virtutis immissio nature active. Quantitas autem, cum

quantitative properties of their objects, how can the dimensions of bodies be transformed into intellectual concepts? The *Perspectiva* will give us the answer.

II. Perception and Intellection of spatial properties in the *Perspectiva*

In the *Questiones supra librum de causis*, in the later *De multiplicatione specierum* and also in the *Perspectiva*, Bacon denies the possibility that a species would have determined dimensions, for two major reasons: (1) a species cannot have proper extension distinct from the medium in which it propagates; (2) matter, which is the source of dimensional properties, does not multiply any species.[46] The result seems to be that we cannot have a perception of quantitative properties, which would be a disaster for Bacon's theory of vision. The question is therefore how Bacon intended to explain the perception of determinate, relative or approximate dimensions of visual objects, and of their spatial relations and distances. One of the main difficulties of the theory of vision by intromission of species is the perception of spatial relations, for there are no species of empty spaces nor of relations, orientations or distances.[47]

Bacon's response to this question in the *Perspectiva* is complex in its construction, but simple and convincing in its principle. Space relations are not seen but estimated and judged on the basis of sensory experience.[48] When seeing a cube of three cubic meters, nobody can know its size by simple visual inspection – at best, one can *estimate* the size if having the *experience* of measuring bodies. Empirical forms and dimensions of bodies are therefore not transmitted to the sensory organ of sight but estimated at the level of the internal senses (common sense and imagination) and constructed as

materie debeatur, non est natura activa, ideo non multiplicat sui speciem, et propter hoc species non est sub dimensionibus quantitativis, set solum refertur ad corpus in quantum substantia est, et ideo in anima intellectiva recipitur."

[46] *DMS*, III chap.1, 181; *Perspectiva* 1.10.2, 151: "it is not given to quantity to act, since quantity is a property of matter to which no activity, but only passivity, belongs..."

[47] This is one of the more interesting objections of Peter of John Olivi against theories of vision by intromission of species. Olivi explains that we cannot perceive a distant object without perceiving our distance to this object: one must explain why all the species arranged on the sense organ do not appear on the same plane. See Katherine Tachau, *Vision and Certitude in the Age of Ockham. Optics, Epistemology and the Foundations of Semantics. 1250-1345* (Leiden-New York-Köln: E.J. Brill, 1988), 39-54; Dominique Demange, "Olivi et les Perspectivi. Les sources de la théorie olivienne de la vision" *Oliviana* 5 (2016): http://journals.openedition.org/oliviana/850; Lukáš Lička, "The Visual Process: Immediate or Successive? Approaches to the Extramission Postulate in 13th Century Theories of Vision", in *Medieval Perceptual Puzzles: Theories of Sense-Perception in the 13th and 14th Centuries*, edited by E. Bălțuță (Leiden: E: J. Brill, 2020), 73-110.

[48] A. Mark Smith devoted a detailed article to this specific question of the representation of space relations in medieval optics: "Spatial Representation in Medieval Visual Theory", in *Représentations et conceptions de l'espace dans la culture médiévale. Repräsentationsformen und Konzeptionen des Raums in der Kultur des Mittelalters*, edited by T. Suarez-Nani and M. Rohde (Berlin-Boston: De Gruyter, 2011), 45-66.

geometrical representations by the means of experience. Therefore, the psychological construction of the spatial properties and relations needs a least four levels: (1) The constitution of the visual field at the optical and physiological level, (2) the action of the discriminative faculty and the internal senses for the construction of the imaginary space (construction of space relations such as positions, distances, depth, etc.), (3) the experience of measuring (or at least estimating) dimensions, by which a more determinate knowledge of quantity can be acquired, (4) the formation of the universal concepts, such as the concepts of geometry, by the logical reasoning.

The first step consists of constructing a luminous and colored image of the visual field by the geometrical operations of the optical system – an image produced at the level of the internal sensory organ of vision.[49] The sensory organ is not the eye itself, but the nervous system beginning in the eyes and terminating in the common nerve at the surface of the brain.[50] At this step, specific optical and physiological conditions are required for the realization of a distinct image.[51]

Whatever the concrete ways of constitution of this internal image, the very act of perception of distance and spatial order is not a pure vision. Bacon makes clear that pure vision does not perceive distance nor form. The sensory organ of sight is only affected by the proper sensibles of light and color. Shape, remoteness, size and position belong to the list of the intentions perceived by the interior senses (imagination, common sense). Distance, size or shape are not objects of vision, but are constructed by imagination and estimation. Properly speaking, space is a fictional construct.[52] At this step, the judgement of distance or size is not intellectual; it is the spontaneous operation of imagination and common sense – and this is the reason why some imaginary errors of vision can occur, such as the famous Moon illusion or the apparent magnification of objects in the water.[53]

The third level is obtained by repeated evaluations of various perceptive situations: the perception of the various positions, orientations and apparent sizes of objects, when considered in mutual relations and in relation to the observer's position, at rest

[49] *Perspectiva,* 1.2-4, 20-59.

[50] *Perspectiva,* 1.5.2, 62: "Oportet igitur quod aliud sit sentiens preter oculos, in quo completur visio, cuius instrumenta sunt oculi, qui reddunt ei speciem visibilis. Et hoc est nervus communis in superficie cerebri...", cap.3, 66: "Et sic patet quod non solum oculi iudicant de visibili; sed incipitur iudicium in eis, et completur per ultimum sentiens, quod est virtus visiva frontalis in nervo communi."

[51] *Perspectiva,* 1.6.2-4, 1.7.1, 74-99. Medieval optics ignores the distinction between real and imaginary image – all images in the optical system are virtual. In other words, one must not imagine a projection of a real picture on the surface of the sensitive organ (for instance the retina), but the introduction of the visual species of light and color in a specific order.

[52] *Perspectiva*, 1.1.3, 8-11. Smith, "Spatial Representation", 57: "the Perspectivist account of spatial perception is the idea that space, as visually perceived, is imaginary and, therefore, a sort of fictional construct."

[53] See the detailed descriptions given by A. M. Smith, "Spatial Representation", 53-55.

or in motion, and in general the appropriate interpretation of the telltale signs given of the visual field. For Bacon, this is the place for explaining how the magnitude of an object is perceived.[54] Most of the Perspectivists held that magnitude is grasped solely from the size of the angle formed at the observer's eye. But this doesn't suffice, Bacon argues, since for the certification of the real magnitude of an object more information is required. It is necessary to integrate the size of the visual pyramid having the object at its basis, an estimation which cannot be obtained without some additional information given in the visual field: "Distance is grasped, therefore, when a sequence of bodies is arranged continuously between the eye and the object, provided that the distance is moderate and that the eye will have inspected those bodies and certified their magnitudes". [55]

At the fourth level, visual perception produces universal representations. Bacon distinguishes three modes of universal knowledge by vision.[56]

The first mode of the universal knowledge by vision is the confused apprehension of a quality or form without distinction (for instance, color in general, without any distinction of the kind of color). The second mode of universal knowledge by vision is of the diffused particularity ('*particulare vagum*'), namely, the identification of a kind of blue, which is known as 'a certain color', distinct from others, but without a clear logical distinction between these kinds. The third mode allows the construction of universal types at the logical level. This is obtained by spontaneous reasoning, such as the immediate inference of a transparent medium when we see through a glass of water: "... this cognition ordinarily occurs suddenly, and we do not perceive that we reason, although we do".[57]

The first mode of universal knowledge is produced by 'sense alone' – an expression used to designate sight in the pupil and the common nerve as far as the common sense.[58] The second mode is described in this way: "... the ability to distinguish universals from one another and from particulars, and particulars from each other by comparison of a thing seen to the same thing previously seen, recollecting that it was previously seen and known to the observer, constitutes a second mode of visual comprehension".[59] Therefore, this mode cannot be achieved without the internal senses of imagination and memory: "For unless imagination and memory of prior vision of the thing are present, comprehension in the second mode cannot occur...".[60] The third mode (spontaneous logical reasoning), "is further removed from sense alone, since in it more things are considered than in the second mode, and its method of argumentation brings

[54] *Perspectiva*, 2.3.5-6, 222-233
[55] *Perspectiva,* 2.3.3, 211.
[56] *Perspectiva,* 1.10.3, 154-159.
[57] *Perspectiva,* 1.10.3, 157.
[58] *Perspectiva,* 1.10.3, 159.
[59] *Perspectiva,* 1.10.3, 157.
[60] *Perspectiva,* 1.10.3, 159.

it closer to a work of reason".[61] Quantity, which is one of the twenty-two intentions received by the sense of sight, "cannot be certified except by this third mode of knowledge".[62]

These three modes of "universal knowledge by vision" belong to the sphere of spontaneous and pre-conscious psychological activity structuring our perception. This is true even for the third mode, about which Bacon says that "...we do not perceive that we reason, although we do".[63] But at the end, this spontaneous pre-conscious reasoning by which we distinguish logical types such as degrees of light, kinds of colors and series of forms at the level of perception, is at the basis of the scientific learning by which we consciously identify all these types and classify them by distinct terms and concepts. According to this passage of the *Perspectiva*, this is the case of logic and this is also the way Bacon, in the *Opus maius*, understands the famous Socratic demonstration, in the *Meno*, of our pre-conscious knowledge of geometry:[64]

> Secondly, an understanding of mathematical truths is almost innate within us. As Tullius relates in the first book of the *Tusculan Disputations*, when Socrates questioned a small boy about geometrical truths, he responded as if he had learned geometry. This has been tried in many cases and it does not occur in the other sciences, as will be more clearly demonstrated by what follows. For that reason, since an understanding of mathematics is almost innate and precedes discovery and learning, as it were, or at least requires them less than the other sciences, it will be first among the sciences, preceding the others and disposing us to them since what is innate or virtually so is disposed to acquiring knowledge.[65]

The exact meaning of these passages of the *Perspectiva* and the *Opus maius*, both introducing the idea of an (almost) *innate* logical or geometrical knowledge, seems to be the following. Bacon doesn't say that logic or geometry doesn't teach us anything and that logical or geometrical knowledge is pure reminiscence. Logic teaches us how to reason properly in general, and geometry how to reason rigorously in the specific case of figures. Nevertheless, these sciences do not create (but only increase and develop) in us the *potency*, *ability* or *capability* to reason or geometrize – and this is the

[61] *Perspectiva,* 1.10.3, 159: "... et magis accredit ad opus rationis propter viam arguendi."
[62] *Perspectiva,* 1.10.3, 159.
[63] *Perspectiva,* 1.10.3, 157. On this spontaneous reasoning by perception and its source in Alhazen, see A. Mark Smith, *From Sight to Light*, *The Passage from Ancient to Modern Optics* (Chicago and London: The University of Chicago Press, 2015), 189-192; A. I. Sabra, "Sensation and inference in Alhazen".
[64] *Perspectiva,* 1.10.3, 159: "...by nature we know the science of arguing, which is logic. But to begin with we are ignorant of the proper terminology, which the first writers on logic invented, but which we learn by instruction. And a treatise and discourse on logic exists not to convey the potency of this science (since it is innate in everybody), but to convey its terminology..." ("...non propter ipsius scientie potestatem, quia hec est cuilibet innata...").
[65] *Opus maius* IV, dist.1 chap.3, ed. Bridges I, 103; transl. Dennis 71.

great difference with the other sciences.[66] The sciences of mechanics or astronomy are in no way innate, for the dispositions for mechanical or astronomical concepts and laws have to be created from nothing, whereas learning of logical or geometrical or arithmetical concepts and laws is only a structuration, rationalization and clarification of pre-conscious intuitions. No doubt therefore that these pre-conscious intuitions appear in the *Perspectiva* at the level of the three modes of "universal knowledge by vision".

Indeed, if we extrapolate a little on the basis of the psychological construction of our universal concepts in the *Perspectiva*, no doubt that we have here all the elements of a convincing model for geometrical leaning. Just like all visual forms and characteristics, the geometrical properties are identified by the geometer at the level of visual figures, not as individual characteristics but as universal properties, gathered from a repeated experience of visual forms. While studying geometry, the student learns to identify the visual forms which enter, as universal properties, into logical demonstrations. He first identifies them as diffused particulars (*particulare vagum*, second mode) and later as logically distinct (third mode). When he identifies them sufficiently, he learns to classify and construct them as distinct scientific concepts. He can therefore judge concerning universal geometrical properties derived from 'false' sensible figures, because he identifies these approximate figures as representing the intellectual exact ones. The classical ways of abstraction and intellectual illumination are completely absent from this theory of scientific learning. The geometer does not need to climb up an abstract noetical level, he does not need to escape from the sensible world and be illuminated by the pure concept of the universal triangle, completely abstracted from sensible and extended matter. He does not need to have such an experience, which would be completely useless for any kind of geometrical demonstration. The geometer's intellectual activity is produced at the level of perception, by an immersion of his thought in the visual field and its universal properties and structures.

[66] Cecilia Panti, "Natural Continuity and the Mathematical Proofs", 174, writes: "Bacon remarks in its *Opus tertium* that we do not know mathematics through nature." I would not put it that way. Mathematics, as a specialized learning, is not known by nature, but is nevertheless based on a natural disposition, and is therefore "almost innate". This is the same as for music. The corresponding passage of the *Opus tertium* says: "Et ideo post linguarum necessitatem pono mathematicam esse in secundo loco necessariam, ad hoc ut sciamus quae scienda sunt; quae non est nota nobis per naturam; sed [mathematica] tamen est prope cognitionem naturalem inter omnes scientias quas scimus per inventionem et doctrinam. Nam ejus speculatio facilior est omnibus scientiis, eo quod pueri statim capiunt has scientias, sicut videmus; et Aristoteles hoc dicit septimo Ethicorum; non sic naturales scientias, et metaphysicas, et alias. Et praeterea laici sciunt de facili figurare, et numerare, et cantare, et uti instrumentis musicalibus, et exultare, et gestus facere conformes cantui et sono instrumentorum; et haec omnia sunt opera mathematicae. Quapropter oportet quod sit facilis scientia, et quasi innata, vel prope cognitionem innatam" (Pars 1, cap.29, ed. Egel, 216).

III. The *Opus tertium*: quantity as first object of human's intellect

I opened this article with a quotation of a passage of Part IV of *Opus maius*. In this text, Bacon asserts that for Aristotle, the first object of human's intellect is quantity and its corresponding species. The complete passage is the following:

> Secondly, the very act of understanding itself is not completed without continuous quantity, because Aristotle, in his book On *Memory and Reminiscence*, states that our whole intellect is associated with continuity and time. From this, we comprehend quantities and bodies by the perception of the intellect, because their forms belong to the intellect. The forms of incorporeal things, however, are not apprehended by our intellect; or if they were formed in the intellect, as Avicenna indicates in the third book of the *Metaphysics*, we nevertheless do not perceive these forms, because our intellect is more strongly oriented around bodies and quantities. Therefore, by our way of argumentation and attention to the corporeal and the quantifiable, we seek knowledge of incorporeal things, as Aristotle does in eleventh book of the *Metaphysics.* Therefore, the intellect develops mostly around quantity itself, and it is in this way, according to the common condition of understanding, that quantities and bodies are apprehended by the human intellect.[67]

Bacon distinguishes two forms of human's intellectual knowledge: (1) the intellectual perception of the sensible bodies, by which our intellect apprehends the forms of quantity, space and time by direct intuition; (2) the way of rational argumentation and reasoning by which we can infer some general truths about the objects which transcend our direct experience of bodies, and for which we have no direct perception: 'the incorporeal things'. The classical opposition between physical and *meta*-physical knowledge is formulated in an original manner: it is not only to say that our physical experience is limited to the forms received in the sensitive soul, but also that physical experience provides an immediate intellectual perception of the mathematical forms intrinsic to bodies, namely, quantity, space and time. This is the way Bacon reads the famous passage of Aristotle's *On Memory and Reminiscence*[68] and this reading is surprising for the modern reader. The idea that the forms of quantity "belong to the intellect", associated with a reference to Avicenna's *Metaphysics,* may lead us to suspect a Platonist or Neoplatonist influence somewhere in Bacon's psychology of mathematics; but whatever the exact nature of this influence, the physical section of the *Opus tertium* sheds light on this passage of the *Opus maius*.[69]

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[67] *Opus maius* IV, dist.1, chap.3, ed. Bridges I, 107, transl. Dennis 77. The Latin text is given in footnote 6.

[68] Quoted in footnote 7.

[69] Bacon seems to refer here to the end of chapter 8 of book III of Avicenna's *Metaphysics* (Avicenna Latinus 162-163). I don't read in this passage anything about our direct intellection of mathematical forms, but the only classical division between the two ways of intellection by abstraction and enlightenment.

The physical section of the *Opus tertium* occupies chapters 38-52.[70] The question of the unity of matter, of the possibility of a vacuum, of the spatial and temporal modes of existence of angels are the main topics. These are classical questions of scholastic philosophy and theology, but Bacon doesn't intend to prove that he is a good Parisian philosopher or theologian. On the contrary, his aim is to present ('to recite') the various positions of the philosophers, theologians and of the 'vulgar' on these questions, in order to demonstrate how human's intellect can err on these difficult matters, and how an appropriate, deep and accurate scientific knowledge (especially of mathematics) is the necessary condition for eliminating the false representations which contaminate theology.[71] In chapters 46-49, the various hypothesis concerning the location of angels in heaven or on earth, in indivisible or divisible spaces are examined and successively eliminated, due to their contradictions and inconsistencies. At last, Bacon concludes that all these contradictions, errors and false representations about the nature of angels are almost inevitable given the nature of our intellect, bound as it is to the inferior world:

> But the arguments for the contrary are much more difficult: this is due to the corporeal representations, into which we are absorbed, because all of our intellect is integrated into the continuum, as Aristotle says in the book *On Memory and Reminiscence*. And therefore, by its first intuition, our intellect doesn't overcome continuum, which is corporeal quantity. And for this reason, it forms by itself corporeal representations of the spiritual [substances] – or similar to these. [...] The same goes for angels, because we talk a lot about them by using figures of speech referring to corporeal things, as this way of speaking corresponds to our proper intellect, which doesn't overcome corporeal things by its first sight and mind intuition...[72]

And once more, in the discussion of chapters 51-52 about angels' temporality (*aevum*), Bacon writes:

> Aristotle says that all our intellect is with continuous and time, because we do not conceive anything at first sight except quantities – such as the things which are measured by an intrinsic continuous quantity, which is the three-dimension, and those

[70] *Opus tertium*, I, cap.38-52, ed. N. Egel, 249-417.

[71] *Opus tertium*, I, cap.47, §289 ed. N. Egel, 360: "Sed in hoc loco volo procedere secundum vias inquisitionis et recitationis, magis quam determinationis et diffinitionis alicujus sententiae, et sine praejudicio melioris sententiae. Atque referam opinionem aliquorum theologorum famosam, cui etiam sapientissimi viri concordabant quos vidimus, licet viam universae carnis ingressi sunt."

[72] *Opus tertium*, I, cap.49, §311-313, ed. N. Egel, 374-376: "Sed difficilia sunt argumenta in contrarium, propter imaginationes corporalium, quibus absorpti sumus, quia omnis intellectus noster est cum continuo, ut dicit Aristoteles, libro de *Memoria et Reminiscentia*. Et ideo primo intuitu non transcendit intellectus noster ultra continuum, quod est quantitas corporalis. Et propter hoc de spiritualibus format sibi imaginationes corporales, aut similes eis; [...] Similiter vero est de angelis, quod multa loquimur de eis secundum similitudines loquendi in corporalibus; quia talis modus loquendi est proprius intellectui nostro, qui corporalia non transcendit, primo aspectu et principali mentis intuitu..." See the same text in *CN*, I, 234-235.

> which extend by extrinsic quantity, such as the temporal things, which are submitted to time. Therefore, we do not perceive spiritual and permanent things in their unchangeable being by the first intuition of the mind, nor by another mode, except when our soul will be extracted from corporeal and temporal things, then we will go beyond them. Therefore, regarding the power of our intellect, without a special illumination we cannot have this sort of intellection [of the spiritual substances], except by a privation of the corporeal and temporal [properties], and not by their affirmation. And for this reason, we perceive the existence of spiritual and permanent things, which is measured by the aevum indivisible and inseparable, with the greatest difficulty...[73]

When confronted with the transcendent truths, when trying to grasp the essence (or mode of existence) of God or the angels, man's intelligence is weak and obscure, and his reasoning sounds like the speech of fool or a child. This is a classical theme, nourished by various philosophical and theological classical *loci*.[74] But this development on angelology of the *Opus tertium*, for which there is no corresponding section in the *Opus maius,* sheds a specific light on Bacon's theology.[75] Bacon enters into a deconstruction of our common representations of the transcendent beings, and the method of this deconstruction appears to have been inspired by Maimonides' *Guide of the perplexed*.[76] The criticism of the imaginary representations and formulas of the common language (taken from our everyday experience of corporeal things) when

[73] *Opus tertium*, I, cap.51 §335, ed N. Egel, 400-402: "Quia Aristoteles dicit quod omnis intellectus noster est cum continuo et tempore, quia nihil primo aspectu concipimus nisi quanta, ut quae quantitate continua mensurantur intrinseca, quae est trina dimensio; et quae quantitate extrinseca extenduntur; ut sunt temporalia, quae sub tempore cadunt. Et ideo spiritualia et permanentia in suo esse invariabili non percipimus primo mentis intuitu, nec aliquo modo, nisi quando abstraxerimus animum a corporalibus et temporalibus, et transiverimus haec. Sed tunc quantum est de potestate intellectus nostri, sine speciali illuminatione non possumus intelligere hujusmodi, nisi per privationem corporalium et temporalium, et non per positionem. Et ideo cum summa difficultate percipimus esse spiritualium et permanentium, quod mensuratur aevo indivisibili et impartibili..." See the same text in *CN* I, 175.

[74] See *De viciis*, 36-37.

[75] But the same text (or parts of the same text) can be found in *CN*, I, pars3, dist.1 cap.8 ("De aevo", 173-182) and dist.2 cap.7-8 ("De loco et motu spiritualium substantiarum", 224-239). See Jeremiah Hackett, "Motion, Time and Aevum in Roger Bacon's *Communia Naturalium*: Context and Content", in *Roger Bacon's Communia Naturalium. A 13th Century Philosopher's Workshop*, edited by P. Bernardini and A. Rodolfi (Micrologus Library) (Firenze: SISMEL-Edizioni di Galluzzo, 2014), 191-213 – especially pages 197-198; Cecilia Panti "*Non abest nec distat*. Place and Movement of Angels according to Robert Grosseteste, Adam Marsh and Roger Bacon." In *Lieu, espace, mouvement: physique, métaphysique et cosmologie (XIIe-XVIe siècles). Actes du Colloque international Université de Fribourg (Suisse), 12-14 mars 2015, edited by* T. Suarez-Nani, O. Ribordy, and A. Petagine (Barcelona and Roma: FIDEM, 2017), 57-77.

[76] I will refer below to the classical translation: Moses Maimonides, *Guide of the perplexed*, translated by Sh. Pines, 2 vols. (Chicago: The University of Chicago Press, 1963).

applied to the description of incorporeal substances;[77] the refutation of atomism;[78] the *via negationis* (it is impossible to truly characterize incorporeal substances by positive properties, we can only characterize them by negation of any corporeal property);[79] all these typically Maimonidian themes are integrated in Bacon's criticism of our common language and our usual representations about angels or God, which infect the most sophisticated theories of the Parisian theologians.[80]

Let us try to formulate Bacon's corresponding psychological theory. All our actual intellectual representations are based on the experience of bodies, and our intellect has great familiarity with the corresponding forms of quantity, for it drew all its experience from them. Since incorporeal beings have a completely different essence and completely different possibilities of existence, their being is almost inaccessible to our understanding in the present state of our life. Indeed, it appears that the term '*intellectus*' in these passages would be better translated by the term 'understanding' of classical philosophy – much more than by the term 'intellect' of the peripatetic tradition. We should say, therefore, that man's understanding is limited to the extended forms of its natural and usual experience; man's intellectual power will be extended to the possibility of intuitions of incorporeal beings in the future life, but its actual understanding cannot exceed these limits, except by the limited power of reason, by which man can infer general truths and many difficult hypothesis or questions about the nature of these transcendent beings. Now, how should we therefore understand the reference to the passage of Aristotle's *On Memory and Reminiscence*? It seems that for Bacon, the idea that our understanding is linked to the forms of quantity is simply one of the basic principles of Aristotle's psychology of space and time in the *Physics*. While

[77] Maimonides, *Guide of the perplexed*, I, chap.1-49, vol.1, 1-110; *Opus tertium* I, cap.49, §312-313 ed. N. Egel, 376: "Nam dicimus quod Deus descendit de coelo; sed secundum modum vulgatum apud nos, quod descendit de alto, relinquit locum illum et adquirit novum locum, quem prius non habuit. Sed haec sunt absurda de Deo. Et cum dicitur: 'Misit Deus Filium suum in terris', non est intelligendum sicut homo mittit filium suum a se ad locum distantem, in quo non est mittens, et quem prius non habuit missus. Haec enim in corporalibus locum habent; et sic de infinitis aliis attributis Deo, secundum sermones vulgatos de corporalibus. Quae aliter intelligenda sunt. Similiter vero est de angelis, quod multa loquimur de eis secundum similitudines loquendi in corporalibus..."

[78] Maimonides, *Guide of the perplexed*, I chap.74-76, vol.1, 215-231; see *Opus tertium* I, cap.46, §286 ed. N. Egel, 356-58.

[79] Maimonides, *Guide of the perplexed*, I, chap.58-60, vol.1, 134-147 – *Opus tertium* I, cap. 47, §229, trad. N. Egel, 366: "Quapropter concludi videtur necessario, quod spiritualis substantia nullum locum, nec divisibilem nec indivisibilem, corporalem requirit, nec debeat habere, propter continentiam, sicut neque propter salute." *Opus tertium* I, cap.49, §320, ed. N. Egel, 384: "...nec oportet quod dicamus quod angelus est simul et semel praesens coelo et terrae, sed per negationem, quod non abest nec distat a coelo nec a terra, et cum est praesens coelo non distat a terra, nec abest ab ea; et, e converso, cum consideratur praesens terrae, non abest nec distat a coelo; ut semper aliqua negatio exprimatur, quia nullam habet rationem distantiae corporalis, cum sit spiritus."

[80] *Opus tertium*, I, cap.50, §326, ed. N. Egel, 392: "Vulgus tamen non capit haec, nec ejus capita multa. Aliqui tamen se confricant ad haec, nulla tamen rationum potentia ducti, sed imaginatione sua in hoc, sicut in aliis, magis falsis quam veris gaudentes."

developing the mathematics of motion, space, and time in books IV-VI of *Physics*, Aristotle doesn't only provide the foundations of the science of nature but also a description of human's immediate understanding of the corresponding physical phenomena. After all, Aristotle defines time as the number of motion, counting is obviously a mathematical operation, and therefore he considers the perception of time as a (pre-conscious) mathematical psychological activity. And the same argument can be formulated concerning the schemes by which we spontaneously perceive, distinguish and compare the three-dimension visual forms with their geometrical properties. As we have seen in the previous section, according to the *Perspectiva*, this spontaneous logical activity at work in perception is produced by sight in the third mode of universal knowledge.[81] All these elements put together, present a convincing picture of Bacon's claim that "we perceive quantity at first sight" (*primo aspectu*). The human intellect is immersed in the sensory world in such a way that he apprehends space, time, and the forms of quantity by an immediate pre-conscious intuition.

IV. Abstraction and the essence of mathematical thought in the *Communia Mathematica* and *Geometria speculativa*

The scope of abstraction in the *Communia mathematica* is much wider than in the previous texts we have examined. Bacon presents a complete account of the mathematical science, including its internal structure and relations to the other sciences, and this goal is achieved through a description of the various modes of abstractive knowledge.[82]

We have five modes of abstraction.[83] The first is the abstraction of universals from particulars, and this mode is common to all sciences, since scientific knowledge is universal. The second mode is absolute abstraction of a being from motion and matter of any kind, and this complete separation is only possible to God. The third mode signifies the ontological separation from corporeal matter and corporeal movement, and this is the case of separated intelligences which, nevertheless, have spiritual matter. The fourth mode of abstraction considers corporeal properties without their relation to corporeal matter and motion, and this is the way speculative mathematics and music (*geometria speculativa, musica speculativa)*, and the speculative science of quantity in general considers its objects. Lastly, the fifth mode considers the abstraction from the sublunary matter of qualitative and quantitative change, generation and corruption, and this is the way mathematical astronomy (*astronomia mathematica),* speculative and practical astrology consider the celestial quantities and

[81] See footnote 64.
[82] See Jeremiah Hackett, "Roger Bacon on the classification of the sciences", in *Roger Bacon and the Sciences. Commemorative essays*, edited by J. Hackett (Leiden-New York-Köln: E. J. Brill, 1997), 49-65
[83] *CM* I, dist.5, 58-65

motions, for they do not depend on the inferior matter undergoing transmutations.[84] Natural astronomy (*astronomia naturalis*), on the contrary, is a natural science, for it studies the physical effects of the celestial motions at the level of the matter of alteration, quantitative change, generation and corruption.

By separating the fourth and fifth modes of abstraction, Bacon is clear enough that we need to create a specific place, apart from physics in the strict sense (the science of the sublunary transformations), for a mathematical science of quantity and motion abstracted from sublunary matter. Motion is a property of a substance; thence mathematics of motion is always linked to a specific matter (sublunary matter or celestial matter), but "quantity in its very essence does not depend on natural change" and mathematics of motion is not a part of physics in the Aristotelian sense.[85] Bacon's argument on this point is the following.[86] Numbers and quantities derive from the genus of body which has been created first, so that they determine the essence of the incorruptible celestial bodies and of the incorruptible elements which are the makeup of the structure of the cosmos. Consequently, quantity, by its very essence, is not submitted to the principles of natural transmutation at the lower level. Quantity is not only present in the geometer's mind as an abstract representation extracted from sensible data, but rather exists in the real world as the property of its perfect and incorruptible structure. Therefore, 'abstraction' is for the geometer a form of perception of the real nature of quantity, as realized in the cosmos. For Bacon, mathematical intuitions are not separated from the perception of the word, for mathematical abstraction is a perception of the ideal structure of the physical world itself. This point is (A) explained by an example in the *Geometria speculativa*, (B) demonstrated by a more general analysis in the *Communia mathematica*, (C) and

[84] *CM* I, dist.5, 61: "Nam Astrologia considerat celestia et similiter Astronomia mathematica, sed tamen non considerant nisi quantitatem corporum et motum non generacionem et corrupcionem non alteracionem non augmentum non diminucionem non loci mutacionem in quantum est causa generacionis et corrupcionis et ceterarum transmutacionum, sed considerant motum localem celestium quantum ad quantitatem illius motus, ut quantus est in hora, quantus in die, quantus in mense, quantus in anno uno, quantus in pluribus, et sic de communibus premissis (aliter omnibus practicis) Mathematice. Verum est quod non considerant materiam corporalem ut est subjecta motibus transmutacionum nec per comparacionem ad illas, et ideo dicuntur abstrahi a materia et motu, id est a materia prout est subjecta transmutacionibus naturalibus et comparata ad illas."

[85] *CM* I, dist.5, cap.2, 62: "quantitas quantum ad suam essenciam non dependet a transmutacione naturali." See Michela Pereira, "Roger Bacon on Nature", in *The philosophy of science of Roger Bacon, Studies in Honour of Jeremiah Hackett*, edited by N. Polloni and Y. Kedar (London and New York: Routledge, 2021), 17-35 – on this question, pages 24-25.

[86] *CM* I, dist.5, cap.2, 62-63: "quantitas continua quam sequetur numerus arismetice procedit in esse secundum creacionem in corporibus primis, et nascitur cum primo genere subalterno quod est corpus, et creata est in celo et in elementis ut sunt partes mundi in quantum sunt incorruptibilia naturaliter, et erunt semper et ab eis, tamen sit in omnia generata. Et ideo quantitas quantum ad suam essenciam non dependet a transmutacione naturali, sed omnia alia accidenda concernunt corpus generabile et corruptibile et generantur et corrumpuntur, eciam lux que est accidens inter corruptibilia nobilissimum, nam in celestibus corrumpitur per eclipses et renovatur."

confirmed by a passage of the *Opus maius* dealing with physical and mathematical continuity.

(A) The example is given in the *Geometria speculativa* for the purpose of explaining the origin of Euclid's first postulate: "A straight line segment can be drawn joining any two points".[87] How does the geometer discover this simple law of geometrical construction? He considers the possible ways and modes of operation of nature, and nature shows him that straight line is the most efficient way of action. The laws of geometry are discovered by the experience of optical rays and planetary perfect trajectories, which are described in the sciences of optics and astronomy. Without considering the ideal structure of the physical world, which is revealed in these mathematical sciences, the geometer would have *never* been able to formulate the ideal laws of speculative geometry, for he would have simply followed his imagination and the "tortuous and deformed" empirical lines of his immediate perception. The *only* way for the geometer to discover geometrical laws, is therefore for him to "imitate the ways of nature". What does it mean? What is he imitating? It means that human's technique is limited to an *approximation* of the ideal operations of nature: "on account of the irregularity or tortuosity or deformity of corporeal matter in these inferior things, the first [postulate] cannot be reduced by man to operation, or scarcely and with great difficulty. But it is possible for operative nature..." Man will never be able to produce a perfect line, whereas nature is able to do so, as optics and astronomy show. Therefore, when drawing a line on the blackboard, the geometer is *imitating* the way nature produces perfect lines – he knows that the empirical line he is drawing approximates the perfect intelligible line *really* produced by nature: "the geometer does not speak of the sensible line but of what is understood through it".[88]

[87] *Geometria speculativa*, §48, 298-301 (transl. G. Molland): "And on account of the irregularity or tortuosity or deformity of corporeal matter in these inferior things, the first [postulate] cannot be reduced by man to operation, or scarcely and with great difficulty. But it is possible for operative nature, as in the multiplication of virtue and species in the things of the world, as in the diffusion of light and rays, which is made multiplicatively by straight lines in a single body, and also the perpendiculars to the first bodies are made in a straight fashion. The geometer therefore considers the possible paths of nature, because geometry was first and essentially constituted for the sake of certifying the works of nature, and thereafter for human works. For the authors of perspective show us that lines and figures declare to us the whole operation of nature, its principles and effects. And this is similarly evident by celestial things, which both natural philosophy and astronomy consider. The geometer therefore does not attend to tortuous sensible matter, but he understands regular nature as it is in celestials and as nature knows how to find in its operations in these inferiors, and he imitates the ways of nature. And thus it was not in the imagination of straight lines, as Aristotle says in the *Posterior Analytics* that the geometer does not speak of the sensible line but of what is understood through it."

[88] *Perspectiva* as a science, needs geometry (geometrical laws are applied in optics) but according to the order of acquisition of knowledge, a first experience of natural figures is necessary for understanding geometrical truths. See *Epistola ad Clementem (Letter to Pope Clement IV)*, ed. N. Egel, chap.10, 165: "In a certain sense, mathematics is also needed for this science [of *perspectiva*], which

(B) The general analysis is given in the introduction of the *Communia mathematica*.[89] According to Bacon, the list of definitions, axioms and postulates by which Euclid opens the *Elements* and elaborates the whole system of geometry is highly lacunar, since Euclid has not given the fundamental concepts and definitions for a true and complete foundation of mathematical science.[90] These concepts are *topological*: simultaneity ('simul'), which has specific applications for space and time; term ('terminus'), which indicates the topological limit of a form or a process; contiguity ('contiguitas') which indicates that the terms of two geometrical forms are joined together and continuity ('continuitas') that these terms are identical; succession ('successivum') which indicate an order; dimension ('dimensio') which indicates the possibility of reduction of a spatial body; position ('positio') and movement ('motus'), as the spatial and timely transition which excludes fixed position; etc. These concepts determine the *very* essence of mathematical objects. All these concepts come from Aristotle's *Physics* and *Metaphysics*: this is a list of fundamental definitions by which Aristotle elaborates his description of physical structures and processes, and Bacon holds that these definitions are the primary foundations of Euclidian geometry[91]. These definitions do not appear in mathematical treatises, because the mathematician doesn't need to know them for the study and practice of geometry. They belong to metaphysics – to the *metaphysics of the continuous matter*.[92] This is what Bacon indicates at the beginning of the *Communia mathematica*: he wants to reveal the metaphysical roots of mathematics – "*Cupiens igitur mathematicam tractare infra radices metaphysice...*".[93]

is why it is subordinate to it in the ranking of nature. But because, according to the opinion of all mathematicians, mathematics deals with other things which do not only extend to this science and are even more important than the things dealt with in *perspectiva*, mathematics is dealt with before the science of perspective, but is immediately subordinate to it in the rank of worthiness and comes later in our understanding." Geometrical intuition is 'abstracted' from experience of natural forms.

[89] *CM*, 19-23.

[90] *CM*, 23: "Et tamen omnes auctores hec omiserunt, sicut Euclides qui incipit a diffinitione puncti, omittens omnia que prescripsi, cum tamen hec naturaliter precedant in ordine discipline."

[91] It seems therefore that Roger Bacon was aware (at least by a general intuition) of the incompleteness of Euclidian axiomatics regarding continuity. On this question see Vincenzo de Risi, "Did Euclid Prove Elements I, 1? The Early Modern Debate on Intersections and Continuity", in *Reading Mathematics in the Early Modern Europe. Studies in the Production, Collection, and Use of Mathematical Books*, ed. P. Beeley, Y. Nasifoglu, B. Wardhaugh, London, Routledge 2020, 12-32. I am very grateful to Paolo Mancosu for this remark and reference.

[92] *CM*, 19: "Quantitatis autem species non possunt haberi nisi premittantur quedam diffiniciones communes necessarie ad intellectum diffinicionum quantitatum. Et sumo hic diffinicionem largo modo, prout sub ea comprehenditur descripcio,quia mathematicus non curat semper observare proprietatem diffinicionis – hoc enim magis ad methaphysicum pertinet."

[93] *CM*, 13: "Cupiens igitur mathematicam tractare infra radices methaphysice sicut feci logicam quam immediate sequitur mathematica, volo sicut debeo ut in pluribus abstinere a demonstracione eorum que verificavi in alia sciencia communi, licet multa ibi verificata que mathematice valent, recitabo per modum narracionis secundum quod congruit mathematice, et aliquando, licet raro, afferam probaciones aliquas methaphysicas, scilicet in casibus certis quando magna necessitas erit,

(C) At last, a passage from the *Opus maius* confirms this view. This metaphysical point of view on mathematical knowledge is applied in the specific case of the demonstration of the continuous unity of prime matter. In this passage, Bacon aims at refuting by "the power of geometry" any real distinction between mathematical and physical continuity by "pointing out the false geometric representations".[94]

> ...mathematical quantity and a physical quantity are the same regarding their being and their reality. They differ only in their point of view because a geometrician considers a physical line, but not insofar as it is physical matter; thus, it is called a mathematical line. A natural philosopher considers this same line as physical matter, as with iron, stone, or other natural matter. And because the same thing is physical and mathematical, according to its being and the reality of its existence, if this were thus one line or one body mathematically, then it would be one in the same way physically.[95]

The physical line and the mathematical line are the same: the difference is only of point of view. Therefore, the one who imagines that geometrical properties do not apply in physics, is simply imagining a false geometry, for geometry is nothing else than the science of matter's intrinsic properties. Bacon demonstrates, against the form of Aristotelian atomism imagined by Averroes, that it would simply contradict the fourteenth proposition of the first book of the *Elements*.[96]

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ne novitate nimia aliquid proponam sine sua racione cujus contrarium pro rato communiter celebratur. Sic enim quasi assumam officium methaphysici ut Aristoteles fecit in *Elenchorum* principio et *Physicorum* et alibi..." See *Geometria speculativa*, §32, ed. Molland, 289: "These things are said here for the sake of exposition according to the opinions of the commentators [of Euclid's *Elements*], although they must be treated otherwise in *Metaphysics* and it must be shown what should be held according to pure truth, supposing that, although these things be true, it is still necessary to speak more certainly of the matter."

[94] "The philosophers before Aristotle claimed that the world is a continuous body, as has been mentioned before, and this assumption results from the supposition of the unity of matter. This is why I explain this not by rejecting what has been rejected before, but by pointing out the false geometric representations, which I was able to resolve extremely successfully at this point, and of which I was also able to show how they must be corrected. Because the position of Democritus and Leucippus, who claimed that everything consists of indivisible atoms, confused Aristotle very much and still confuses the natural philosophers by their sophistry, I also completely destroy their view by the power of geometry. But since the arrangement of geometric bodies is a passion of matter, and since both theologians and philosophers carefully examine the geometric figure of the heavens and the fundamental areas of the world, and since this consideration is quite beautiful, I show all that is necessary at this point." *Epistola ad Clementem*, transl. N. Egel, chap.10, 168. On this subject, see Panti, "Natural Continuity and the Mathematical Proofs" (complete reference footnote 2).

[95] *Opus maius* IV, chap.9, transl. Dennis, 144.

[96] "If, however, in those joined bodies two lines are drawn from those points within the bodies, and one line falls to their extremities at right angles, it is necessary that lines extended in bodies would be one continuous line, according to Proposition XIV of the first book of Euclid's *Elements*. And thus it is also for the bodies, for such is the conclusion of that proposition" (*Opus maius* IV, chap.9, transl. Dennis, 144).

These pages of the *Opus maius*, the *Communia mathematica* and the *Geometria speculativa* are of the utmost importance, because they provide strong arguments for refuting the interpretation according to which mathematical physics in Roger Bacon is nothing but a baroque (and sterile) mixture of Euclidian mathematics and Aristotelian physics. Bacon considers speculative geometry as the science of the intrinsic structure of matter in general (without any distinction, at this level, between sublunary or celestial matter). Bacon's claim that all sciences (including theology) require mathematics endows mathematics a metaphysical status. The mathematical (topological, structural, and dynamic) properties of matter are universal. This scheme, presented in the introduction of the *Communia mathematica*, is not Pythagorean nor Platonic. It seems to me (but this question would need extended attention) that the way Bacon identifies mathematical and metaphysical structures of matter fits much more with some views of modern mathematicians on the topological foundations of Aristotle's physics.[97]

Conclusions

Roger Bacon has always taken for granted Aristotle's axiom according to which our universal knowledge comes from sensory experience, and that numbers and figures have no real existence apart from the sensible substances from which they are abstracted. However, in his mature writings (especially in the *Perspectiva*) Bacon completely abandons the classical psychology of abstraction, and it is even doubtful that he ever really adopted such a theory. Instead, the *Perspectiva* presents a convincing model of the formation of perceptual concepts (colors, forms, spatial relations) on the basis of repeated and varied situations and experiences, by the action of the internal senses. At the psychological level, space appears as a fictional construct: The spatial properties (distance, size, depth, etc.) are formed as perceptive representations by the action of common sense, imagination, and memory. Moreover, the spontaneous logical reasoning at work in perception of confused particularities produces a first discrimination of the visual forms, which will be later consciously and clearly distinguished and reconstructed *rationaliter* in the geometrical demonstrations. One can call this complex construction a "theory of abstraction" if he wants to, but it has nothing to do with the schemes usually so labeled in medieval theories.

In the *Opus maius* and the *Opus tertium*, Roger Bacon holds that quantity is the first and natural object of human's intellect. This thesis comes from an original reading of a passage of Aristotle's *On Memory and Reminiscence*: human's intellect in the present state, is immersed in the sensible world, and has a direct intuition of the mathematical forms of time and space, so that the forms of quantity "belong to the intellect" and our

[97] See René Thom, *Esquisse d'une sémiophysique. Physique aristotélicienne et théorie des catastrophes* (Inter Editions, 1988); translated into English by Vendla Meyers, *Semiophysics: A Sketch*, *Aristotelian Physics and Catastrophe Theory* (Redwood City, California: Addison-Wesley, 1990).

intellectual knowledge of these forms is "almost innate". This is not a platonic psychology of knowledge of ideal objects, but an original view about the way human's intellect forms abstract concepts from his intuitive experience of the physical world. The introduction of the *Communia mathematica* presents the metaphysical side of this theory: the reduction of Euclidian geometry to the elementary concepts of physics (continuity, succession, position, movement, etc.). Immersed in the physical world, the intellect is therefore immersed in these intuitive structures as well.

It seems therefore that Bacon considered mathematical understanding as a mode of perception of the structure of the physical world. This universal structure is at the same time also the real nature of the world; this is Bacon's realism. According to the *Geometria speculativa*, nature produces ideal structures. Light produces perfect lines, planetary motions perfect curves. Therefore, 'abstracting' means having the intellectual perception of the ideal (metaphysical) intrinsic structure of the physical world. But the most efficient way for this perception of the 'abstract' structure is experimentation; in the case of optics, the laws of geometry become *visible* for the intellect at the level of sensory experience when an experimental apparatus makes them *appear*.

One of the most evident sources of this idea appears to be Alkindi's introduction to optics in the first pages of his *De aspectibus*.[98] For optics is a physical science, in which geometrical properties are to be *demonstrated* by the physical properties of visual or light rays. This is the reason why Euclid was wrong in his presentation of the laws of optics: he thought he could simply postulate ideal geometrical laws, in a Platonic manner, without demonstrating them at the level of physical experience.[99] But the optician must demonstrate the primary geometrical laws of vision, such as the rectilinear propagation of rays, by the experimental apparatus which make them *appear* at the level of direct observation, so that physics will perform geometry. This way of considering the *verification* and *realization* of geometry by natural experimentation is the key to Alhazen's and Bacon's mathematical physics.

It is clear, therefore, that David Lindberg was right in claiming that Bacon was neither a Platonist nor Aristotelian on the question of applicability of mathematics to nature.[100] The idea of the natural multiplication of forces "according to the laws of

[98] Al-Kindi, *L'optique et la Catoptrique*, edited by R. Rashed (Œuvres Philosophiques et Scientifiques d'Al-Kindi, vol.1; Islamic Philosophy Theology and Science. Texts and Studies. Vol. 29) (Leiden: E. J. Brill, 1997) – *Liber Jacob Alkindi De causis diversitatum aspectus et dandis demonstrationibus geometris super eas*, 439-523.

[99] See David C. Lindberg, *Theories of Vision from Al-Kindi to Kepler* (Chicago and London: The University of Chicago Press, 1976), 18-22; Smith, *From Sight to Light*, 166-169.

[100] Lindberg, "On the Applicability of Mathematics to Nature", 24: "Where did Bacon fall on the Platonist-Aristotelian spectrum of opinion regarding the applicability of mathematics to nature, and what did he contribute to the debate? Strictly speaking, Bacon was neither Platonist nor Aristotelian on this question. When we have cut through the rhetoric, it seems clear that Bacon

geometry", which is the most important application of mathematics to physics for Bacon, never appears in Plato nor in Aristotle. It rather comes from Grosseteste and Alkindi. A Platonist would claim that the very essence of things is the ideal figure or ideal number separated as an object of intuition from sensory matter, from movement and real action. An Aristotelian would claim that mathematical forms are mental representations abstracted from sensory matter, real motion and action. Something very different is said by Alkindi, Alhazen, Grosseteste and Bacon, namely, that physical agents *act* according to the laws of geometry. Properly speaking, geometry is not 'applied' in physics; physics performs geometry, and therefore demonstrates its power and efficiency.

Dominique Demange
dominique.demange@gmail.com

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granted mathematics a considerably larger role in physics that did Aristotle, but that he fell short of the Platonic reduction of physics to mathematics."