THE PARADOXES PRODUCED BY THE DIFFERENT WAYS OF DETERMINING THE RAPIDITY OF MOTION IN THE ANONYMOUS TREATISE *DE SEX INCONVENIENTIBUS*

LAS PARADOJAS CAUSADAS POR LAS DIFERENTES FORMAS DE DETERMINAR LA VELOCIDAD DEL MOVIMIENTO EN EL TRATADO ANÓNIMO *DE SEX INCONVENIENTIBUS*

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Abstract

The anonymous treatise *De sex inconvenientibus* is a good example of the *calculatores*' approach when dealing with motion. It is organized around four main questions relating to the determination of rapidity in four kinds of changes, i.e. in the generation of substantial forms, in alteration, in increase, and in local motion. In some arguments the author points out the paradoxes to which the two ways of determining the rapidity of a motion can lead: rapidity is determined by the effect produced (the degree of quality generated, the space covered, etc.) or it results from the ratio between the moving power and the resistance of the mobile or patient. While this twofold approach to determining rapidity appears in the majority of calculator texts, the two points of view – the analysis according to its effects and the analysis according to its causes – have rarely been confronted.

Keywords

Natural philosophy; Motion; Rapidity; 14th Century

Resumen

El tratado anónimo *De sex inconvenientibus* es un buen ejemplo del enfoque de los calculadores al tratar el movimiento. Está organizado en torno a cuatro cuestiones principales relativas a la determinación de la velocidad en los cuatro tipos de cambios, es decir, en la generación de formas sustanciales, en la alteración, en el aumento y en el movimiento local. En algunos argumentos, el autor señala las paradojas a las que pueden conducir las dos formas de determinar la velocidad de un movimiento: la velocidad se mide por el efecto producido (el grado de calidad generado, el espacio cubierto, etc.) o resulta de la relación entre la potencia del movimiento y la resistencia del móvil o del paciente. Aunque este doble enfoque de la determinación de la velocidad aparece en la mayoría de los textos de cálculo, los dos puntos de vista – el análisis según sus efectos y el análisis según sus causas – rara vez se han confrontado.

Palabras clave

Filosofía natural; movimiento; velocidad; siglo XIV

Introduction

In the 14th century, several masters at Oxford University, known as the *Calculatores*, introduced mathematical tools, notably the theory of proportions, mixed with logic, to study motion in the framework of Aristotelian physics (the most famous are Thomas Bradwardine, Richard Kilvington, William Heytesbury, John Dumbleton, and Richard Swineshead, and after them, many scholars in all European universities are taking up their ideas).[1] Aristotle, in *Physics* VII (250a 4-6), explained that if a motor moves a mobile with a certain rapidity, a motor of double power moves a mobile with double resistance with the same rapidity. It is tempting to deduce that velocity follows immediately from the ratio of the power to the resistance, but Aristotle himself shows what paradoxes such an identification leads to: for example, any motor can move a mobile of any resistance, however great it may be, or a man could move a boat that twenty men move, even if it is twenty times slower, while experience proves that he would not be able to move it (Aristotle, *Physics*, VII, v, 250 a 10-18).

Referring to Aristotle and seeking to determine velocity from the ratio between the power of the motor and the resistance of the mobile, Thomas Bradwardine sets out, in his famous treatise *De proportionibus velocitatum in motibus*, an alternative rule, which was generally accepted until the 17th century:[2]

> The ratio between the rapidities of motions derives from the ratio between motive powers and resistive powers, and conversely,

or

> The ratios between motive powers and resistive powers and the rapidities of motions are proportional in the same order, and conversely.[3]

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[1] There are many studies on these authors. See in particular John E. Murdoch, "The Science of Motion", in *Science in the Middle Ages*, edited by D. C. Lindberg (Chicago: The University of Chicago Press, 1978), 206-264.

[2] See for example Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Milwaukee and London: The University of Wisconsin Press, 1959), ch. 7.

[3] H. Lamar Crosby, *Thomas of Bradwardine. His* Tractatus de Proportionibus. *Its Significance for the Development of Mathematical Physics* (Madison: The University of Wisconsin Press, 1961), 112:

According to this rule, the rapidities are proportional to the ratios between powers and resistances. I will not discuss here the mathematical problems posed by this formulation, which will be resolved by Nicole Oresme. [4] I notice that in the introduction to his treatise, Bradwardine says that he felt it necessary to present in a first chapter the little-known theory of proportions, which he uses for the study of motion, pointing out, following Boethius, that "whoever omits mathematical studies has destroyed the whole of philosophic knowledge."[5]

In this same treatise, Bradwardine explains that the rapidity of any local motion is also determined by the greatest distance traveled in a given time, or that the rapidity of a locally moved body is determined by the rapidity of the fastest moving point or the fastest moving points in that body.[6]

Unlike Bradwardine, who only studies local motion, William Heytesbury discusses, in the *Regule solvendi sophismata*, the three kinds of changes established by Aristotle: local motion, alteration, and increase or diminution.[7] Heytesbury takes up Bradwardine's rule of motion,[8] but he also explains, for each of the three kinds of changes, how to determine the rapidity from the space covered for local motion, from the latitude of the form acquired for alteration (e.g. the heat acquired during heating), from the quantity acquired for increase.[9] He then introduces the expressions "*quo ad*

"Proportio velocitatum in motibus sequitur proportionem potentiarum moventium ad potentias resistivas, et etiam econtrario" or "Proportiones potentiarum moventium ad potentias resistivas, et velocitates in motibus, eodem ordine proportionales existunt, et similiter econtrario"; the English translation is mine.

[4] See Sabine Rommevaux-Tani, *Les nouvelles théories des rapports mathématiques du XIV^e au XVI^e siècle* (Turnhout: Brepols, 2014), 15-34.

[5] Crosby, *Thomas of Bradwardine. His* Tractatus de Proportionibus, 64: "quisquis scientias mathematicales praetermiserit, constat eum omnem philosophiae periddisse doctrinam"; English translation, 65.

[6] Crosby, *Thomas of Bradwardine. His* Tractatus de Proportionibus, 130: "Cuiuslibet motus localis, velocitas secundum maximum spatium lineale ab aliquo puncto sui moti descriptum accipitur"; "Ideo videtur magis rationabiliter dici quod velocitas motus localis attenditur penes velocitatem puncti velocissime moti in corpore moto localiter".

[7] William Heytesbury, *Regule solvendi sophismata* (Venice: Bonetus Locatelllus, 1494), see the sixth chapter "De tribus predicamentis", fols. 37ra-52rb.

[8] Heytesbury, *Regule solvendi sophismata*, fol. 44vb: "(...) secundum proportionem potentie motoris ad potentiam resistivam generaliter attenditur velocitas in quocumque motu (...)".

[9] Heytesbury, *Regule solvendi sophismata*, fol. 38vb: "In motu autem locali difformi in quocumque instanti attenditur velocitas penes lineam quam describeret punctus velocissime motus si per tempus moveretur uniformiter illo gradu velocitatis: quo movetur in eodem instanti: quocumque instanti dato"; fol. 45ra: "Ideo sequitur tertia positio quam inter alias in ista materia reputo veriorem scilicet quod universaliter omnis velocitas talis motus attenditur penes proportionem quantitatis de novo uniformiter acquirende in tanto tempore vel in tanto ad quantitatem prius habitam (...)"; fol. 51ra:"Ideo sequitur tertia positio et ultima quam magis probabiliter meo iudicio poterit sustineri: videlicet quod omnis velocitas in alteratione attenditur penes maximam latitudinem talis

effectum", to qualify the determination of rapidity from the space covered, the heat or the quantity acquired, and "*quo ad causam*" to qualify the determination of rapidity from the ratio of power to resistance in the case of local motion.[10] It is indeed necessary that the power of the motor or the agent be greater than the resistance of the mobile or the patient for there to be motion, and the action of the motor or agent on the mobile or patient causes the change. Thus, Nicole Oresme, in his treatise *De proportionibus proportionum*, speaks of the ratio from which rapidity arises ("*proportio a qua venit velocitas*"[11]) and of the rapidity that comes from such a ratio ("*velocitas que a tali proportione oriatur*"[12]). Moreover, the effect produced is either the space covered, for local motion, or the form acquired, for alteration, or the quantity acquired, for increase.

In most of the treatises of the *Calculatores* tradition to which Bradwardine and Heytesbury belong,[13] where the question of determining rapidity for the different kinds of change is raised, the two ways of understanding rapidity, from the effect or from the cause, are treated separately.[14]

Several remarks are called for at this point:

– It is anachronistic to identify these two ways of studying rapidity with the division between dynamics and kinematics in modern physics; I agree with Daniel A. Di Liscia, who has explained this point perfectly.[15]

– To express the dependency between rapidity and cause or the ratio between power and resistance, Bradwardine uses the verb "*sequitur*", which Crosby translates as "varies in accordance with";[16] and to express the dependency between rapidity and

forme sue qualitatis que uniformiter acquiretur alicui subiecto maiori seu minori in tanto tempore vel in tanto correspondenter."

[10] He ends the chapter on local motion with these words (Heytesbury, *Regule solvendi sophismata*, fol. 44rb): "Ideo viso iam generaliter penes quid tamquam quo ad effectum attendatur velocitas in motu locali: quia secundum proportionem potentie motoris ad potentiam resistitivam generaliter attenditur velocitas in quocumque motu tanquam quo ad eius causam sequitur secunda pars huius capituli in qua perscrutabit penes quid quo ad effectum attendatur velocitas in augmentatione et diminutione, communiter dictis que rarefactio et condensatio appellantur."

[11] Nicole Oresme, *De proportionibus proportionibus* and *Ad pauca respicientes*, edited with Introductions, English Translations and Critical Notes by E. Grant (Madison: University of Wisconsin Press, 1966), 288, l. 357.

[12] Nicole Oresme, *De proportionibus proportionibus* and *Ad pauca respicientes*, 290, l. 374.

[13] There are many works on this tradition. See for example: Edith Dudley Sylla, "The Oxford calculators", in *The Cambridge History of Later Medieval Philosophy. Vol. 1: From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100–1600*, edited by N. Kretzmann, A. Kenny, J. Pinborg and E. Stump (Cambridge: Cambridge University Press, 1982), 540-563.

[14] See Daniel A. Di Liscia, "Velocidad *quo ad effectus* y velocidad *quo ad causas*: la tradición de los calculadores y la metodología aristotélica", in *Method and Order in Renaissance Philosophy of Nature. The Aristotle Commentary Tradition*, edited by D. A. Di Liscia, E. Kessler and Ch. Methuen (Aldershot: Ashgate, 1997), 143-176.

[15] Di Liscia, "Velocidad *quo ad effectus* y velocidad *quo ad causas*", 173-176.

[16] Crosby, *Thomas of Bradwardine. His* Tractatus de Proportionibus, 112 and 113.

distance traveled, Bradwardine uses the expression "*attenditur penes*" translated by Crosby as "is to be determined by". Grant, in his edition of Nicole Oresme's *De configurationibus qualitatum et motuum*, translates "*attenditur penes*" as "is a function of" in most cases[17] or "is measured by"[18] or sometimes "is attended", specifying in the latter case "i.e. is measured by".[19] I note that in his treatise *De configurationibus qualitatum et motuum*, Nicole Oresme uses the verb "*mensuratur*" only in the case of determining the rapidity from the space traveled in the local motion.[20] Further study of these expressions and their uses in the *Calculatores* treatises would be necessary to determine their exact meanings; in particular, it would be necessary to see whether the term "*mensura*" is usually used in this context.

– I note that Bradwardine's rule and the rules which express rapidity in terms of space covered, form or quantity acquired, are rarely used in these texts in order to calculate rapidity (even though Bradwardine's rule is a mathematical statement). These rules are generally used, in the comparison of two motions, to explain that one is faster than the other, or to decide whether a motion is uniform or uniformly difform, for example. Thus, if the ratio of power to resistance for one change is greater than the ratio of the same type for the other change, the former is said to be faster than the latter. And if, during a change, the ratio of power to resistance remains unchanged, the change is said to be uniform; it is uniformly difform if this ratio increases or decreases continuously. Moreover, if more space is covered by the first mobile than by the second in the same time, the first will be said to be faster than the second. All these considerations are qualitative.

– To my knowledge, the link is never made by Bradwardine or Heytesbury between these two approaches to determining rapidity, *quo ad effectum* and *quo ad causam*. And this seems to be the case for the other members of the *Calculatores* tradition,[21] with the exception of the author of *De sex inconvenientibus*, as we shall now see.

The anonymous treatise *De sex inconvenientibus*

In a treatise entitled *De sex inconvenientibus*, written between 1335 and 1339, the anonymous author, who is perfectly familiar with the works of Bradwardine,

[17] Marshall Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions. A Treatise an the Uniformity and Difformity of Intensities known as* Tractatus de configurationibus qualitatum et motuum (Madison, Milwaukee and London: University of Wisconsin Press, 1968), 169, 221, 223, 225, 245.

[18] Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions*, 215, 277.

[19] Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions*, 279.

[20] Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions*, 168, l. 9-11: "Item sicut velocitas in motu locali secundum longitudinem spatii mensuratur, ita in alteratione velocitas attenditur penes intensionem."

[21] See Daniel A. Di Liscia, "Velocidad *quo ad effectus* y velocidad *quo ad causas*...".

Heytesbury, whom he quotes, and undoubtedly Kilvington, even though he does not quote him, raises the question of determining the rapidity for the three usual kinds of changes, i.e., alteration, increase and local motion; but the author adds to these the generation of substantial forms.[22] The treatise thus consists of four main questions, to which are attached secondary questions or articles (three per main question).[23] The treatise is thus composed of *Questiones*. In this type of text, the *quod non* or *quod sic* arguments that open the *Questio* are not very developed, the most important part being the *determinatio,* in which the author develops his answer to the question asked.[24] This is not the case in *De sex inconvenientibus*. The *determinationes* are reduced; for the most part, the author simply says which opinion he agrees with, with a few exceptions (for example, for the article concerning the theorem of the middle degree, he gives several demonstrations of this theorem[25]). On the other hand, he presents three opinions for each main question and produces for each of them a set of six *inconvenientes* or difficulties that would result from the acceptance of these opinions, even for the one (the third) that he finally accepts. And for each article, he produces six difficulties that would result from a positive or negative answer (as the case may be) to the question posed. Hence the title of the treatise is *De sex inconvenientibus* (About six difficulties). It is several of these *inconvenientes* that we will examine here, those in which the author confronts the points of view on the case treated.

Contradiction between the two ways of determining rapidity

The first main question of the treatise asks whether the rapidity of generation of substantial forms can be determined. The cases considered in this question are of two kinds: a hot body *a* warms another less hot or a cold body *b*, introducing its form of heat until *b* becomes as hot as *a*; a body becomes hotter by the destruction of its internal coldness by its internal heat. These are thus processes of alteration, but described as the generation of heat from other heat or by the destruction of cold.

The author records three positions regarding the determination of the rapidity of these processes. He rejects the first two and accepts the third, but presents six

[22] See J. Papiernik, "How to measure different movements? The 14th-century treatise *De sex inconvenientibus*", *Przegląd Tomistyczny* 25 (2019): 445-460.

[23] See Sabine Rommevaux-Tani, *"De sex inconvenientibus". Un traité anonyme de philosophie naturelle au XIVe siècle* (Paris: Vrin, 2022); this book contains the critical edition of the *De sex inconvenientibus* and a doctrinal analysis of the treatise.

[24] For the genre of the Questio see, for example: Olga Weijers, *La 'disputatio' dans les facultés des arts au moyen âge* (Turnhout: Brepols, 2002) and more recently Olga Weijers, *In Search of the Truth. A History or Disputation Techniques from Antiquity to Early Modern Times* (Turnhout: Brepols, 2013).

[25] See Sabine Rommevaux-Tani, "The influence of the Oxford Calculatores on the Understanding or Local Motion: The Example of the *Tractatus de sex inconvenientibus*", in *Quantifying Aristotle. The Impact, Spread and Decline of the Calculatores Tradition*, edited by D. A. Di Liscia and E. Sylla (Leiden: Brill, 2022), 153-185.

difficulties raised by each of these three positions. According to the first position, the rapidity of such a kind of generation would depend on the form or quality introduced by what generates it.[26] And the explanation that follows the statement of this position is the following:

> (...) in heating, for example, where a more intense form of fire is introduced, the motion by which that form was introduced is faster than another motion in which a weaker form is introduced.[27]

Then the author considers the case of an extreme heat *a* (a fire) approximated by a less hot body *b*.[28] He assumes that *b* is uniformly difformly hot, i.e. he considers a linear body whose heat is continuously distributed from one end to the other, from degree 0, or non-degree of heat, to an extreme degree, and he further assumes that the hottest end is of extreme heat exclusively (it has a degree of heat lower than the extreme degree by an indivisible).

On the one hand, the author notes that, since the heat of *b* increases due to the action of *a* on *b*, the resistance of *b* to heating by *a* diminishes continuously. Thus, the ratio between the power of *a* and the resistance of *b* increases continuously during the whole heating process. And since, according to Bradwardine´s rule, the rapidity depends on the ratio of the power of *a* to the resistance of *b*, then the rapidity increases; the motion is uniformly accelerated or uniformly difform.

But on the other hand, it may be noticed that *a* introduces during the whole period of the motion an extreme constant heat. Since the rapidity is, according to this first opinion, determined by the degree of heat introduced, the rapidity is constant; the motion is uniform.

Then the following difficulty arises, which leads the author to reject this first: "*a*, which generates, will continuously generate with a greater and greater ratio, and yet, continuously, it will generate uniformly."[29] The motion is uniform if the rapidity is determined by the generated form, i.e. by the effect produced, but the motion is uniformly difform if we consider it from the point of view of the cause, that is, the ratio between power and resistance (even if the author of *De sex inconvenientibus* does not use the terms 'effect' and 'cause' in this treatise).

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[26] Rommevaux-Tani, "*De sex inconvenientibus*", 136: "secundum sectam positionis prime sequitur quod talis velocitas attenditur penes formam inductam vel inducendam a generante."

[27] Rommevaux-Tani, "*De sex inconvenientibus*", 135: "quando generans inducit vel incipit inducere formam suam, verbi gratia in calefactione, ubicumque inducitur forma ignis intensior, motus iste, quo inducitur, est velocior aliquo alio motu, quo forma remissior inducitur."

[28] See the second difficulty raised by this position: Rommevaux-Tani, "*De sex inconvenientibus*", 138.

[29] Rommevaux-Tani, "*De sex inconvenientibus*", 136: "*a* generans continue generabit a proportione maiori et maiori, et tamen ipsum continue uniformiter generabit."

The paradoxes caused by the two ways of determining rapidity are also at the heart of the difficulties raised by the second position presented by the author regarding the determination of the rapidity of generation:

> such rapidity would depend on the latitude of the form to be acquired and on the quantity in which the latitude of this form to be acquired is extended.[30]

The term "latitude" refers to the variation in intensity of forms, qualities, quantities, rapidities, etc., during a change in a subject.[31] We also find in *De sex inconvenientibus* the term "degree", referring to the latitude. Thus, when a body is heated, its heat increases from degree *a* to degree *b*, passing through all intermediate degrees; all these degrees constitute the latitude *ab*.

Moreover, the statement should be understood as follows: the rapidity is determined by the quantity of the altered subject when equal forms are introduced, and it is determined by the latitude of the form introduced in equal subjects. The author never considers the case where latitudes of different forms are introduced in subjects of different quantities.

We are not going to examine all the difficulties arising from this second opinion, in which, as has been stated, the author compares the two modes of determining rapidity. Let us look at the fifth.[32] The author presents the case of two heats, *a* and *b,* acting respectively on *c* and *d*, introducing equal latitudes of heat. It is assumed that *a* is one hundred times hotter than *b* and that *c* and *d* are of equal quantities.

The two heats act with equal rapidities, since the rapidity is, according to this second opinion, determined from the latitudes introduced in equal subjects, and it was assumed that *a* and *b* introduced equal forms in their patients.

Furthermore, it was assumed that *a* has a power to act one hundred times greater than *b*, and the patients *c* and *d* have equal resistances, so that *a* alters *c* one hundred times faster than *b* alters *d*.

In conclusion: "*a* and *b*, things that generate, alter their patients equally, and yet *a* does so a hundred times faster."[33] Again, equal effects imply equal rapidities, but unequal ratios lead to the conclusion that the changes are unequal.

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[30] Rommevaux-Tani, "*De sex inconvenientibus*", 142: "talis velocitas attenderetur penes latitudinem forme acquirende et penes quantitatem per quam extenditur ista latitudo istius forme acquirende, sicut ponit seconda positio."

[31] For this notion see: Edith Sylla, "Medieval concepts of the latitude of forms. The Oxford calculators", *Archives doctrinales et littéraire du Moyen Age* 40 (1973) : 223-283, in particular 251-257.

[32] Rommevaux-Tani, "De sex inconvenientibus", 145.

[33] *Ibid*., 143: "*a* et *b* generantia equaliter alterant sua passa, *a* tamen in centuplo velocius."

It should be noted in this case that the numerical value "one hundred" is of no importance in the reasoning here. The only thing that matters is that *a* acts more strongly than *b*.

Let us now consider one of the arguments that the author opposes to the second opinion concerning the determination of the rapidity of alteration. According to this opinion, the rapidity would depend on the quantity or the extension of the subject in which the alteration takes place.

The author then considers the case of a fire *a* heating water *b* in such a way that *a* continuously introduces as much heat into any part of *b* during the process. The alteration is thus assumed to be uniform, according to this opinion. But the author notes that the resistance of *b* to the action of *a* decreases as *b* becomes hotter and hotter. So, the ratio increases between the power of *a*, which is constant, and the resistance of b, which decreases. Hence the difficulty that "something that alters will continually alter with an ever-increasing ratio and yet it will continually alter uniformly."[34] Here again, the author rejects this opinion because of this contradiction.

Distinction between "being moved or changed by faster motion or change" and "being moved or changed more quickly"

Let us return to the first opinion, considered in relation to the determination of the rapidity of generation: the rapidity would depend on the form or quality introduced. Let us consider the same case as above: the generation of heat produced by an extreme heat *a* in a uniformly difformly hot body *b*, the most intense end of which is in the extreme form of heat exclusively. Now compare it with the generation of heat produced by *c,* half as hot as *a,* on a uniformly difformly hot body *d*, similar to *b* but half as hot.[35] The heat introduced by *c* is twice as low, therefore, according to this first opinion, as *a* generates; the time taken for the action of *a* in *b* is half the time taken for the action of *c* in *d*.

It may be observed, moreover, that the ratio of the power of *a* to the resistance of *b* is the same as the ratio of the power of *c* to the resistance of *d*. The author does not explicitly make this last remark, but it can be deduced from what he says: "*a* and *c* will generate their forms precisely as quickly ('*eque cito*'). Indeed, each of them immediately after that will introduce its form into the patient that resists it, and only by a motion of generation."[36]

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[34] *Ibid*, 208: " aliquod alterans alterabit continue a maiori proportione et maiori, et tamen continue uniformiter alterabit"; proof on page 209.

[35] See the fifth *inconveniens*: Rommevaux-Tani, "*De sex inconvenientibus*", 140-141.

[36] Rommevaux-Tani, "*De sex inconvenientibus*", 141: "eque cito precise generabunt *a* et *c* formas suas, quia utrumque istorum immediate post hoc inducet formam suam in passum sibi resistens, et solum motu generationis."

The difficulty is summarized as follows: "*a* and *c* are two things that generate similar forms in patients *b* and *d*, and *a* will generate its form in half the time of *c* and they begin to generate at the same time, yet *c* generates its form precisely as fast as *a*, all other things being equal."[37] So, the motion is accelerated, if we consider the effect produced, but it is uniform if we consider the cause, i.e. the ratio of power to resistance.

Note that in this argument, the author uses two different terminologies: he speaks of a motion that is "faster" ("*velocior*") than another on the one hand, and on the other hand, he talks about mobiles that reach the final heat "as quickly" ("*eque cito*"); the first notion, "faster", refers to the determination of rapidity from the effect produced, and "as quickly" to the ratios between power and resistance, which are equal here.

Note, moreover, that for the reasoning to be valid, the author could have said that *c* is less hot than *a*, and *b* is less hot than *d* according to the same ratio, without specifying what ratio. And he would have come to the paradoxical conclusion that *a* and *c* are two things which generate similar forms in patients *b* and *d*, and *a* will generate its form in less time than *c*, and yet *c* generates its form precisely as quickly as *a*. That *a* will generate its form in half the time of *c* in the case under consideration is anecdotal.

Still, with regard to the generation, the author agrees with the third position, according to which the rapidity depends only on the latitude of the form to be acquired, without taking into consideration the extension of the subject of generation.[38] But here again, he proposes six difficulties that may arise if this position is accepted.

He thus considers two identical bodies, *a* and *b*, slightly hot. These two bodies are heated by the destruction of their intrinsic coldness by their intrinsic heat. And it is assumed that all the coldness is thus destroyed in *b*, so that the whole of *b* becomes extremely hot; in *a*, however, one half remains unchanged, while the other half becomes extremely hot by the destruction of the coldness in it.[39]

The altered subjects are *b* and half of *a*. Since *b* and *a* have been assumed to be identical, the altered subjects are in a double ratio. And the same extreme heat is acquired by both. It is deduced that the alteration by which *b* is changed is twice as fast as that by which *a* is changed. But the author notes also that the alterations in *a* and *b* begin and cease at the same time (the two halves of *b* begin and cease to heat up at the same time as the half of *a*).

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[37] Rommevaux-Tani, "*De sex inconvenientibus*", 137: "*a* et *c* sunt duo generantia que generabunt ex *b* et *d* passis formas illis consimiles, et *a* in duplo minori tempore generabit formam suam quam *c*, et simul incipiunt generare, et tamen eque cito precise generabit *c* formam suam sicut *a* formam suam, ceteris paribus".

[38] Rommevaux-Tani, "*De sex inconvenientibus*", 147: "si in generatione formarum sit certa ponenda velocitas, igitur talis velocitas attenderetur solum penes latitudinem forme acquirende, sicut ponit tertia positio et tenet tota scola oxoniensis."

[39] Rommevaux-Tani, "*De sex inconvenientibus*", 151.

Thus we have the following difficulty (the fifth):

> "in the intrinsic generation of an element, two uniform slightly hot things, of equal quantities and heats, are altered during precisely the same time, until each of them becomes extremely hot, in such a way that they begin to be altered as quickly and cease to be altered as quickly, and yet the entire alteration by which *b* will be continuously altered will be continuously twice as rapid as the alteration by which *a* will be continuously altered."[40]

The author must respond to the argument because he accepts this opinion.[41] He rejects, of course, the argument that *b* moves twice as fast as *a* because the subject in which the alteration in *b* takes place is double that in which the alteration in *a* takes place. Indeed, according to this position, only the latitude of the acquired form is to be taken into consideration and not the extension of the subject where the alteration takes place.

But he also takes advantage of this case to distinguish between "be altered by faster motion" ("*velociori motu alteratur*") and being "altered more quickly" ("*velocius alteratur*): *a* is altered by a motion twice as fast, when the ratio of power to resistance is twice as great, and *a* is said to be altered twice as fast, if the latitude acquired by *a* is double. And the author accepts that an alteration can be said to be twice as fast, while the thing altered is not twice as fast. He therefore rejects the following implication: "*b* is altered by a motion twice as fast as the motion by which *a* is altered, so *b* is altered twice as fast as *a*." He thus acknowledges that the two ways of determining rapidity can lead to different conclusions, at least for this type of alteration.

Note again that the fact that *b* is altered precisely twice as fast as *a* is unnecessary; the paradox is that *b* is altered faster than *a*.

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40 Rommevaux-Tani, "*De sex inconvenientibus*", 147: "QUINTO, quod in generatione intrinseca elementi, aliqua sunt duo calida remissa uniformia equalis quantitatis et eque calida, que alterabuntur per idem tempus precise, quousque utrumque istorum fuerit calidum in summo, ita quod eque cito incipiunt alterari et eque cito desinent alterari, et tamen tota alteratio qua *b* continue alterabitur erit continue in duplo velocior quam alteratio qua *a* continue alterabitur."

41 Rommevaux-Tani, "*De sex inconvenientibus*", 201: "AD QUINTUM ET SEXTUM, que in modico discrepant, dicitur concedendo illas conclusiones contra, quarum primam arguitur sic: « *a* et *b* iam sunt per omnia similia et utrumque alterabitur uniformiter, quousque ipsum fuerit summum; et *b* continue alterabitur in duplo velocius; igitur *b* erit citius summum quam *a* », conceditur consequentia et negatur antecedens pro ista parte: « *b* continue alterabitur in duplo velocius *a* ». Sed contra: « duplo velociori motu alteratur *b* quam *a* », conceditur. « Igitur in duplo velocius alteratur *b* quam *a* », non sequitur. Et si adhuc arguitur: « a maiori proportione alteratur *b* quam *a*, igitur velocius alteratur *b* quam *a* », adhuc non sequitur. Sed illud sequitur quod velociori motu alteratur *b* quam *a*, quoniam motus sequitur proportionem, ipsum autem velocius, tardius vel equaliter sequitur magnitudinem spatii in eodem tempore vel equali descripti, sicut clarius patebit in questione de augmentatione."

This distinction between "having a faster motion" and "being moved more quickly" can be found in the third main question concerning increase in the arguments against the third opinion, which is the one accepted by the author. According to this opinion, the rapidity of the increase results from the ratio between the latitudes of rarity (i.e. the quantities of bodies at the beginning and at the end of the increase), and one increase is faster if the linear space described by the fastest moving point or points is longer in the same time.[42] In the case of increase, in the absence of an identified agent, rapidity cannot result from the ratio of the power of the agent to the resistance of the patient. The difficulties raised relate to the apparent contradiction that may arise between the two ways of determining rapidity stated by this position.

Difficulties may arise in particular from comparing the increase of a finite body *a* and an infinitesimally small body *b*, for example, an infinitesimally small part of *a* (one can imagine that *a* and *b* are linear quantities). Since the body *b* is infinitesimally small, it begins to move infinitely slowly, because the distance covered by its fastest moving point is infinitesimally small. This statement is justified as follows: if one body begins to rarefy along a certain distance with a certain rapidity, another body would begin to rarefy along half that distance with half the rapidity, and another along a third of that distance with three times the rapidity, etc., because the ratio between the rapidities depends on the ratio between the distances travelled by the fastest moving point. So, since *b* has been assumed to be infinitesimally small, it starts to become infinitely rarefied. If, on the other hand, it is assumed that the finite body *a* begins to rarefy with the same degree of rapidity, i.e., with the same ratio between the latitude of final rarity and the latitude of initial rarity, we have the first difficulty: "*a* and *b* begin rarefied with the same ratio, and yet *b* begins rarefied infinitely more slowly than *a.*"[43]

Moreover, it follows from the same case that, whatever the degree of rapidity with which this same continuous *a* becomes rarefied (i.e. whatever the ratio between the latitudes of rarity), it begins to become rarefied infinitely slowly. Indeed, it is supposed that *a* is continuously rarefied, therefore part by part. Now, given a part of *a*, it is always possible to find one that is infinitely smaller than it. And according to the previous case, this infinitely small part will increase infinitely slowly. So, *a* will start to increase infinitely slowly. Thus, we have the second difficulty: "whatever ratio *a* begins to rarefy, it begins to rarefy infinitely slowly."[44]

[42] Rommevaux-Tani, "*De sex inconvenientibus*", 267: "velocitas in tali motu augmentationis attendetur penes proportionem latitudinum raritatis, et ipsum velocius penes proportionem quantitatum linealium a puncto velocissime moto vel a punctis velocissime motis in tanto vel in tanto tempore descriptarum."

[43] Rommevaux-Tani, "*De sex inconvenientibus*", 267: "*a* et *b* ab eadem proportione incipiunt rarefieri, et tamen in infinitum tardius incipit *b* rarefieri quam *a.*"; proof on pages 267-268.

[44] Rommevaux-Tani, "*De sex inconvenientibus*", 267: "quecumque fuerit proportio a qua *a* incipit rarefieri, in infinitum tarde ipsum incipit rarefieri."; proof on page 268.

To these two difficulties the author responds by admitting the conclusions, and he adds: “as far as I can see, they have been clearly demonstrated and are not in contradiction with the rules of the ratios.”[45] He then explains that the rapidity or slowness of an increase results from the ratio between the latitudes of rarity, and whether one increase is faster or slower than another depends on the ratio between the distances travelled by the fastest moving point or points in the bodies, in the same time.

He thus distinguishes two ways of qualifying the increase, which can lead to different conclusions, without this being a difficulty. And he adds that, in so doing, augmentation is distinguished from local motion:

> And although, perhaps, this is against the rules of ratios for local motion, it is not against these demonstrative rules for the rarefaction and increase, since these motions differ specifically from local motion.[46]

Conclusion

The author of the *De sex inconvenientibus* is well aware that there are different ways of determining the rapidity of a motion or change. In particular, rapidity can be determined by the effect produced, and any motion or change is more or less rapid according to the ratio between the power of the agent or motor and the resistance of the mobile or patient. The author makes evident this double way of considering rapidity by using different expressions to qualify them. He distinguishes between “having a faster motion or change” and “being moved or changing faster.” The motion or change is said to be faster when the ratio of power to resistance is greater, and *a* is said to be moved or change faster than *b*, if the effect of the motion of change for *a* is more intense than for *b*. In the first case, the object considered is the motion or change which results from the ratio between power and resistance. In the second case, the object is the subject that is moved or changed, and the effect of the motion or the change on this subject tells us how quickly the subject is affected. The confrontation of these two points of view leads the author to reject certain opinions regarding generation, alteration, and increase.

Furthermore, we have noticed on several occasions that the numerical values present in the different cases are of no importance. The paradoxes raised arise from the fact that the rapidities are unequal, whereas the data of the problem should have led to

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[45] Rommevaux-Tani, “*De sex inconvenientibus*”, 295: “ET TUNC AD ARGUMENTA IN OPPOSITUM dico quod conclusiones adducte pro maiori parte sunt vere, videlicet prima, secunda, quarta et quinta, que, sicut mihi apparet, sunt clarius demonstrate, nec ille sunt contra regulas proportionum, quoniam, licet motus et ipsa velocitas et tarditas proportionem sequantur, ipsum tamen velocius et tardius sequitur proportionem spaciarum linealium in eodem tempore descriptarum.”

[46] Rommevaux-Tani, “*De sex inconvenientibus*”, 295: “Et licet forte hoc foret contra regulas proportionum de motu locali, non tamen foret hoc contra regulas demonstrativas in motu rarefactionis et augmentationis, cum illi motus specifice differant a motu locali.”

the fact that they are equal, or vice versa. All these considerations in the *De sex inconvenientibus* are fundamentally qualitative.

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Bibliography

Primary Sources

Clagett, Marshall, *Nicole Oresme and the Medieval Geometry of Qualities and Motions. A Treatise on the Uniformity and Difformity of Intensities known as* Tractatus de configurationibus qualitatum et motuum (Madison, Milwaukee and London: The University of Wisconsin Press, 1968).

Crosby, H. Lamar, *Thomas of Bradwardine. His* Tractatus de Proportionibus. *Its Significance for the Development of Mathematical Physics* (Madison: The University of Wisconsin Press, 1961).

Nicole Oresme, *De proportionibus proportionibus* and *Ad pauca* respicientes, edited with Introductions, English Translations and Critical Notes by E. Grant (Madison, Milwaukee and London: The University of Wisconsin Press, 1966).

Rommevaux-Tani, Sabine, *"De sex inconvenientibus". Un traité anonyme de philosophie naturelle au xiv[e] siècle* (Paris: Vrin, 2022).

William Heytesbury, *Regule solvendi sophismata* (Venice: Bonetus Locatelllus, 1494).

Secondary Sources

Clagett, Marshall, *The Science of Mechanics in the Middle Ages* (Madison, The University of Wisconsin Press, 1959).

Di Liscia, Daniel A., "Velocidad *quo ad effectus* y velocidad *quo ad causas*: la tradición de los calculadores y la metodología aristotélica", in *Method and Order in Renaissance Philosophy of Nature. The Aristotle Commentary Tradition*, edited by D. A. Di Liscia, E. Kessler and Ch. Methuen (Aldershot, Brookfield, Singapore and Sidney: Ashgate, 1997).

Murdoch, John E., "The Science of Motion", in *Science in the Middle Ages*, edited by D. C. Lindberg (Chicago: The University of Chicago Press, 1978), 206-264.

Papiernik, Joanna, "How to Measure Different Movements? The 14th-Century Treatise *De sex inconvenientibus*", *Przegląd Tomistyczny* 25 (2019): 445-460.

Rommevaux-Tani, Sabine, "The Influence of the Oxford *Calculatores* on the Understanding or Local Motion: The Example of the *Tractatus de sex inconvenientibus*", in *Quantifying Aristotle. The Impact, Spread and Decline of the Calculatores Tradition*, edited by D. A. Di Liscia and E. Sylla (Leiden: Brill, 2022), 153-185.

Sylla, Edith, "Medieval Concepts of the Latitude of Forms. The Oxford Calculators", *Archives doctrinales et littéraire du Moyen Age* 40 (1973): 223-283.
–– "The Oxford Calculators", in *The Cambridge History of Later Medieval Philosophy. Vol. 1: From the Rediscovery of Aristotle to the Disintegration of Scholasticism 1100–1600*, edited by N. Kretzmann, A. Kenny, J. Pinborg and E. Stump (Cambridge: Cambridge University Press, 1982).
Weijers, Olga, *La 'disputatio' dans les facultés des arts au moyen âge* (Turnhout: Brepols, 2002).
–– *In Search of the Truth. A History or Disputation Techniques from Antiquity to Early Modern Times* (Turnhout: Brepols, 2013).