

NICOLE ORESME ON MOTION AND THE ATOMIZATION OF THE CONTINUUM

NICOLE ORESME SOBRE EL MOVIMIENTO Y LA ATOMIZACIÓN DEL CONTINUUM

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Abstract

As Aristotle classically defined it, continuity is the property of being infinitely divisible into ever-divisible parts. How has this conception been affected by the process of mathematization of motion during the 14th century? This paper focuses on Nicole Oresme, who extensively commented on Aristotle's *Physics*, but also made decisive contributions to the mathematics of motion. Oresme's attitude about continuity seems ambivalent: on the one hand, he never really departs from Aristotle's conception, but on the other hand, he uses it in a completely new way in his mathematics, particularly in his *Questions on Euclidean geometry*, a tantamount way to an atomization of motion. If the *fluxus* theory of natural motion involves that continuity is an essential property of real motion, defined as a *res successiva*, the ontological and mathematical structure of this continuity implies that continuum is *in some way* "composed" of an infinite number of indivisibles. In fact, Oresme's analysis opened the path to a completely new kind of mathematical continuity.

Keywords

Continuity; Nicole Oresme; Mathematics; Motion; Fluxus Theory; Indivisibles; *res successiva*; Ontology; Infinitely Small

Resumen

De acuerdo con la definición clásica de Aristóteles, la continuidad es la pertenencia de ser infinitamente divisible dentro de las partes siempre divisibles. ¿Cómo ha afectado este concepto al proceso de matematización del movimiento durante el siglo XIV? Este artículo se centra en Nicole Oresme, quién ha extensamente comentado la *Física* de Aristóteles y, al mismo tiempo, llevó a cabo contribuciones decisivas relativas a las matemáticas del movimiento. La actitud de Oresme con respecto a la continuidad parece indecisa: por un lado, él nunca se aleja de la

concepción de Aristóteles; por otro lado, la utiliza de una manera completamente nueva en su matemática particularmente en sus *Cuestiones sobre la Geometría de Euclides*, una manera que es equivalente a una atomización del movimiento. Si teoría del *fluxus* del movimiento natural implica que la continuidad es una propiedad esencial del movimiento real, definida como una *res successiva*, la estructura ontológica y matemática de esta continuidad insinúa que esta continuidad está de alguna manera “compuesta” de un número infinito de indivisibles. De hecho, el análisis de Oresme abrió el paso a una nueva forma total de continuidad matemática.

Palabras clave

Continuidad; Nicole Oresme; matemáticas; movimiento; teoría del *fluxus*; indivisibles; *res successiva*; ontología; infinitamente pequeño

Introduction

By teaching how to represent motion by geometrical diagrams in his *Tractatus de configurationibus qualitatum et motuum*,¹ Nicole Oresme² made a decisive contribution to the mathematization of motion.³ Moreover, such a mathematization has ontological counterparts. Mathematizing motion requires us to identify it with a whole whose properties are determined by its very parts. The continuity of motion requires moreover that it is conceived as infinitely divisible in divisible parts, according to the classical

¹ Marshall Clagett, *Nicole Oresme and the Medieval Geometry of Qualities and Motions: a treatise on the uniformity and difformity of intensities known as Tractatus De configurationibus qualitatum et motuum* (Madison, Milwaukee and London: The University of Wisconsin Press, 1968). See also Marshall Clagett, *The science of mechanics in the Middle Ages* (Madison: University of Wisconsin Press, 1959); Peter Damerow, Gideon Freudenthal, Peter Mclaughlin and Jürgen Renn, *Exploring the limits of preclassical mechanics: a study of conceptual development in early modern science: free fall and compounded motion in the work of Descartes, Galileo, and Beeckman* (New York: Springer, 2004).

² On the life of Nicole Oresme, see Max Lejbowicz, “Nicole Oresme ‘spectateur engagé’”, in *Nicole Oresme philosophe: Philosophie de la nature et philosophie de la connaissance à Paris au XIVe siècle*, edited by J. Celeyrette and Ch. Grellard (Turnout: Brepols, 2014), 21-61.

³ See the references in footnote 1. See also Philippe Debroise, *Mathématiques de l'intensité et Merveilles de la nature, Étude sur le Tractatus de configurationibus qualitatum et motuum* (Paris: Universitè Paris Diderot, Doctoral thesis, 2019); Pierre Duhem, *Études sur Léonard de Vinci*, 3 (Paris: Hermann, 1984 [1908]), 314-350; Pierre Duhem, *Le Système du monde. Histoire des doctrines de Platon à Copernic*, vol. 7 (Paris: Hermann, 1956), 462-633; Anneliese Maier, “La doctrine de Nicolas d’Oresme sur les ‘configurationes intensionum’”, *Revue des sciences philosophiques et théologiques* 32, 1/2 (1948): 52-67, also in Anneliese Maier, *Ausgehendes Mittelalter 1: Gesammelte Aufsätze zur Geistesgeschichte des 14. Jahrhunderts* (Rome: Edizioni di Storia e Letteratura, 1964), 335-352; John E. Murdoch and Edith D. Sylla, “The Science of Motion”, in *Science in the Middle Ages*, edited by D. C. Lindberg (Chicago and London: University of Chicago Press, 1978), 206-264; Pierre Souffrin and Jean-Pierre Weiss, “Le Traité des configurations des qualités et des mouvements. Remarques sur quelques problèmes d’interprétation et de traduction”, in *Nicolas Oresme. Tradition et innovation chez un intellectuel du XIVe siècle*, edited by P. Souffrin and J.-P. Segonds (Paris: Les Belles Lettres, 1988), 125-134.

definition Aristotle gave of continuity in his *Physics*.⁴ Could such an *understanding* of the continuum be maintained when applied to a mathematized motion? Superficially, Oresme never departs from this classical understanding of the continuum, particularly in his extended studies that are to be found in his *Questions on Physics*.⁵ However, when one looks more precisely at his arguments and compares them to other, more mathematical works, like his *Questions on Euclidean geometry*,⁶ one cannot fail to be struck by the completely original conception Oresme had of the continuum, and by the way he could turn those ontological views into strong mathematical techniques. In fact, those striking mathematical techniques are tantamount to an atomization of the continuum, an atomization that Oresme always *explicitly* refuses, but *implicitly practices*.⁷ The goal of this paper is not to propose a general survey of Oresme's conception of the continuity of motion. More limited, its purpose is to present, compare, and comment on surprising texts that show how Oresme's understanding of the continuum was original, profound, and yet very ambivalent.⁸ I shall first show that Oresme's *fluxus* theory, his identification of motion with a continuous motion, is in part due to the necessity of distinguishing real from apparent motion. However, we will then see that the ontology of real motion, identified with an absolutely successive being, retains some strong analogies with apparent motion, in particular its atomization. Finally, I will show how Oresme drew very paradoxical mathematical consequences of this kind of atomization, which allowed him to define a very fine-grained idea of an infinitely small increase.

⁴ Aristotle, *Physics*, III.1, 200b18-20.

⁵ Nicole Oresme, *Questiones super physicam. Books I-VII*, edited by S. Caroti, J. Celeyrette, S. Kirschner and E. Mazet (Leiden: Brill, 2013).

⁶ Hubert L. L. Busard, *Nicole Oresme, Questiones super geometriam Euclidis* (Stuttgart: F. Steiner, 2010).

⁷ This is the reason why Oresme's mathematics is relevant for the history of the calculus. See Carl B. Boyer, *The history of the calculus and its conceptual development* (New York: Dover Publications Inc, 1959), 61-95 (1st ed 1939); Adolf P. Youshkevitch, *Geschichte der Mathematik im Mittelalter* (Leipzig: Teubner, 1963); Charles Henry Edwards Jr., *The historical development of the calculus* (New York: Springer Verlag, 1979). One medieval studies following archimedean methods, see Marshall Clagett, *Archimedes in the Middle Ages*, 5 vols. (Madison and Philadelphia: University of Wisconsin Press and American Philosophical Society, 1964-1984).

⁸ Oresme's deepness in mathematics is well established. His skill is mainly known for his doctrine of configurations and his theory of ratios of ratios. For a general survey, one can see on the first subject, see the bibliography in footnote 3. On the ratio of ratio, see Nicole Oresme, *De proportionibus proportionum and Ad pauca respicientes*, edited by E. Grant (Madison: University of Wisconsin Press, 1966); Sabine Rommevaux, *Les nouvelles théories des rapports mathématiques du XIVe au XVIe siècle* (Turnhout: Brepols, 2014).

1. Motion and continuity

1.1 Mathematical and ontological *fluxus*

Motion is a recurrent matter of concern in many of Oresme's works.⁹ Oresme contributed to the development of a *mathematical* science of motion by defining a geometrical model for the study of the variation of motion, be it local or qualitative.¹⁰ Variations of motion were only a special instance of the general theory of the latitude of forms, a science Oresme deepened in his *Questiones super geometriam euclidis* and synthetized in his *Tractatus de configurationibus qualitatum et motuum*. There, motion is mathematically assimilated to a kind of intensive quality, affecting a mobile whose specific intensity and velocity can vary according to space and time. The mathematical analysis of heating, for example, would distinguish two kinds of intensity at work: first, the intensity of heat, second, the intensity of "heating", of the motion itself, that is, the velocity of heating. Motion is called a "*fluxus*" more than one time,¹¹ and by such a *fluxus*, Oresme can indeed refer to concrete motions, or to mathematical and imaginary motions, like the *fluxus* of a line above another one, used to symbolize the simultaneous intensification of a whole subject.¹² Although never defined in Oresme's mathematical works, this notion of a *fluxus* is, on the contrary, the main concept of his ontological studies.

Indeed, Oresme also sought to understand more adequately the *essence* of motion, from a *gnoseological* and *ontological* point of view. This problem is mainly studied in his *Questiones super Physicam*, particularly in questions III.1 to 7.¹³ In the first extended study of Oresme's commentary, III.1-7, Stefano Caroti acknowledged the originality of Oresme's position concerning the essence of motion, and the need for more analysis.¹⁴

In the background of this discussion, there is, in the Latin West, the classical distinction, due to Albert the Great, between two notions of motion: motion as a *forma*

⁹ Stefan Kirschner, "Oresme's Theory of Motion", in *Nicole Oresme philosophe: Philosophie de la nature et philosophie de la connaissance à Paris au XIVe siècle*, *Studia Artistarum* 39, edited by J. Celeyrette and Ch. Grellard, (Turnhout: Brepols Publishers, 2014), 83-104; Johannes M. M. H. Thijssen, "The Debate Over the Nature of Motion: John Buridan, Nicole Oresme and Albert of Saxony. With an Edition of John Buridan's *Quaestiones Super Libros Physicorum, Secundum Ultimam Lecturam*, Book III, Q. 7", in *Evidence and Interpretation in Studies on Early Science and Medicine*, edited by E. Sylla and W. R. Newman, 14, 1-3 (2009): 186-210; Stefano Caroti, "La position de Nicole Oresme sur la nature du mouvement (*Questiones super Physicam* III, 1-8): problèmes gnoséologiques, ontologiques et sémantiques", *Archives d'histoire doctrinale et littéraire du Moyen Age* 61 (1994): 303-385; Stefano Caroti, "Oresme on Motion (*Questiones super Physicam* III, 2-7)", *Vivarium* 31, 1 (1993): 8-36.

¹⁰ For a more extended presentation, see Debroise, *Mathématiques de l'intensité et Merveilles de la nature*.

¹¹ See for example Nicole Oresme, *De configurationibus*, II.4, 276.

¹² See for example Nicole Oresme, *De configurationibus*, II.4, 394.

¹³ Nicole Oresme, *Questiones super Physicam*, 293-341.

¹⁴ Caroti, "Oresme on Motion".

fluens, or as a *fluxus formae*.¹⁵ It was usual to find a contradiction in Aristotle's work concerning the ontology of motion. From *Physics* III, it could be deduced that motion was not a specific category, but that each kind of motion belonged to the same category as what is gained by the motion, the *res acquisita*, as it appears in Oresme's text. But from the *Categories*, it could be deduced that motion belonged in general to the category of *passio* or affection. In his commentary, Averroès explained this contradiction by distinguishing two different approaches to motion: motion could be studied from a teleological point of view, according to the *terminus ad quem*, toward which the mobile goes (a site, a quality, a size), or from a processional point of view, according to the enduring process itself. In this second case only, motion belonged to a specific category of motion, distinct from the category of the reality being acquired. Those two aspects of motion became two different theories about its nature, the teleological one of motion as *forma fluens*, and the processional one of motion as *fluxus formae*.

As it is well known since Stefano Caroti's studies, Oresme's main conclusion is that motion is a *fluxus*, as it was for his Parisian contemporaries Buridan and Albert of Saxony. However, Oresme's conception of velocities strongly departs, though implicitly, from Buridan's position by adding that this *fluxus* must not be understood as an entity *added* to the mobile and *inherent* to it (a *res superaddita*): such a being would be essentially contradictory, just as *to be* is in contradiction with *to become*.¹⁶ But in fact, even God cannot create such a contradiction: this *fluxus* is a way of being of the mobile, a *modus seu condicio* of the mobile.¹⁷ Moreover, Oresme generalizes the *fluxus* theory to all kinds of motion, whether local or qualitative, when Buridan limits this idea to local motion. Therefore, one can say that in Oresme's works, the *fluxus* theory has an unprecedented extent.

One can guess that mathematical *fluxus* and ontological *fluxus* are related one way or another. Obviously, the geometrical figuration of motion, as distinct to the mobile, is a mathematical expression of the ontological reality of motion as such: motion can be easily represented only because motion is something in its own right, distinct from

¹⁵ Anneliese Maier, "Die scholastische Wesensbestimmung der Bewegung als *forma fluens* oder *fluxus formae* und ihre Beziehung zu Albertus Magnus", *Angelicum* 21 (1944): 97-111. For the Arabic origin of this distinction, see John McGinnis, "A Medieval Arabic Analysis of Motion at an Instant: The Avicennian Sources to the *forma fluens/fluxus formae* Debate", *The British Journal for the History of Science* 39, 2 (2006): 189-205.

¹⁶ "Nec est fluxus distinctus, quia tale haberet unam partem preteritam et aliam futuram, et ita non esset, nec esset subiectum in quo sue partes essent (...)", Nicole Oresme, *Questiones super Physicam*, 312.

¹⁷ On the *condicio* theory, see: Stefano Caroti, "Nicole Oresme and Modi Rerum", *Noctua* 1, 1 (2014): 1-27. See also Stefan Kirschner, "Oresme on Intension and Remission of Qualities in His Commentary on Aristotle's 'Physics'", *Vivarium* 38, 2 (2000): 255-274; Stefano Caroti, "Modi rerum and materialism: a note on a quotation of a condemned articulus in some fourteenth-century Parisian *De anima* commentaries", *Traditio*, 55 (2000): 211-234; Stefan Kirschner, "A Possible Trace of Oresme's *Conditio*-Theory of Accidents in an Anonymous Commentary on Aristotle's *Meteorology*". *Vivarium*, 48, 3 (2010): 349-367.

the mobile and *res aquisita*, even if this reality is neither a substance added, nor an accident, but a new kind of ontological being, a *condicio* of the mobile. The configuration theory is a mathematical counterpart of Oresme's ontological stance: being symbolized by geometrical figures, uniform or difform motions become objects *per se* of a new mathematical science.¹⁸

This identification of motion to a *fluxus* in both mathematical and ontological contexts reflects Oresme's effort to mathematize *continuous* processes.

1.2 Discrete and continuous mathematics

It has been a matter of debate, and should still be, whether Oresme was a pure continuist, or whether he admitted, one way or another, the idea of an atomization of the continuum.

In Aristotle's *Physics*, the continuity of motion plays a key role.¹⁹ As Barbara M. Sattler recalls, Aristotle gives two different definitions of continuity: first, two things are continuous whose limit at which they touch each other is one; secondly, one thing is continuous if it is divisible into ever-divisible parts.²⁰ However, continuity of motion is harder to understand than the continuity of a magnitude. A magnitude can be infinitely divided in infinitely divisible parts because it is a whole whose parts are all simultaneously given. Motion, on the contrary, is an ongoing process: to walk in a park has a beginning, and this motion is fulfilled only when it has come to an end. Thus, it cannot be conceived as a whole to be divided before it is accomplished, and when it is accomplished, the motion is already past: it is never a given whole. This is why Zeno could deny the very possibility of motion: if motion is to be understood as a continuous whole, it has to be on the one hand infinitely divisible, but on the other, generated one part after the other. Consequently, a mobile seems to span an *infinite* number of places in a *finite* time. For this reason, Aristotle's study of the continuity of motion analyzes

¹⁸ However, Oresme's contribution to the science of latitudes of forms should not be limited to this geometrical symbolisation. Daniel A. Di Liscia has shown how Oresme could be ingenious even in the *calculatores* or rhetoric style of mathematics. See Daniel A. Di Liscia, "La conclusio pulchra, mirabilis et bona: una ingeniosa demostración atribuible a Nicole Oresme", *Mediaevalia. Textos e estudos* 37 (2018): 139-168. This important contribution should be added to any reflection on Oresme's mathematical style (See George Molland, "The Oresmian Style: Semi-Mathematical but Also Semi-Holistic in Oresme I", *Cahiers du Séminaire d'Epistémologie et d'Histoire des Sciences-Université de Nice* 18 (1985): 7-12; Edmond Mazet, "Richard Swineshead et Nicole Oresme: deux styles mathématiques", in *Nicole Oresme philosophe: philosophie de la nature et philosophie de la connaissance à Paris au XIVe siècle*, edited by J. Celeyrette and Ch. Grellard (Turnhout: Brepols, 2014), 105-137.

¹⁹ Barbara M. Sattler, *The Concept of Motion in Ancient Greek Thought, Foundations in Logic, Method and Mathematics* (Cambridge: Cambridge University Press, 2020).

²⁰ Sattler, *The Concept of Motion*, 295. For the two definitions, see Aristotle, *Physics*, III.1, 200b18-20 and VI.1, 231a22.

the way the infinite number of parts of space are related with the infinite number of parts of time in one continuous motion.²¹

Obviously, Oresme uses geometrical and static continuity to mathematize the continuity of motion: to spread this processional being on a surface, to symbolize a successive or changing being by a permanent being is the basic idea of the second part of his *De configurationibus*. However, in a series of papers,²² Stillman Drake argued that the medieval approach to motion, contrary to a Galilean one, required an atomization of it in discrete successive parts. He suggested, in particular, that philosophers of what is sometime called the “Parisian school” defending the impetus theory, Buridan or Albert of Saxony, were also defending a “quantum theory of free fall, with a succession of extremely short but increasing uniform speeds succeeding one another contiguously.”²³ The case of Nicole Oresme is ambivalent: if Drake acknowledged that Oresme, thanks to his configurational doctrine, could think of a mathematically continuous motion, and could therefore be thought of as an “exception”,²⁴ he still included Oresme in his general idea of “medieval writers”. In any case, he suggested that, due to deficiencies in Campanus’s translation of Eudox’s continuous theory of proportionality, medieval mathematicians developed an arithmetical theory of proportion “brought by Oresme to a point almost equivalent to our own arithmetization of the continuum.”²⁵

Oresme’s idea of *intensio velocitatis* could seem to be in favor of such an atomization of motion: this concept would seem to burst motion into an infinite number of successive instantaneous velocities. Of course, Oresme did not define anything like instantaneous velocity, a notion that would require the method of derivation of space through time.²⁶ But thanks to his geometrical theory of motion, he was able to analyze the mathematical behavior of instantaneous change, such as the beginning or the end of a motion, a maximum or a minimum, a continuous acceleration or deceleration. However, Georges Molland was right when objecting that Oresme’s approach to motion

²¹ Sattler, *The Concept of Motion*, 277-334.

²² Stillman Drake, “Impetus Theory and Quanta of Speed before and after Galileo”, *Physics* 16 (1974): 47-65; Stillman Drake, “Impetus Theory Reappraised”, *Journal of the History of Ideas* 36 (1975): 27-46; Stillman Drake, “Free Fall from Albert of Saxony to Honoré Fabri”, *Studies in History and Philosophy of Science* 5 (1975): 347-366; Stillman Drake, “A Further Reappraisal of Impetus Theory”, *Studies in History and Philosophy of Science* 7 (1976): 319-336.

²³ Drake, “Free Fall from Albert of Saxony”, 351.

²⁴ Drake, “Impetus Theory Reappraised”, 38 n. 2.

²⁵ Drake, “Impetus Theory Reappraised”, 41.

²⁶ Pierre Souffrin, “La quantification du mouvement chez les scolastiques. La vitesse instantanée chez Nicole Oresme”, in *Autour de Nicole Oresme, Actes du colloque Oresme organisé à l’Université de Paris XII*, edited by J. Quillet (Paris: Vrin, 1990), 63-83.

is essentially continuous.²⁷ But he was undoubtedly wrong when he added that Oresme's continuity was "essentially Aristotelian".²⁸

As Molland argues, for Oresme, when a body starts to move, as when a heavy body is dropped, it does not suddenly acquire a definite speed, however small: nothing in nature happens suddenly. Indeed, the acceleration of the body should be analyzed like this: for any given degree of speed of the mobile, there was a previous instant when the mobile had a lesser velocity.²⁹ As Oresme notes in his *Livre du Ciel et du Monde*, this is the way the technical formula "to begin *a non gradu*" should be analyzed.³⁰

We could add to Molland's argument that points or instants, for Oresme, are only mathematical commodities. Thus, if a mathematical argument concludes demonstratively the existence of an instantaneous motion, as a mobile being at rest at any instant before an instant T, and at a finite and determinate speed at instant T, then the mathematical argument must be rejected as in contradiction with natural motion.³¹ Thus, it is clear that for Oresme, motion is a continuous being. But what kind of continuity is it? How does Oresme understand this continuity of motion? We shall progressively see that Oresme's continuity is clearly not that of Aristotle's.³²

²⁷ Georges A. Molland, "The Atomisation of Motion: A Facet of the Scientific Revolution", *Studies in History and Philosophy of Science* 13, 1 (1982): 31-54.

²⁸ "Oresme's view of continuity was essentially Aristotelian", Molland, "The Atomisation of Motion", 41.

²⁹ See for instance: "Et pour entendre les causes de ces choses, je di premièrement que tout mouvement de chose pesante ou legiere, quelcunque il soit, commence en enforçant telement que quelcunque degré de ysneleté donney ou signey en lui, il convient que il eust devant mendre ysneleté et mendre et mendre outre toute proporcion ; et est ce que l'en seult appeller commencer *a non gradu*", Nicole Oresme, *Le Livre du ciel et du monde*, edited by A. D. Menut (Madison: University of Wisconsin Press, 1968), 414.

³⁰ Nicole Oresme, *Le Livre du ciel et du monde*. And of course, the same thing can be said for the opposite, "to end *ad non gradum*". This is why "*non gradum*" cannot simply be translated by "zero", and why Oresme, even in his French work, kept the Latin formula without translation.

³¹ See for example: "Also, since the former case of alteration of subject AB does not seem to be naturally impossible and yet it is naturally impossible for something to become suddenly hot in a maximum degree after being very cold in a maximum degree (and similarly for other cases), so an argument can be made for proving that a point is not something really indivisible, nor is a line or a surface something, although the imagination of these [entities] is convenient for better understanding the measures of things, as was noted in the first chapter of the first part", Nicole Oresme, *De Configurationibus*, 403.

³² I don't mean to revive Drake's opinion. Drake's idea was based on the comparison between a "medieval approach" and a Galilean one. I am only concerned with Oresme's understanding of the continuity of motion.

1.3 Gnoseology and visual illusions

Oresme's understanding of the continuity of motion stems from his analysis of the perception of motion, which is to be found in Question 1 of the third book of his questions on Aristotle's *Physics*.³³ Indeed, an originality of Oresme's ontological study consists of starting from the standpoint of the perception of motion, in the perspectivist tradition. So his main goal, in question 1, is to prove that we don't have a direct vision of a motion: we only see successive states or relations that, by comparison, our inner sense or *virtus distinctiva* will use to judge what is really moving and what is not.

This is why he starts from the description of motion proposed by Witelo in his *Perspectiva*: "*moveri est aliter se habere nunc quam prius*."³⁴ In his commented translation in French of the *De Caelo*, Oresme keeps approximately the same description: "l'en ne apparçoit mouvement fors telement comme l'en apparçoit .i. corps soy avoir autrement ou resgart d'un autre."³⁵ But, as this last quote makes clear, this definition only applies to *apparent* motion: it does not allow to distinguish between apparent motion and real motion. In particular, if two mobiles are in a relative motion, one with the other, it isn't enough to decide which one is actually in motion. This is the main reason why, in question III.7, where Oresme definitely settles the ontological question of what a motion is, he also gives a new definition – or "description" as he says – of motion in terms of an internal reference mark: to be absolutely in motion, a mobile doesn't need the existence of another body relative to which it finds itself in a new state or position: it only has to be different from what it was previously.³⁶ But in fact, another understanding of motion is already at work in this first question. In a corollary, he specifies that, if "to move", for a body, means "to behave differently than before", "it means, moreover, a mutation of it (*significat ultra permutationem ipsius*)."³⁷ This addition expresses, in fact, what we mean by a *real* motion, as distinct from an apparent motion.

Absoluteness is only one aspect of this second definition. The other main aspect is *continuity*. Thus, when Oresme defines motion in the beginning of the ontological questions, in question III.3, he writes: "motion is a connotative name that is used for the sake of brevity in the place of a proposition, like this one or one similar: 'the mobile behaves continually differently than before, relative to something immobile'."³⁸ As we

³³ Nicole Oresme, *Questiones super Physicam*, 293-303.

³⁴ Nicole Oresme, *Questiones super Physicam*, 296. Same formula in Witelo, *Perspectiva*, IV, 110, *Opticae Thesaurus*, edited by F. Risner (Basel: Per Episcopios, 1572), 167.

³⁵ Nicole Oresme, *Le Livre du ciel et du monde*, 522.

³⁶ "Quinta est descriptio melior et vera <est> quod moveri est aliter se habere continue quam ipsum mobile prius se habebat respectu sui et non respectu cuiuscumque extrinseci", Nicole Oresme, *Questiones super Physicam*, 337. See also below.

³⁷ Nicole Oresme, *Questiones super Physicam*, 296.

³⁸ "'Motus' est nomen connotativum et quasi propter breviliquium ponitur loco unius orationis, sicut illius vel consimilis: 'mobile se habet continue aliter quam prius respectu cuiuslibet non moti';

see, this definition is still relative, but it adds the important specification of continuity. The reason for this is that the continuity of a motion can be illusory, as a better understanding of the perception of motion shows.

Jean Celeyrette has already described the gnoseological process that Oresme supported.³⁹ Oresme's main idea is that one does not *see* motion itself, as one immediately sees the color of a wall. First of all, to "see" a motion requires a capacity to compare the mutual states of two objects, one at rest and the other in motion, and two different periods, the present and the past. This comparative capacity goes beyond the external senses and needs to be fulfilled by the activity of an inner sense, the *virtus distinctiva*. However, because this comparison is not enough to judge which of the objects is actually moving, another operation is needed, which Oresme calls a *discursum*⁴⁰ of the intellect: a logical deduction which concludes from actual knowledge which body is actually moving. Thus, one only "perceives that things are not related as before (*solum sentitur aliter se habere quam prius*)":⁴¹ real motion as a *permutatio* of the mobile is invisible.

Oresme is conscious that his analysis of the perception of motion goes beyond common opinion: don't I see someone running in front of me?⁴² But in fact, what we see is that he is now in a place different from where he was earlier. Thus, we saw he has moved in a very near past, but cannot see him moving in the present: this would require us to see that the mobile will be in another place *later*. Otherwise, it could be presently at rest. To know that something is moving now, at the very moment of the perception itself, would require the ability to know the future: the continuity of motion links the past to the future.⁴³ Thus, the impossibility to see the future makes it impossible to see with obviousness that something is actually moving.

From this gnoseology, basic deceptions can be described. Deceptions due to relational illusion are simple: if something is moving relative to another, there is no certainty as to which one is moving, and which one is at rest. More interesting here are deceptions due to sequential illusions. For example, Oresme says, "it is possible that something be divided in a thousand instants, each imperceptible, and move in one, then rest in another alternately. From this, it follows moreover that, by imagination, it is

et hoc vel secundum qualitatem vel secundum locum, et sic de aliis", Nicole Oresme, *Questiones super Physicam*, 313.

³⁹ Jean Celeyrette, "Apparences et imaginations chez Nicole Oresme: Question III. 1 sur la Physique et question sur l'apparence d'une chose", *Revue d'histoire des sciences* 60, 1 (2007): 83-100.

⁴⁰ Nicole Oresme, *Questiones super Physicam*, 296.

⁴¹ Nicole Oresme, *Questiones super Physicam*, 300.

⁴² "Dubitatur contra illam conclusionem, quia experientia est quod ego video Sor currere, et hoc est moveri (...)", Nicole Oresme, *Questiones super Physicam*, 299.

⁴³ "Ad primum dico quod non video evidenter etc., sed quia tempus propinquum instanti presenti, scilicet preteritum, iudicatur quasi presens, ideo dicitur quasi esset de presenti quod video <Sor> moveri, licet non videam <nisi> quod immediate <ante> vel statim movebatur", Nicole Oresme, *Questiones super Physicam*, 300.

possible that a well-disposed vision judges something moving that never moves: if at each instant, something is instantaneously moved, the period between each being imperceptible.”⁴⁴

Therefore, there are two different cases described here. In the first one, a period is divided into a great number of imperceptible intervals, and the mobile alternatively moves and stops. In the second one, the mobile instantaneously jumps from one position to another, while the periods during which it is at rest are imperceptible. In the second case, vision will judge the mobile to be in motion, while it is not, and one can guess the same about the first case. Both rely on the important idea that vision requires time: there is no instantaneous vision. Thus, there is a *minimum sensibile* such that a discrete succession of positions or states can be mistaken for a continuous motion.

He goes back to a similar point a few lines later: “Third, it is obvious (...) that continuity, without which there is no motion, is not experimented. For this reason, by imagination or power of God, if something were instantaneously moved in instants – the time between those instants being imperceptible – and if it were not moving during those [imperceptible] times, the thing would not seem other than if the mobile were continuously moving. Thus, continuity is not known by experiment.”⁴⁵ Jean Celeyrette, who commented on those cases in a different perspective, mentions similar arguments in another work attributed to Oresme, the *De apparentia dei*.⁴⁶

Those examples are striking. They are similar but not identical to the most quoted illusion of the rotating spinning top described by Boethius in his *Institutio musicae*, and some other usual illusions of the same kind.⁴⁷ In those illusions, a moving point creates the illusion that it occupies continuously a static continuous line, a circle for example. But here, Oresme describes cartoon-like illusions, where the observer believes he sees a continuous motion, while in reality, there is only a very quick succession of discrete

⁴⁴ “Sequitur corollarie quod possibile est quod aliquid dividatur per mille instantia, quorum quodlibet sit imperceptibile, et moveatur in uno et quiescat in alio alternatim; ex quo etiam sequitur ultra quod per ymaginationem possibile est quod visus bene dispositus iudicet aliquid moveri quod numquam movetur, ut si per instantia aliquid subito transferretur, inter que esset tempus insensibile”, Nicole Oresme, *Questiones super Physicam*, 297. This text is also commented by Jean Celeyrette in Celeyrette, “Apparences et imaginations chez Nicole Oresme”.

⁴⁵ “Tertio, patet ex corollario quarte supposito>nis quod continuatio, sine qua non est motus, non experitur. Unde per ymaginationem aut per potentiam Dei, si aliquid transferretur subito per instantia inter que esset tempus insensibile, et non per tempus moveretur, non appareret aliter quam si continue moveretur, igitur continuatio non patet experiment”, Nicole Oresme, *Questiones super Physicam*, 300.

⁴⁶ Celeyrette, “Apparences et imaginations chez Nicole Oresme”.

⁴⁷ Boethius, *De institutione arithmetica libri duo, De institutione musica libri quinque. Accedit Geometria quae fertur Boetii* I, 3, edited by G. Friedlein (Leipzig: Teubner, 1867), 190, lines 11-21. Oresme refers to this deception in relation to the apparent continuity of a sound. See Nicole Oresme, *De Configurationibus*, 305.

phases of a motion. This really is the basic idea behind the phenakistiscope invented by Joseph Plateau in 1832, the illusion being due to the retinal persistence.

Thus, the gnoseological analysis of motion implies a strong distinction between apparent motion, and real motion. The first simply means that something behaves differently than before relatively to something else, but real motion goes beyond: it means a real *permutatio* that affects the mobile. But this means that motion is a kind of reality distinct to the mobile and to the thing acquired. The purpose of the ontological study that follows immediately is to establish the ontological necessity of the supposition of such a *permutatio*.

1.4 Ontology

Indeed, the first ontological problem raised about motion concerns its continuity. In the second question, III.2, where Oresme asks whether motion is something or not, the first argument *quod non* notes is that motion as a whole has two parts: a past one, which is not anymore, and a future one which is not yet.⁴⁸ One solution would be: to answer that motion is a successive being, and not a permanent being. It doesn't exist *tota simul*.⁴⁹ However, such an explanation is not enough, because even a successive being requires an *existing* part. Thus, he suggests to define a "present part (*pars presens*)", "composed of something past and something future."⁵⁰ This solution is not absolutely satisfactory, and Oresme will refine this answer as we will see later.⁵¹ Anyway, it highlights the main problem: if motion is to be mathematized, it has to be understood as a whole composed of parts. How are those relations to be understood, since those parts cannot be simultaneous?

Although Oresme is a supporter of the *fluxus* theory, he first denies that motion is a *fluxus*. Indeed, there is only three possible opinions: first, motion could be the mobile, second, motion could be the thing acquired, and third, motion could be a *fluxus*.⁵² But if he calls defenders of the *fluxus* theory his "adversaries",⁵³ it is only because they

⁴⁸ "Et arguitur primo quod non, quia pars preterita motus non est nec pars futura, ergo motus non est; consequentia tenet, quia totum non est aliud quam sue partes", Nicole Oresme, *Questiones super Physicam*, 304.

⁴⁹ On the distinction between *res permanentes* and *res successiva*, see the second part below.

⁵⁰ "Dico quod motus habet aliam partem quam medietatem preteritam et medietatem futuram, scilicet partem presentem, que componitur ex aliquo preterito et aliquo futuro", Nicole Oresme, *Questiones super Physicam*, 308.

⁵¹ Caroti, "Oresme on Motion", 17.

⁵² Nicole Oresme, *Questiones super Physicam*, 312. Oresme identifies five "rational opinions" about motion, the two first of which are quickly dismissed. See Nicole Oresme, *Questiones super Physicam*, 305.

⁵³ Nicole Oresme, *Questiones super Physicam*, 313.

understand this *fluxus* as “*distinctus*”,⁵⁴ a “*res superaddita*”,⁵⁵ a being *added* to the mobile. Such an idea implies an ontological inflation. For example, if water becomes hot, it also becomes becoming-hot: it becomes in motion. Thus, it also becomes becoming-becoming-hot, and this one single situation of heating would imply an infinite number of beings.⁵⁶

Why, then, does Oresme finally support the *fluxus* theory? His arguments to show that motion is a *fluxus* or, as he calls it, a “successive being absolutely distinct from permanent beings (*res successiva distincta simpliciter a permanentibus*)”,⁵⁷ are primarily logical and concern the truth-makers of propositions involving a reference to motion: it is logically necessary to suppose such a being, in addition to the mobile and space spanned (in the case of local motion) to establish the truth of the proposition: “This is moving (*hoc movetur*).”⁵⁸ If motion is to be real, “*in re*”, and not only the appearance corresponding to the observational fact that things are now related otherwise than they were before, it has to be a *fluxus*. Ontologically, if something is moving, it has to be “*aliter et aliter*”, successively one thing and another, and this very mode of being is precisely the ontological aspect of the general situation of a moving body that Oresme calls its *fluxus*: even God cannot create a motion without this additional being-other-and-other – this way of being that is characteristic to motion.⁵⁹

Now, this *fluxus* theory is not to be confused, as I said earlier, with Buridan’s theory: the *fluxus* is not a *being* added to the mobile, but a mode of being of the mobile itself. This idea is definitely established as the most probable in question III.7, where Oresme exposes his own opinion. To do so, he needs to fix, once again, the definition of motion: none of the previously given definitions, neither the one inspired by Witelon (III.1) nor the one specifying the continuity of motion (III.3) were suitable enough to describe real motion as absolute. For this reason, Oresme introduces the new kind of definition mentioned above, involving what Stefano Caroti calls an “internal reference mark:”⁶⁰ “To move is to behave continuously differently from how the mobile itself behaved

⁵⁴ Nicole Oresme, *Questiones super Physicam*, 331.

⁵⁵ Nicole Oresme, *Questiones super Physicam*, 312.

⁵⁶ “Contra: sit *a* mobile, et *b* sit ille *fluxus*; tunc sic: prius est verum quod *b* non est in *a* et postea quod *b* est in *a*, ergo *a* est mutatum ad ipsum *b*, ergo per suppositionem secundam hoc est per mutationem distinctam a subiecto et termino, quia propter aliud non ponitur <alis *fluxus*>, ergo motus erit motus, et sic proceditur in infinitum, quod est contra Aristotelem septimo *huius*”, Nicole Oresme, *Questiones super Physicam*, 313.

⁵⁷ Nicole Oresme, *Questiones super Physicam*, 334.

⁵⁸ “Et probatur, <quia>, quando due res non sufficiunt ad hoc quod aliqua propositio sit vera, oportet ponere aliam rem vel saltem alium modum rei; patet statim, quia, si sufficiebat ante, iam fuisset vera, sed posito mobili et spatio non sufficit ad hoc quod hec sit vera: ‘hoc movetur’, ergo, quando fuit vera, aliud ponitur”, Nicole Oresme, *Questiones super Physicam*, 334.

⁵⁹ “Sicut Deus non potest facere quod aqua calefieret successive quin haberet se aliter et aliter, ita nec potest tollere illum modum se habendi in casu posito”, Nicole Oresme, *Questiones super Physicam*, 335.

⁶⁰ Caroti, “Oresme on Motion”, 28.

before relative to itself and not to anything extrinsic (*moveri est aliter se habere continue quam ipsum mobile prius se habebat respectu sui et non respectu cuiuscumque extrinseci*).⁶¹ Then, he is able to conclude that motion is indeed a *fluxus*: “motion is some mutation distinct from permanent beings, a mutation that is successive, supposing ‘successive’ as before (*motus est quedam mutatio preter res permanentes, que est successiva, exponendo ‘successivum’ sicut prius*).”⁶² Both points, the new description and the *fluxus* theory, are based on the same kind of argument, the one-body argument which can also be found in Buridan’s studies on motion.⁶³

Indeed, if we suppose only one body in the world, that is the world itself. It could happen that this body would be rotating around its own axis. Thus, the meaning of this “motion” cannot be a varying relation of the mobile with something else: “motion” must mean an *internal* change or mutation. The motive of this distinction is obviously to distinguish real motion, an internal mutation, from apparent motion, a varying relation. But one must also observe that this new description makes the *continuity* an essential aspect of what we call a real motion.

The semantical question left aside, the one-body argument is also required to establish ontologically that motion is a *fluxus*: let’s now suppose this body to be in motion for one hour, then at rest for the next hour, and again indefinitely moving and resting successively. There is no thing nor “place” to which it could be compared to define this motion. In the same way, all its parts are always in the same relation to one another. Thus, the body has two different behaviors: motion and rest, but nothing extrinsic relative to which this difference could be defined. Therefore, motion cannot be anything else than an internal “*condicio*”, a way of being, a mutation which affects the body when it is moving, and not when it is at rest.⁶⁴

The *condicio* theory of motion has another consequence. Oresme doesn’t absolutely reject the identification of motion with a mobile. Of course, motion is not the mobile of which it is a way of being. But there is another sense in which motion is, in a way, a mobile: motion can be itself in motion, as it is in the case of acceleration and deceleration. This is the meaning of the argument *quod sic* of question III.3: “some motion behaves continually differently than before due to its own mutation. Thus, this motion is moved, and consequently it is something moved or a mobile.”⁶⁵ Indeed,

⁶¹ Nicole Oresme, *Questiones super Physicam*, 337.

⁶² Nicole Oresme, *Questiones super Physicam*, 338. For the meaning of “successive” here, see below.

⁶³ Thijssen, “The Debate Over the Nature of Motion”.

⁶⁴ “Hiis positis arguendo ad conclusionem, ymaginetur in mundo unum corpus solum et sit *a*, et moveatur in una hora, et in alia quiescat, et sic alternatim; tunc *a* movetur in prima hora et non in secunda, et postea in tertia, nec partes eius nec ipsum ad aliud se habet aliter quam prius, ergo in se ipso habet aliquam condicionem, que non erat ante; et hoc vocatur ‘motus’, et quando non habet quiescit. Patet statim per suppositiones”, Nicole Oresme, *Questiones super Physicam*, 338.

⁶⁵ “Aliquis motus continue se habet aliter quam prius per sui mutationem, igitur ille motus movetur, et per consequens est res mota seu mobile”, Nicole Oresme, *Questiones super Physicam*, 311.

acceleration is a variation of the motion, a motion of the motion or, as he calls it in his *De configurationibus*, a kind of “succession in motion (*successio in motu*).”⁶⁶ Another case of “succession in motion”, extensively studied in this last treatise, is the motion of the “beginning (*incipit*)” of the motion, a notion tantamount to the variation of the derivative of velocity with respect to space $\frac{dv}{dx}$.⁶⁷ Because motion is not a being, the ontological inflation I mentioned earlier is not to be feared anymore: acceleration could be in turn in motion, and so on indefinitely. Once again, this *condicio* theory legitimizes mathematical techniques: in fact, his mathematical doctrine typically authorizes an inflation of graphs or geometrical figures. But one should not believe naïvely that there is a being corresponding to each graph. Thus, Oresme does not reject absolutely this identification of motion to a mobile: he only rejects it in the ontological sense used in the question asked. Indeed, for the motion to be in motion in this sense, it would have to be, according to the definition of the motion, different from what it was: it would have to retain the *same being*. But on the contrary, an accelerated motion is a successive being never identical to itself, “because one part would be faster and the other slower.”⁶⁸ Motion cannot be a “mobile”, something that retains a permanent being while moving and changing.

Thus, Oresme’s ontological study aims at separating apparent and real motion. *Continuity* is an essential property of real motion as distinct to apparent motion. An infinite succession of states can look like a continuous motion if *leaps* are imperceivable. And the leaps are imperceivable if the time between two different states is imperceivable. This *apparent* continuity based on the imperceptibility of leaps should be different from *real* continuity, supposedly a process without any leaps. But as we are now going to see, this is not absolutely the case: the “continuity” that characterizes real motion is absolutely not what we would expect.

2. The problem of unity and multiplicity of a *res successiva*

To be a continuous *fluxus*, for Oresme, is the same as to be a *res successiva*, a successive being.⁶⁹ Although he uses the expression frequently, as opposed to *subita* and to *permanens*, he doesn’t give any precise definition of it before question III.6. Thus, when Oresme faces the real continuity of motion, he has to tackle the classic distinction between *permanent* beings and *successive* beings. A permanent being is a whole whose parts all exist simultaneously. A successive being is a whole whose parts exist one after the other, like a word: it never exists as a whole, but only one syllable after the other.

⁶⁶ Nicole Oresme, *De configurationibus*, II.V, 280.

⁶⁷ Nicole Oresme, *De configurationibus*, II.V, 280. See also Debroise, *Mathématiques de l’intensité et Merveilles de la nature*, 515.

⁶⁸ Nicole Oresme, *Questiones super Physicam*, 318.

⁶⁹ See the title of question III.6: “Utrum motus sit res successiva sive fluxus distinctus a rebus permanentibus”, Nicole Oresme, *Questiones super Physicam*, 331.

A successive being is an ongoing whole. Originally meant to grasp the contrast between the instability of the creature and the stability of the Creator, this pair of concepts had a fixed meaning by the end of the 13th century. However, Robert Pasnau noted the originality of Oresme's understanding of those concepts. In particular, for a being to be permanent, Oresme required not only the simultaneity of existence of all the parts, but also that this existence last for a time, excluding any instantaneous being.⁷⁰

Oresme not only asserts that motion is a successive being; he also wants to show that it is an *absolutely* successive being. For Aristotle, any change requires something unchanged, a *substratum*. This is the paradox of change he insists on in *Physics*: "What comes to be must do so either from Being or from non-Being, and both are impossible. For Being cannot come to being, since it already is, and nothing can come to be from non-Being, since something must be underlying."⁷¹ On the contrary, Oresme is looking for a highly paradoxical concept: an *absolutely* successive being, a succession without any permanent part or counterpart. I shall insist here on the mathematical implications of such an idea, and on its paradoxical nature acknowledged by Oresme himself.

As we saw, continuity is essential to motion, Oresme insists.⁷² But continuity of motion is in fact the same thing as its unity.⁷³ Or should we add, the mixture of alterity and unity. Thus, a motion can be more or less one: "a regular motion is more one than an irregular one."⁷⁴ An irregular motion is a motion whose velocity varies. Although Oresme follows Averroès on this remark, he insists: "However, I say that variation in velocity doesn't destroy the unity of motion",⁷⁵ precisely because it doesn't destroy its continuity. Thus, the reality of motion as a successive being introduces an important metaphysical problem: how can a successive being keep its unity? Doesn't its successive multiplicity destroy its unity? Oresme tackles this question in an overwhelming ontological chapter, the only one of the sort, of his *De configurationibus qualitatum et motuum*. A treatise, one should recall, structured by the distinction between permanent beings (part I) and the successive beings (part II), of which motion is the main example, but not the only one.

Here, Oresme notes that "certain things are so successive that they cannot last in any way, such as time and motion."⁷⁶ Anyway, this *absolute* successivity of motion does

⁷⁰ Robert Pasnau, *Metaphysical themes: 1274-1671* (Oxford: Clarendon Press, 2011), 378-379.

⁷¹ Aristotle, *Physics*, 191a28-31. See Sattler, *The Concept of Motion*, 281.

⁷² "(...) continuitas est intrinseca motui, ut dicitur tertio huius, quia apparet ex descriptione motus", Nicole Oresme, *Questiones super Physicam*, V.10, 636.

⁷³ "Tertio, pono illam descriptionem quod motum esse unum vel aliquid moveri uno motu non est nisi aliquid moveri continue", Nicole Oresme, *Questiones super Physicam*, V.10, 633.

⁷⁴ "Sexta conclusio est quod motus regularis magis est unus quam motus irregularis", Nicole Oresme, *Questiones super Physicam*, V.10, 636.

⁷⁵ "Dico tamen incidenter quod diversitas in velocitate non tollit unitatem motus", Nicole Oresme, *Questiones super Physicam*, V.10, 636.

⁷⁶ "Rerum quedam sunt ita successive quod non possunt aliquo modo permanere, sicut tempus et motus", Nicole Oresme, *De Configurationibus*, 298. Clagett's translation slightly modified.

not prevent this motion from keeping its unity. In particular, the intension and remission of velocity never destroys unity, as we can guess from Oresme's examples of the unity of varying curvature or rarity: "For just as in the intensity of curvature or rarity, there is continually different curvature or different rarity while in the whole time it consists of one successive curvature or rarity, and similarly in the cases of augmenting a ratio or a dissimilarity, so I imagine it to be in the case of the intension of any intensible quality such as heat or whiteness, and similarly for the case of the remission of the same quality."⁷⁷ Thus, the *identity* of the whole is nothing else than its *continuity*. Still, the case of motion is more complicated than that of those intensive qualities, because it can *in no way* be a *permanent* being.

We thus see that it was a major concern for Oresme to understand how motion could keep its unity while being "made of" multiplicity: this is the issue of the nature of absolute succession, an issue Oresme addresses more precisely in two major texts: *Quaestiones super de generatione et corruptione*, I.13 ; *Questiones super Physicam*, III.6. In both, the question is the same: are successive beings distinct from permanent beings, or can they be reduced to such beings? His answer is very similar in both, but overall more precise in the first one.

2.1 *Questiones super Physicam III.6*⁷⁸

The question asked is whether motion is a successive being or a *fluxus* distinct from permanent beings, that is the mobile and the being acquired during the process.⁷⁹

Oresme starts his study on successivity by determining what it means to be "successive", and suggests three different meanings: the first is *improper*, and simply names a permanent being, always equal to itself, but changing location. The second is *less improper*, and names a thing of which one part exists already, and of which another part is being acquired, as is some heat being acquired. In this sense, one could speak of succession of a river as compared to the permanence of the riverbed: it is a succession *secundum quid*, the whole entity being divided between permanent parts which guarantee unity, and other successive parts. None of these two kinds of succession is really problematic.

The difficulty starts with the third kind of succession, the succession *simpliciter* or *absolute* succession: "Third, [succession can be said] for this which never behaves in

⁷⁷ "Sicut enim in intensione curvitatibus vel raritatis est continue alia et alia curvitas vel alia et alia raritas et in toto tempore illo est una curvitas vel raritas successiva et conformiter in augmento proportionis vel dissimilitudinis, ita ymaginor in intensione cuiuscunque qualitatis intensibilis, sicut caliditatis vel albedinis, et similiter in eiusdem qualitatis remissione", Nicole Oresme, *De Configurationibus*, 300.

⁷⁸ Nicole Oresme, *De Configurationibus*, 331-335.

⁷⁹ "Utrum motus sit res successiva sive *fluxus* distinctus a rebus permanentibus, cuiusmodi sunt mobile et res acquisita, ad quam est motus", Nicole Oresme, *Questiones super Physicam*, 331.

such way that what was in the first part is in the second part, but for any given period, in any part of it, there is something of this successive being, and in another part of it, there is a completely other thing.”⁸⁰ Time is an example of such an absolute successive being, and for this reason it is said to be in “*continuous fluxus*”. Indeed, one must distinguish between a locative way to flow (*fluere secundum locum*), as a river does, and an ontological way (*fluere secundum esse*): if something flows ontologically, “during the whole period, it does not have the same *esse*.”⁸¹

The four conclusions are not of equal values. After defining successivity, Oresme wants to prove that there are indeed such things as successive beings. The reality of successive being *secundum quid* is unproblematic. But Oresme insists on the fact that these kinds of beings are those referred to by Plato in the *Timeo*.⁸² The same reference is made in his *Questiones super generationem et corruptionem*, in the same context.⁸³ Implicitly, Oresme is suggesting that when Plato asserted that everything is in a continuous flow of change, he was only thinking of this relative kind of succession: he didn’t know the reality of *absolutely* successive being, a reality asserted in the second conclusion, with *motion* as a first example.⁸⁴ Indeed, the mobile is obviously “continually in one place and another”, and is “continuously in one state and in another.” This is why Plato thought that one cannot say, about a successive being, “this” or “that”: its unity and identity, as Oresme concludes in his questions on the *De generatione*, is only improper.⁸⁵

⁸⁰ “Et tertio pro eo quod in nullo tempore sic se habet quod illud quod fuit in prima parte est in secunda parte, sed quolibet tempore accepto in una parte illius est aliquid tale illius successivi, et <in> alia totaliter aliud; sic ymaginatur de tempore, quia prima pars non est quando secunda est, ideo tale dicitur non permanens, sed in continuo fluxu et transitu. Verbi gratia, illud dicitur fluere secundum locum, quod in aliquo eodem loco proprio non est per tempus; ita dicitur aliquid fluere secundum esse, quod in aliquo toto tempore non habet idem esse; et propter hoc dicitur tempus preterit more fluentis aque. Et permanens per oppositum dicitur, quando est aliquid tempus, et in pluribus eius est idem et totum simul in aliquo instanti et usque ad aliud instans”, Nicole Oresme, *Questiones super Physicam*, 331-332.

⁸¹ See the note above.

⁸² “Et ideo de talibus dicit Plato in *Timeo* quod sunt in continuo fluxu nec expectant demonstrationem, que sit per illa pronomina ‘hoc’ vel ‘illud’, quia continue est aliud et aliud”, Nicole Oresme, *Questiones super Physicam*, 332.

⁸³ “Et similiter intelligitur dictum Platonis in *Timeo* ubi dicit quod propter talem successionem non potest idem bis demonstrari in talibus que continue transmutantur”, Nicole Oresme, *Questiones super de generatione et corruptione*, edited by S. Caroti (München: Bayerische Akademie der Wissenschaften and C. H. Beck, 1996), 116.

⁸⁴ “Secunda conclusio est de successivo simpliciter, quod est aliqua condicio simpliciter successiva. Probatur primo de motu, et est manifestum in motu locali quod mobile continue est in alio et alio loco, et quod continue se habet aliter et aliter; et similiter de tempore”, Nicole Oresme, *Questiones super Physicam*, 332.

⁸⁵ In the case of inanimated beings, Oresme concludes: “non manet idem proprie et simpliciter (...), tamen, si maior pars maneat, potest dici idem (...)”. In the case of animated beings, he concludes: “quod in animalibus in quibus quedam partes fluunt etc. Adhuc magis proprie manet idem totum

The very problematic nature of the kind of succession Oresme has in mind is even more obvious with the other conclusions. Let's examine first the fourth one: "It does not imply any contradiction nor is it absolutely impossible that a substance be absolutely successive."⁸⁶ Of course, Plato had already said in the *Timeo* that substances were always changing, so every man is, indeed, a successive being, with his hairs and nails always growing. But once again, this is not what Oresme has in mind, and what he calls a "successive man (*homo successivus*)"⁸⁷ is something only God could create thanks to His absolute power.⁸⁸ *Natural* men are successive, but not absolutely successive beings.

The argument goes like this: if A, which is double of B, successively decreases, it is not contradictory that God would create one substance or a man who would exist precisely when A will be double, thus for only one instant, and in the same way when A is sesquialtera (in ratio 3:2), and thus continually for the other ratios. And here, maybe Oresme means *rational* ratios. Now the *sum* "composed" of all those instantaneous men would be *one* absolutely successive man, an absolutely successive substance. Nothing of it which would have existed in any part of time would still exist in the future.⁸⁹ What kind of man is this cartoon-like man, created like an apparent motion whose illusion emerges from successive flapping papers, or the rotation of a phenakistiscope? God would be creating a man just like a geometer would draw a line point by point. And indeed, the comparison is Oresme's: an *instantaneous* man would be to the *aggregate*, the single absolute successive man, just "as a point is to a line", or "an instant to time".⁹⁰ When commenting on this passage, Stefano Caroti admitted that it was "difficult to see how this aggregation could be considered a single man, as the text seems to suggest."⁹¹ We cannot but agree, except for the fact that Oresme doesn't mean that this heavenly creation is an ordinary man: it is something never seen, an absolutely successive man, whose existence is continuous, although created one phase after the other.

quam in rebus inanimatis, licet non sit idem simpliciter", Nicole Oresme, *Questiones super Physicam*, 116.

⁸⁶ "non implicat contradictionem nec <est> simpliciter impossibile quod sit aliqua substantia simpliciter successive", Nicole Oresme, *Questiones super Physicam*, 333.

⁸⁷ Nicole Oresme, *Questiones super Physicam*, 333.

⁸⁸ This doesn't mean, of course, that such an absolute successive being is a mere fiction that can be thought without contradiction: if God were to create such a successive man, it would be a *real* being, not a *chimera*.

⁸⁹ "Verbi gratia: si a, quod est duplum ad b, diminuatur successive, non est contradictio quod Deus creet unam substantiam vel hominem, qui precise durabit quamdiu a erit duplum, scilicet per solum instans, et similiter quando erit sesquialterum, et sic de qualibet alia proportione; igitur tale aggregatum ex omnibus istis esset homo vel substantia successiva, cuius nihil quod erat in aliqua parte temporis fuerit in sequenti. Et dico corollarie quod in talibus illud quod est solum per instans, non est pars illius successivi, sed se habet ad illud sicut punctus ad lineam et instans ad tempus", Nicole Oresme, *Questiones super Physicam*, 333-334.

⁹⁰ Nicole Oresme, *Questiones super Physicam*, 333-334.

⁹¹ Caroti, "Oresme on Motion", 26.

We should halt a moment on the expression “*aggregatum*”. It is a common way to designate an arithmetical sum.⁹² It expresses a specific relation between “parts” and wholes. As we see, Oresme defines instantaneous beings, and then collects this infinity of beings as an aggregate to “compose”, in a way, a successive man. Strictly speaking, the instantaneous men are not parts of the aggregated successive man, just like points are not parts of a line. In his *De configurationibus*, he also uses the same expression to express the relation between a long sound such as a cantilena and partial sounds separated by perceivable sensible pauses, as when the singer is breathing. Both sounds have unity, but the latter has a unity of the second mode (only cut by imperceptible pauses), when the former has a unity of the third mode, improper and “*ex aggregatione*”.⁹³

Other examples express this higher mode of unity formed by unities of a lesser mode. In the second conclusion, after having given motion as a first example of an absolutely successive thing, and then time, Oresme gives a very abstract and mathematical example: ratios. Let there be a greater quantity A, he says, and a lesser one B, and let A decrease successively.⁹⁴ Then, in any instant, A and B will have a ratio, *always other and other*, and so the “total ratio (*totalis proportio*)”, that is concerning the whole time, is called “successive”.⁹⁵ Thus, this *total* ratio is composed of an infinite succession of *instantaneous* ratios, a continuous succession: this is quite exactly what we would call a *varying* ratio.

Totalis proportio is a surprising formula, analogous to the *aggregatum* mentioned before. Studying the first two questions on Geometry, Edmont Mazet noticed that Oresme is the first to call “a total” the sum of an infinite series, that is, geometrically speaking, the sum of the infinite number of parts in which a magnitude could be divided into a finite duration.⁹⁶ This notion is tantamount to a practical use of actual infinite, even though, when Oresme addresses the philosophical question of the possibility of such an actual infinite few questions later, he rejects it.⁹⁷ It is very important to have in

⁹² For example: “square numbers are always the result of the sum of odd numbers (*ex (...) aggregatione numerorum imparium semper resultant numeri quadrati*)”, Nicole Oresme, *Questiones super geometriam Euclidis*, 153.

⁹³ Nicole Oresme, *De Configurationibus*, 306.

⁹⁴ “Secundo, sit a quantitas maior et b minor, et diminuatur a successive; tunc in quolibet instanti a ad b habet aliam et aliam proportionem, ergo totalis proportio, que est per totum tempus, dicitur successiva et quidam modus <se> habendi successivus, et in nulla parte temporis habet taliter esse qualiter se habet in sequenti, ergo est simpliciter successivum, iuxta expositionem prius positam etc.” Nicole Oresme, *Questiones super Physicam*, 333.

⁹⁵ See the preceding note.

⁹⁶ “Oresme introduit, sous le nom de ‘tout’, la notion même de somme de la série, et cela non seulement dans le cas d’une série convergente, mais aussi – et c’est là qu’intervient le dépassement du point de vue physique d’Aristote – dans le cas d’une série divergente”, Edmond Mazet, “La théorie des séries de Nicole Oresme dans sa perspective aristotélicienne. ‘Questions 1 et 2 sur la Géométrie d’Euclide’”, *Revue d’histoire des mathématiques* 9, 1 (2003): 77.

⁹⁷ “Actu et categorematice nihil est infinitum”, Nicole Oresme, *Questiones super Physicam*, 361.

mind, when we read those ontological arguments, that Oresme is probably the first to propose a general “theory of series”, that he taught general a general method to calculate infinite series, and that he is probably the first to demonstrate the divergence of the harmonic series.⁹⁸

Lastly, the third conclusion concerns the reality of absolutely successive *qualities*.⁹⁹ The image in a mirror, *species* and light, and finally sound. These cases are neither possible creations of God, nor mathematical beings, but physical and quite ordinary things. But the unity of each has to be understood as the result of a kind of aggregation of instantaneous units: if an object is moving, its “total image (*totalis ymago*)”¹⁰⁰ in a mirror is a continuous being, but in a way made of an infinite number of instantaneous images, because “at each instant, there is a new image.”¹⁰¹ Another example: if a coin, a *denarium*, is deep in a flowing river, there will be continually new *species* or “images” of the coin in the river. Finally, if a sound is intensified and goes continuously higher and higher, nothing remains of the lower degrees, otherwise the same sound would produce continually concord and discord, which is never the case.¹⁰²

⁹⁸ John E. Murdoch, “Review of Nicole Oresme’s ‘Quaestiones super geometriam Euclidis’, edited by H.L.L. Busard”, *Scripta Mathematica* 27 (1964): 67-91.

⁹⁹ “Tertia conclusio est quod est aliqua qualitas simpliciter successiva. Probatur primo, si ymago sit aliquid in speculo, tunc, si obiectum moveatur, facilius potest ostendi quod in quolibet instanti est nova ymago secundum se et quolibet sui propter novum motum vel continuam mutationem situs obiecti ad speculum, ergo totalis ymago, que est per tempus, est res successiva. Secundo, si conceditur quod species est in medio et medium continue moveatur, sicut <si> denarius sit in fundo aque currentis, tunc per idem probaretur quod continue in illa aqua, que sup<er>ponitur denario, est nova species. Eodem modo est de lumine secundum aliquos: bene lumen intenditur, <et> dicunt quod est continue novum secundum quodlibet sui, et ita dicunt de caliditate. Tertio, arguitur fortius de sono, quia conceditur quod est quedam qualitas sensibilis distincta a medio vel subiecto. Sed tunc arguitur: sonus consequitur motum, ut patet in secundo *De anima* et etiam in *Musica* Boethii, ergo sonus est successivus eo modo sicut motus. Etiam patet quod una syllaba non est, sed iam transit quando venit alia, et ita de partibus syllabe; et propter hoc sonus mensuratur aliter tempore et duratione quam alie qualitates <scilicet> permanentes. Etiam si aliquis sonus continue intendatur, tunc, si aliqua pars permaneat, tunc grave et acutum esse<n>t simul, et sic ex uno sono proveniret dissonantia vel consonantia, quod est contra Boethium in *Musica* sua”, Nicole Oresme, *Questiones super Physicam*, 333.

¹⁰⁰ See the footnote above.

¹⁰¹ See the footnote above.

¹⁰² See the footnote above. This case should be compared to Oresme’s solution to the ontology of intensive variation. As it is well-known, Oresme admits an intensive quality to be *composed* of (simultaneous) degrees only by mathematical imagination. In reality, when a substance is becoming whiter, it is not composed of simultaneous degrees successively added one to the other, but has continuously another being-white: “ideo quando subiectum dicitur intendi vel fieri magis album, continue habet aliud et aliud esse album. Unde totaliter est aliud esse album intense <et> aliud esse album remisse, nec unum componitur ex alio”, Nicole Oresme, *Questiones super Physicam*, 42. Thus, concerning the ontology of degrees, Oresme adopts the *successive* theory, and admits the *additive* theory only by mathematical imagination and for mathematical sake. This shows that the adverbial-indivisibles *succeeding* one another concerns the *ontology* of the *res successiva*, not their

Oresme is perfectly aware of the difficulty: isn't he *composing* the continuum as a totality generated by the succession of an infinite number of indivisibles? This is in part the meaning of the sixth objection: if motion is a *res successiva* as defined, then "in a small period, there will be an infinite number of things, like [an infinite number] of being-changed (*in parvo tempore fierent infinita, sicut infinita mutata esse*)."¹⁰³ And Oresme *concedes* this objection: "it is not a difficulty (*non est inconueniens*)."¹⁰⁴ Indeed, this infinite number of "being-changed" is not an infinite number of *entities*, but an infinite number of *modifications*. The *condicio* theory enables Oresme to atomize the continuum in an infinite number of indivisibles, because those indivisibles are not *beings*, but *modes* of beings. This is why there is no paradoxes in the fact that the *totalis proportio* we saw above is really "composed" of an infinite number of ratios. This must be kept in mind when coming to Oresme's study of continuity in book VI, as we shall see below.

2.2 *Quaestiones super de generatione et corruptione, I.13*¹⁰⁵

The same kind of discussion is to be found in Oresme's *Quaestiones de generatione et corruptione*, question I.13: "Does the thing increased remain the same in the beginning of the increase and in the end? And the same question for the case of decreasing."¹⁰⁶ As before, Oresme is thus asking how a successive being can have some unity and keep its identity during time. If the study has a general perspective, Oresme focuses himself on the case of a substance continuously gaining or losing parts, as would a living animal.

Oresme suggests two ways to solve the problem, and then his own. The second is very simple and simply states that a successive thing has an identity because it keeps a permanent and essential part.¹⁰⁷ Oresme doesn't even discuss this answer, obviously because it doesn't solve the problem of *absolutely* successive beings. Indeed, immediately after the formulation of this solution, which Oresme dismisses, he distinguishes between three ways to understand the words "*unum*" and "*idem*": for totally permanent things, for absolutely successive things, and for mixed things. He goes on to say that the unity of an absolutely successive being, such as the motion of

mathematical *imagination*. See also Kirschner, "Oresme on intension and remission of qualities"; Debrouse, *Mathématiques de l'intensité et Merveilles de la nature*, 305-327.

¹⁰³ Nicole Oresme, *Quaestiones super Physicam*, 335.

¹⁰⁴ Nicole Oresme, *Quaestiones super Physicam*, 335.

¹⁰⁵ Nicole Oresme, *Quaestiones super de generatione et corruptione*, 111-118.

¹⁰⁶ "Queritur tertiodecimo utrum augmentatum maneat idem in principio augmentations et in fine ipsius, et similiter de diminution", Nicole Oresme, *Quaestiones super de generatione et corruptione*, 111.

¹⁰⁷ "Alia via est quod in composito, saltem animate, quedam sunt partes necessario per se requisite ad esse illius compositi, et animal est proprie tales partes, et ille non fluunt et refluent; sed alie sunt partes que non requiruntur per se sed dicuntur accidentales, sicut accidit homini habere digitum et posset esse et abesse, et tunc hoc totum non est tales partes nisi per accidens", Nicole Oresme, *Quaestiones super de generatione et corruptione*, 115.

heaven, is due to “its successive continuity.”¹⁰⁸ The Parisian river Seine is a case of an absolute *fluxus*: the water of the Seine is not today the same as it was two years ago.¹⁰⁹ Anyway, the Seine is the same Seine, and this is only due to continuity: “the whole is one continuum (*totum est unum continuum*).”¹¹⁰ Thus, once again, his own solution is really to make the continuity the real cause of the unity of a successive being.

However, the first *via* Oresme suggests is most interesting, and most impressive. Oresme dismisses it, but only because it lacks generality and could be used to prove paradoxes. He carefully deduces difficult conclusions from this solution, as if one of Oresme’s purposes in this question was precisely to demonstrate his skill in manipulating the logical and quasi mathematical concepts involved.

This first *via* has a theological stance, and rests upon the general principle: “one being is many beings (*una res est plures res*).”¹¹¹ For permanent beings, the mixture of unity and multiplicity is due to the “divisibility at the same time.”¹¹² But in the same way, “one being is many successive beings (*una res est plures successiva*).”¹¹³ What Oresme is talking about is not absolutely clear, but he immediately adds: the first case is possible only “*supernaturaliter et in divinis*”, but the second case is true *naturaliter*. Stefano Caroti supposes in his commentary¹¹⁴ that Oresme is referring to the mystery of Trinity.¹¹⁵ Thus, what Oresme is suggesting here is that successive things, particularly motion, could be some kind of temporal images of the trinity, just as *impossible* to rationally understand as the mystery of religion.

The case he uses to illustrate this natural unity is odd at first glance: Socrates “is [now] some parts, and then other parts will be, while he is the same, and himself before was other parts.” That is, a man “who now is body and soul, and after death will be only soul.”¹¹⁶ From this case, Oresme will now suggest rules to solve paradoxes of identity of

¹⁰⁸ Nicole Oresme, *Quaestiones super de generatione et corruptione*, 115.

¹⁰⁹ “Non est eadem aqua Secane nunc, que erat quod sunt duo anni”, Nicole Oresme, *Quaestiones super de generatione et corruptione*, 116.

¹¹⁰ Nicole Oresme, *Quaestiones super de generatione et corruptione*, 116.

¹¹¹ “Nunc pro solutione difficultatum multi sunt modi dicendi. Unus est quod sicut una res est plures res divisim simul tempore, sic etiam una res est plures successive. Primum tamen est possibile solum supernaturaliter et in divinis, sed secundum est verum naturaliter. Et idéo Sortes, qui modo est aliqua partes, postea erunt alie partes ipse idem, et ipse ante fuit alie, sicut aliqui dicunt quod homo, qui nunc est corpus et anima, post mortem erit sola anima”, Nicole Oresme, *Quaestiones super de generatione et corruptione*, 113.

¹¹² “Divisim simul tempore” Nicole Oresme, *Quaestiones super de generatione et corruptione*, 113.

¹¹³ Nicole Oresme, *Quaestiones super de generatione et corruptione*, 113.

¹¹⁴ Nicole Oresme, *Quaestiones super de generatione et corruptione*, 112*-118*.

¹¹⁵ “Im ersten Fall bewahrt man die Einheit Gottes innerhalb der Dreifaltigkeit; im zweiten die Identität der natürlichen Dinge, die der zeitlichen Veränderung unterworfen sind”, Nicole Oresme, *Quaestiones super de generatione et corruptione*, 114*.

¹¹⁶ “Et ideo Sortes, qui modo est aliqua partes, postea erunt alie partes ipse idem, et ipse ante fuit alie, sicut aliqui dicunt quod homo, qui nunc est corpus et anima, post mortem erit sola anima”, Nicole Oresme, *Quaestiones super de generatione et corruptione*, 113.

a general form. Thus, if a totality composed of two parts, A and B, loses one part, say B, then, two composed totalities will successively exist, first *A and B*, and then *only A*. But those two realities are in fact only one and the same reality: when Socrates is dead, he is a soul deprived of its body, but he remains the same being, Socrates. The paradox emerges from the confrontation between this unity and the succession of time: “This whole will be tomorrow, this whole is A and B, thus A and B will be tomorrow; and let’s suppose that B is a part to be suppressed. We answer by conceding that A and B will be tomorrow, but it doesn’t follow that B will be tomorrow, because A and B will be B.”¹¹⁷

Oresme’s solution to the paradoxes relies on the logical operation called “exchange of names (*communicatio ydiomaticis*)”, an operation Oresme explained in a theological treatise named *De communicatione ydiomatum*: if there is an identity between two realities, the properties of one can be stated of the other.¹¹⁸ For example, if the man Jesus is God, as Jesus is mortal, God is mortal, and as God is immortal, Jesus is immortal. The contradiction of such a statement doesn’t destroy the argument, but only expresses the mystery of Incarnation and the incapacity of human reason to understand it. What is important to note is that the nature of the paradoxes Oresme studies in this theological treatise is exactly the same as the seven cases Oresme analyzes in this section of question 1.13.

Even if this first *via* is not Oresme’s final answer, it illustrates the perplexities Oresme had to face when studying the unity of an absolutely successive being such as motion: the logical techniques he uses are the same as those he needed to analyze paradoxes of the theological mystery of Incarnation, as if the continuity of motion was just as difficult to understand as the unity of God.

Thus, Oresme’s analysis is paradoxical: on the one hand, he has distinguished real motion from apparent one by its essential continuity: motion is a *continuous flux*. But on the other hand, this continuous flux can be analyzed as a whole “composed” of an infinite number of indivisibles. It is not a composition properly speaking, because those indivisibles are not the *parts* of the continuum: no part of motion is instantaneous. But for any two given successive parts, however small, there is no instant in the first part when the motion is in the same “state” as in any instant of the second. Fundamentally, the structure of continuity is not only defined by a whole/part relationship, but a whole/part/point relationship. The ontological paradoxes involved in the idea of a composition of the continuum by an infinite number of indivisibles are avoided thanks to the *condicio* theory. However, as we saw, Oresme judged this continuity sufficiently

¹¹⁷ “Arguitur primo sic: hoc totum erit cras, hoc totum est a et b, ergo a et b erunt cras; et sit ita quod b sit pars resolvenda. Responditur concedendo quod a et b erunt cras nec ex hoc sequitur quod b erit cras, quia a et b erunt cras a”, Nicole Oresme, *Quaestiones super de generatione et corruptione*, 113.

¹¹⁸ Ernst Borchert, *Der Einfluss des Nominalismus auf die Christologie der Spätscholastik: nach dem Traktat De communicatione idiomatum des Nicolaus Oresme* (Münster: Aschendorff, 1940).

paradoxical to be compared to the mysteries of Religion. But in fact, he also deduced from this notion astonishing mathematical corollaries, as we shall now see.

3. A new kind of mathematical continuity

Any scholastic discussion about continuity and atomism rests upon Aristotle's classical rejection of the idea that a continuous magnitude is composed of *indivisibles* parts.¹¹⁹ As John Murdoch has extensively shown, in the 14th century, atomism was revived in Western universities by philosophers such as Henry of Harclay, Walter Chatton, Gerard of Odon, or Nicholas Autrecourt.¹²⁰ This intellectual movement incited defenders of the continuum to renew their argument, as we can see with Thomas Bradwardine's treatise *De continuo* where he conscientiously refutes the idea that the continuum would be composed of *extensionless* indivisibles, whether finite in number or *infinite*. Murdoch has insisted on the fact that the critics against this new kind of atomism went "beyond Aristotle", "providing *new* conceptions and *new* arguments for their cause."¹²¹ It is not the goal of this paper to reevaluate the relation between Oresme and contemporary atomism.¹²² However, this global renewal implied for Oresme a real deepening of what continuum is.

Oresme studies continuity directly in two main works: the first three questions on book VI of Aristotle's *Physics*,¹²³ and the eighth question on Euclid's *Geometry*.¹²⁴ There are strong analogies between those two difficult studies, and I think that his analysis in *Physics* is better understood in the light of the questions on Euclid's geometry.

At first sight, Oresme's conclusions about continuity are not original at all. He first denies that the continuum is composed of indivisibles, whether this continuity concerns spatial and permanent entities (VI.1) or successive entities (VI.2). In the same way, the immediate conclusion of the next question (VI.3) is expected: a continuum, he says, is always divisible in divisible things (*divisibilia*). However, the arguments he uses

¹¹⁹ On medieval atomism, see Bernhard Pabst, *Atomentheorien des lateinischen Mittelalters* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1994); Andrew Pyle, *Atomism and its Critics. problem areas associated with the development of the atomic theory of matter from Democritus to Newton* (Bristol: Toemmes Press, 1995); Christophe Grellard and Aurélien Robert, *Atomism in Late Medieval Philosophy and Theology* (Leiden and Boston: Brill, 2009).

¹²⁰ John E. Murdoch, "Beyond Aristotle: Indivisibles and Infinite Divisibility in the Later Middle Ages", in *Atomism in Late Medieval Philosophy and Theology*, edited by Ch. Grellard and A. Robert (Leiden and Boston: Brill, 2009), 15-38.

¹²¹ Murdoch, "Beyond Aristotle: Indivisibles and Infinite Divisibility", 17.

¹²² For Oresme's relation to atomism, one should start with Stefano Caroti "Configuratio, ymaginatio, atomisme et modi rerum dans quelques écrits de Nicole Oresme", in *Méthodes et statuts des sciences à la fin du Moyen Âge*, edited by Ch. Grellard (Villeneuve d'Ascq: Presse universitaire du Septentrion, 2004), 127-140.

¹²³ Nicole Oresme, *Questiones super Physicam*, 658-677.

¹²⁴ Nicole Oresme, *Questiones super geometriam Euclidis*, 125-128.

are much more surprising, and reveal a truly original and profound understanding of the nature of the *continuum*. Before examining those arguments, I shall first present the content of the first two questions.

3.1 *Questiones super Physicam VI.1 and 2*

At the beginning of question VI.1, Oresme gives a very traditional definition of continuity: “Something is continuous whose parts are joined to one another and make one (*continuum est cuius partes copulantur ad aliquem terminum et faciunt unum*).”¹²⁵ He then distinguishes different kinds of continuity: the continuity *primo divisibile*, meaning the *quantitas extensa*, and the continuity *secundario divisibile*, meaning intensively divisible. This intensive continuity is thought of by analogy with “distance”, extensive continuity. Finally, he distinguishes *continuum permanens* and *continuum successivum*, the continuity of a whole whose parts are not simultaneous.¹²⁶

The first two questions only concern the supposition of a composition of a *finite* number of indivisibles. The first one is limited to extensive quantities and gives a very general conclusion: “no continuum is composed of indivisibles (*nullum continuum est ex indivisibilibus*).”¹²⁷ His general arguments rely heavily on Aristotle and the impossibility for indivisible things to be mutually in contact. Then Oresme specifies this general conclusion to the case of straight lines, circular lines, surfaces and bodies, using geometrical arguments. For example, the composition *ex indivisibilis* would not be compatible with continuous divisibility or incommensurability. It would also imply that the smaller magnitude would be equal to the greater, because both would be composed of the same quantity of points.

In the second question, Oresme studies the case of successive continuity, such as time, motion, and consequences of motion, like sound. Obviously, such a continuity cannot be defined by the mutual contact of its parts, because for a successive being, parts only exist one after the other: a being cannot be in contact with a nonbeing. However, Oresme gives a new understanding of continuity only in his final answers to preliminary arguments. Instants, he says, are concerned with continuity only *syncategorematic*, because after one instant, there is another instant “*sine intermissione*.”¹²⁸ Indeed, in the course of this second question, he implicitly identifies successive continuity with the negation of any quantitative instantaneous “leaps”.¹²⁹

¹²⁵ Nicole Oresme, *Questiones super Physicam*, 659.

¹²⁶ Nicole Oresme, *Questiones super Physicam*, 659.

¹²⁷ Nicole Oresme, *Questiones super Physicam*, 659.

¹²⁸ Nicole Oresme, *Questiones super Physicam*, 669.

¹²⁹ For example: “Tertia conclusio est quod nec motus intensionis, ut intensio albedinis, componitur ex indivisibilibus, sicut aliqui ymaginantur gradus indivisibiles. Probatur, quia sequitur quod talis intensio non esset continua; patet consequentia, quia indivisibile non aquiritur nisi in

This raises questions about his analysis of the continuity of time. Indeed, his first conclusion states without ambiguity that time is not composed of a finite number of instants, just like a line is not composed of points.¹³⁰ Moreover, if time was composed of instants, and if a mobile was in motion during a period composed of three instants, A, B, and C, then the mobile would not be in motion in any instant: motion is a successive being, with *prior* and *posterior* as Oresme says, but if an instant is indivisible, there cannot be *prior* and *posterior* in it. Therefore, there would be no motion. Still, he concedes at the end of the question that “there will continually be an instant after an instant (*continue post instans erit instans*).”¹³¹ In particular, he concedes that, for a duration A, all the instants that “endure (*continuant*)” during A are immediate to the term or last instant of A.¹³² Doesn’t that imply that time is composed of instants? At least, it doesn’t imply, Oresme adds, that there are two instants immediately successive one after the other: the set of all instants before the last instant is an infinite set *without a last term*.¹³³ This argument introduces the

<instanti>, ergo non fieret intensio nisi per instantia, ex quibus non componitur tempus per primam conclusionem”, Nicole Oresme, *Questiones super Physicam*, 667.

¹³⁰ “Prima est quod tempus non componitur ex instantibus, saltim finitis”, Nicole Oresme, *Questiones super Physicam*, 666.

¹³¹ “Secundo: si non ita est, sit ergo tempus *a b c* compositum ex tribus instantibus, tunc moveatur aliquid <in> illo tempore; tunc in nullo instanti movetur, ergo in tempore non movetur. Tenet consequentia, quia quod non movetur in aliqua parte temporis non movetur primo in illo tempore; et patet antecedens, quia in instanti non est prius aut posterius, quia iam esset divisibile; ergo in instanti non est motus qui est successivus”, Nicole Oresme, *Questiones super Physicam*, 669.

¹³² These are the arguments *quod sic* opening the question and arguing that instants are continuous: “Secundo, instantia sunt continua in tempore, ergo tempus componitur ex illis. Tenet consequentia, quia ex eis non fit aliud quam tempus, et si sint continua faciunt unum continuum. Probatur antecedens tripliciter: primo, quia continue et semper est instans in tempore, ergo illa <que> sic sibi succedunt in tempore sunt continua. Secundo, confirmatur auctoritate: inter omnia instantia que continuant horam et instans terminans est aliquod medium aut nullum. <Si aliquod>, sic dicendo similiter <hoc medium erit> tempus, et in illo sunt instantia, ergo non essent <omnia> alia assumpta. Si nullum, ergo omnia illa et instans terminans sunt immediata. Tertio, confirmatur: inter *a* instans et non esse illius nullum est medium, sed non esse illius terminatur aliquo instanti, ergo illud est immediatum *a*”, Nicole Oresme, *Questiones super Physicam*, 664, and here is Oresme’s final answer to these arguments: “Ad secundam, negatur antecedens, scilicet quod instantia sint continua. Et cum dicitur quod sunt continue, distinguendum est, quia potest intelligi quod se habeant continue, ita quod unum sit alteri continuum; et tunc est negandum. Alio modo quod ly ‘continue’ tenetur syncategorematicè et valeat tantum sicut: ‘sine intermissione post instans est instans’; et tunc conceditur quod continue post instans erit instans, et tamen nullum instans continue sequitur post hoc instans nec ex hoc sequitur quod aliqua sunt immediata vel continua etc. Et quando confirmatur ultra: ‘inter omnia continuantia et terminans etc.’, dico quod si ly ‘omnia’ potest teneri collective pro infinitis, tunc concedo quod terminans et omnia alia sunt immediata. Et ex hoc non sequitur quod aliqua duo sunt immediata, quia inter illa continuantia non est dare aliquod ultimum. Ad aliam confirmationem, cum dicitur: ‘inter *a* instans et suum non esse etc.’, verum est secundum <quod> suum non esse non habet primum instans, immo in quocumque instanti non erit prius non fuerit”, Nicole Oresme, *Questiones super Physicam*, 669.

¹³³ “Et ex hoc non sequitur quod aliqua duo sunt immediata, quia inter illa continuantia non est dare aliquod ultimum”, Nicole Oresme, *Questiones super Physicam*, 669.

hypothesis of an infinite number of indivisibles, and thus goes beyond the purpose of this second question: we will go back to it when we shall study the third one. Anyhow, we see that continuity is not understood as a *relation* between *two parts* anymore, but as a *propriety* of a *whole*, an *infinite* whole.

Let's go back to the other successive continuities. Local motion, as one expects now, is not composed of "*mutatis esse indivisibilibus*" (second conclusion).¹³⁴ The same conclusion is repeated for the *motus intensionis*, the intensive motion (third conclusion).¹³⁵ Then, the fourth conclusion concerns the successive things which are consequences of motion, such as sound, proportion, sickness, intension, velocity, "*et similia*".¹³⁶ Once again, the same conclusion is drawn, with an interesting corollary about which I will say more below. Oresme finally goes back to permanent qualities, asserting that they are not composed of finite indivisible degrees either.¹³⁷

Except for the corollary I just mentioned, those first conclusions do not seem very original. However, we already know from III.6 that in fact, Oresme considers it unproblematic to atomize local motion in an infinite number of indivisible *mutata esse*, provided that those indivisibles are not understood as beings, but as modes of beings: clearly, Oresme doesn't tell us the whole story here. For this reason, we should not be surprised that his argumentation gets more complicated in the next question.

3.2 *Questiones super Physicam VI.3*¹³⁸

The question that is now raised is surprising. He asks whether a continuum is divisible into ever-divisible "things" (*divisibilia*).¹³⁹ In fact, he divides this question in two topics: first, he asks whether the continuum is divisible into ever-divisible things;

¹³⁴ "Secunda conclusio est quod nec motus localis vel extensionis componitur ex mutatis esse indivisibilibus. Probo, quia motus dividitur ad divisionem temporis a quo habet successionem per notabile prius dictum; ergo in tot dividitur sicut tempus, quod non componitur ex talibus per primam conclusionem", Nicole Oresme, *Questiones super Physicam*, 666.

¹³⁵ "Tertia conclusio est quod nec motus intensionis, ut intensio albedinis, componitur ex indivisibilibus, sicut aliqui ymaginantur gradus indivisibiles. Probatur, quia sequitur quod talis intensio non esset continua; patet consequentia, quia indivisibile non acquiritur nisi in <instanti>, ergo non fieret intensio nisi per instantia, ex quibus non componitur tempus per primam conclusionem", Nicole Oresme, *Questiones super Physicam*, 667.

¹³⁶ "Quarta conclusio est quod nullum successivum sequens motum componitur ex indivisibilibus", Nicole Oresme, *Questiones super Physicam*, 666 and 667.

¹³⁷ Nicole Oresme, *Questiones super Physicam*, 668.

¹³⁸ "Quinta conclusio est quod <n>ulla qualitas permanens, si est, componitur ex indivisibilibus intensive, supposito quod habeat partes secundum intensionem, sicut calor vel albedo, quia tunc motus intensionis componeretur ex indivisibilibus, quod est contra tertiam conclusionem", Nicole Oresme, *Questiones super Physicam*, 671-677.

¹³⁹ "Consequenter queritur utrum continuum Sit divisibile in semper divisibilia, intelligendo quod dividatur in aliqua et illa in alia, et sic semper", Nicole Oresme, *Questiones super Physicam*, 671.

second, he studies whether it is composed of *infinite* indivisibles. Therefore, the main difference with the two former questions is that the number of indivisibles is now supposed to be infinite. However, the question of infinite divisibility might seem to have been already settled: obviously if a continuum is not composed of indivisibles, should it not be thought of as always divisible in divisible parts? But Oresme's purpose is different, and his main conclusions are much more complicated. There is a neat contrast between the apparent classical general conclusions he draws and the complexity of the corollaries he adds very allusively.

The first general conclusion concerns the first topic, and is quite expected: a continuum is indeed divisible into ever-divisible parts (*continuum est divisibile in semper divisibilia*), at least “by signations of parts”, although no actual separation of the parts could occur.¹⁴⁰ The arguments do not teach the reader anything new: first, a part of a continuum is not indivisible, and conversely a point is not part of the continuum; secondly, the signature process used in arithmetic or astronomy is infinite; third, and more originally I think, the musical tonus cannot be divided in two equal halves.

Now, this general conclusion is immediately followed by two very surprising series of corollaries. But before turning to them, I shall comment on the general conclusions of the second topic.

Of the three general conclusions he comes to, the last one is the most disturbing:

- Continuum is not composed of an infinite number of indivisibles;¹⁴¹
- There are no such indivisibles in a continuum;¹⁴²
- But: “the being of indivisibles must not be denied, taking ‘being’ in a large and equivocal meaning (*non est negandum indivisibilia esse, large et equivoque capiendo ‘esse’*).”¹⁴³

Thus, we immediately see that Oresme's conclusion is ambivalent: on the one hand, he completes Aristotle's doctrine by extending his rejection of the composition of the

¹⁴⁰ “De primo est conclusio quod continuum est divisibile in semper divisibilia per signationem partium, quamvis non sit separatio actualis”, Nicole Oresme, *Questiones super Physicam*, 672.

¹⁴¹ “Prima conclusio est quod continuum non componitur ex indivisibilibus infinitis”, Nicole Oresme, *Questiones super Physicam*, 675.

¹⁴² “Secunda conclusio est quod non sunt talia indivisibilia in continuo, quia non substantia, ut probatum est, nec accidens, nec tales forme, quia tunc quere<re>tur de subiecto. Et secundo, videtur quod aggregatum ex omnibus illis esset continuum, et quod componeretur ex illis”, Nicole Oresme, *Questiones super Physicam*, 675.

¹⁴³ “Tertia conclusio est quod non est negandum indivisibilia esse, large et equivoce capiendo ‘esse’, et ymaginando aliter quam mathematicus ymaginatur, quia talia sunt significabilia aut complexe aut similitudinaria”, Nicole Oresme, *Questiones super Physicam*, 675. It is interesting to note that the being of indivisibles must be “imagined”, but not in the way mathematician do. The common distinction between reality and mathematical imagination is not enough: there is room for an imagination which is not mathematical, that is to say which is not, in that case, merely fictional.

continuum to the case of an infinite number of indivisibles. He even adds that those indivisibles don't exist in the continuum. But, on the other hand, he justifies the logical and mathematical use of indivisibles by clarifying their meanings. A natural philosopher cannot avoid points, surfaces, or instants, but he must exactly understand the meaning of these words. Indeed, what Oresme means by this “*large et equivoce*” meaning of *esse* is that a point must be understood as “here in-an-indivisible-manner (*hic indivisibiliter*)”, an instant as “now in-an-indivisible-manner (*nunc indivisibiliter*)”, so that indivisibles are not beings, or *res*, but modes of beings more adequately named by adverbs.¹⁴⁴ As we see, Oresme doesn't dismiss the reality of indivisibles as mere mathematical fictions: the *esse* of indivisibles *must not be denied*. However, indivisibles are not what the mathematician thinks they are, that is to say: *beings*.

His answers to objections in favor of the composition of the continuum are very significant. The fourth argument *quod non* states that if a sphere tangent to a surface moves on it, the motion will describe a line on this surface, so that “a line is composed of points, but infinite in number.”¹⁴⁵ Oresme's answer is straightforward: “I concede the whole case.”¹⁴⁶ But one must not think that there is indeed a point where the sphere touches the surface, “as mathematician imagines (*sicut mathematicus ymaginatur*)”: what is true is that the sphere touches the surface “somewhere in an indivisible manner (*indivisibiliter alicubi*).”¹⁴⁷ The use of indivisibles, even when a continuum is mathematically *imagined* as composed of such indivisibles, is mathematically legitimate. Yet, the way the mathematician imagines things does not reflect reality.

Again, he objects to himself the possibility to imagine a body composed of an infinite number of indivisible surfaces. Doesn't that mean that a quantity can be composed of an infinite number of *non quanta*?¹⁴⁸ Indeed, answers Oresme, but only in

¹⁴⁴ Nicole Oresme, *Questiones super Physicam*, 675. This propositional analysis is related to the *condicio* theory. See the bibliography above (note 18). See also Laurent Cesalli, “Ontologie ‘nominale’ et ‘adverbiale’ chez Nicole Oresme”, in *Nicole Oresme philosophe: Philosophie de la nature et philosophie de la connaissance à Paris au XIVe siècle*, edited by J. Celeyrette and Ch. Grellard (Turnout: Brepols, 2014), 163-183.

¹⁴⁵ “Quarto, unumquodque dividitur in ea ex quibus componitur, sed non componitur ex semper divisibilibus, immo indivisibilibus, quod patet si sphaera super planum moveatur, que tangit planum in puncto, et continue punctus talis est supra <punctum> spatii, et tamen describit lineam tali motu, et cum in fine fuerit super totam, et non nisi super puncta, videtur quod linea sit composita ex punctis, saltem infinitis”, Nicole Oresme, *Questiones super Physicam*, 671.

¹⁴⁶ “Ad quartam, de sphaera mota super planum, conceditur totus casus. Nec est ibi punctus aliquis secundum quem tangat, sicut mathematicus ymaginatur, sed tangit indivisibiliter alicubi, ideo non oportet, nec etiam describit lineam, igitur etc.”, Nicole Oresme, *Questiones super Physicam*, 677.

¹⁴⁷ See the footnote above.

¹⁴⁸ “Ad ymaginationem superficies finita lata pedalis componitur ex infinitis superficiebus indivisibiliter latis, ergo non est impossibile quod quantum componatur ex infinitis non quantis. Antecedens patet in figura supra *a b c d*”, Nicole Oresme, *Questiones super Physicam*, 676.

imagination, not *secundum rem*.¹⁴⁹ And he adds: “(...) anyway, this is a beautiful argument (*pulchra persuasio*) for those who defend the opposite.”¹⁵⁰ But the identification of a body with an infinite number of surfaces is just the kind of mathematical method Oresme uses with high skill, for example in his *De visione stellarum*, where he assimilates an atmospheric volume to an infinite set of thin refractive layers.¹⁵¹ We thus have the feeling that, if Oresme denies the reality of composition of the continuum, it is mainly to *justify* its mathematical use *secundum ymaginationem* in new mathematical techniques invented by Oresme himself.

3.3 The first series of corollaries of question VI.3

This skill is in fact suggested in the two series of corollaries in question VI.3 mentioned before. Here is the first series:¹⁵²

- (1.1) the past being considered in the divisive sense, it is possible that if the world was eternal, any part of a continuum would have been divided.
- (1.2) the continuum cannot be divided in every manner in all [indivisibles] in which it can be divided.
- (1.3) the past being considered in the divisive sense, this is possible: if the world had been eternal, any part of a continuum would have been divided and no part would remain undivided, meaning a part which was previously not divided, although parts would still be joined.
- (1.4) there are an infinite number of points on this continuum, where there never was a division, but that can be divided in an infinite number of other ways.

Oresme is very allusive, but obviously, those corollaries are not at all expected as the general conclusions we saw before.

The first corollary projects the division of a continuum in an eternal past: it rests on the logic of time. If it is supposed that the world had not been created and was eternal, then it is possible that a continuum was divided in the past in such a way that

¹⁴⁹ “Ad secundam, conceditur antecedens ad ymaginationem; tamen non propter hoc tales superficies secundum rem sunt <partes> indivisibiliter late. Et ideo non est omnino simile, licet esset pulchra persuasio tenentibus oppositum”, Nicole Oresme, *Questiones super Physicam*, 676.

¹⁵⁰ See the footnote above.

¹⁵¹ Dan Burton, *Nicole Oresme’s “De visione stellarum (On seeing the stars)”*: A Critical Edition of Oresme’s *Treatise on Optics and Atmospheric Refraction* (Leiden and Boston: Brill, 2007), 158-160.

¹⁵² “De preteritis in sensu diviso hoc est possibilis, si mundus fuit eternus quelibet pars continui fuit divisa”; “non potest esse divisum omnimode in omnia in que est divisibile”; “de preteritis in sensu diviso hec est possibilis: si mundus fuit eternus, quelibet pars huius continui fuit divisa et nulla remanet indivisa, hoc est que prius non fuerit divisa, quamvis iterum partes sint unite”; “infinita sunt puncta in isto, ubi numquam fuit divisio, et quod infinitis aliis modis potest dividi, ergo”, Nicole Oresme, *Questiones super Physicam*, 672-673.

each part of it *had been* divided. The proposition is true only “*de preteritis de sensu diviso*”, and not *composito*: in the composite sense of the past, the proposition would mean that there was an instant in the past when all the parts of the continuum *were* actually divided. In the divisive sense, it means that for each part of continuum, there was an instant when this part was divided, while some other parts still remained undivided.¹⁵³ In particular, if A is such a divided part, it was divided in parts B and C still left undivided. However, there was also an instant when B was divided, and another for C.

Because of the past tense of the proposition, the reader is left with the quite strange idea that a continuum *is actually wholly* divided. The third corollary is very clear, specifying that “no part remains undivided (*nulla [pars] remanet indivisa*).”¹⁵⁴ At first sight, such an idea is absolutely in contradiction with what we usually understand by the infinite divisibility of the continuum: precisely, we mean that *there is always something left to be divided*.¹⁵⁵ Indeed, the meaning of this proposition is that there is no instant when the division leads to such small quantities that they cannot in turn be divided. Now, if the division is supposed to have been done in the past, it should be thought of as *fully accomplished* in the present. And of course, we have difficulties to understand in what kind of state could be a *wholly* divided continuum!

This paradoxical idea is not unusual in Oresme’s work. In fact, this is the basis of a new kind of exhaustion principle Oresme used and probably invented, a principle I called a “complete exhaustion along proportional parts of time.”¹⁵⁶ If a continuous magnitude is continually divided in proportional parts, for example in a ratio of 2: 1, but in such a way that the division process is done in one hour, the first half being divided in the first half of the hour, then a quarter in the next quarter of the hour, and thus continually for each proportional parts of the hour, then the division should be thought of as *complete* at the end of the hour: nothing is left to be divided. As Edmond Mazet showed, this mathematical method, which Oresme presents in the first question of his *Questiones super geometriam Euclidis*, is very important to calculate the sum of infinite series of determinate ratio, a major topic in Oresme’s mathematical accomplishments.¹⁵⁷

The two other corollaries, the second and the fourth, are much harder to understand. What does it mean, that the continuum cannot be divided “in any manner in anything where it is divisible (*omnimode in omnia que est divisibile*)”? Why is it

¹⁵³ This interpretation of mine is based on Curtis Wilson, *William heytesbury. Medieval Logic and the Rise of Mathematical Physics* (Madison: University of Wisconsin Press, 1960), 17.

¹⁵⁴ Nicole Oresme, *Questiones super Physicam*, 672-673.

¹⁵⁵ Mazet, “La théorie des séries de Nicole Oresme”.

¹⁵⁶ Debrouse, *Mathématiques de l’intensité et Merveilles de la nature*, 595-616.

¹⁵⁷ About the originality of this kind of exhaustion, Edmont Mazet notices: “Sur ce point, Oresme opère un renversement complet, qui ne consiste en rien de moins qu’à passer du point de vue strictement aristotélicien d’un processus se poursuivant indéfiniment à celui d’un processus actuellement poussé à l’infini”, Mazet, “La théorie des séries de Nicole Oresme”, 58.

important to notice that there was an infinite number of points where no division occurred? I think the key is to be found in the analogous arguments, and slightly more precisely, that one can find the eighth question of his questions on Euclidean geometry.

3.4 Question 8 on Euclidean Geometry

In question 8,¹⁵⁸ Oresme asks a seemingly harmless question: is the diagonal of a square commensurable to its side?¹⁵⁹ Having argued that it is not, he concludes two corollaries and, as he says, two “difficulties”.¹⁶⁰ The two difficulties are thus formulated:

- (1) It could be proved that a magnitude A, yet of the same kind as any line between C and D, and smaller, could become greater than any of these lines by a continuous increase, and never would be equal to any of them.¹⁶¹
- (2) From this, it could be proven that it is possible that a *continuum* be composed of an infinite number of indivisibles.¹⁶²

As usual in those questions, Oresme doesn't justify those two statements. Of course, they are startling: the second one just states the contrary to what we would expect. Moreover, as we shall see, the arguments are very similar to those we can find in VI.3 of his *Questions on Physics*. The first one will be fully explained below, but we can already notice that it is supposed to conceptualize a “continuous increase (*continua augmentatio*)”.¹⁶³

Both difficulties are necessary consequences of the two imaginations he had proposed as corollaries, of which the first is:

- (1) “Any continuum, as a line, can be divided in two incommensurable [parts]. From this, it follows that if a line were so divided, that one part is to the other like the diagonal to the side of a square, and once again those two parts divided in the same way, and so on infinitely, and if this line had been divided along all those imagined points, on which such a division can be done, then there would remain something to be divided, and there would be an infinite number of points on which no division would have been performed. Indeed, the

¹⁵⁸ Nicole Oresme, *Questiones super geometriam Euclidis*, 125–128.

¹⁵⁹ “Utrum dyiameter sit commensurabilis coste”, Nicole Oresme, *Questiones super geometriam Euclidis*, 125.

¹⁶⁰ Nicole Oresme, *Questiones super geometriam Euclidis*, 128.

¹⁶¹ “(...) poterit probari, quod a, quod est eiusdem rationis cum qualibet linea, que est inter c et d, et minus, fiet maius qualibet illarum et hoc per continuam augmentationem et nulli earum fiet equale”, Nicole Oresme, *Questiones super geometriam Euclidis*, 128.

¹⁶² “(...) ex hoc probatur, quod possibile est, quod continuum componeretur ex indivisibilis infinitis”, Nicole Oresme, *Questiones super geometriam Euclidis*, 128.

¹⁶³ Nicole Oresme, *Questiones super geometriam Euclidis*, 128, l.78.

division would have fallen on no point dividing the line into two commensurable parts.”¹⁶⁴

For the argument to be well understood, it must be noticed that the infinite process of division is supposed to be achieved. Implicitly, Oresme is asking to measure the process of division of time, for example, one hour, to divide the whole hour according to a continuous proportion, and to consider the state of the divided line at the end of the hour. This is a case of the complete exhaustion along proportional parts of time I mentioned above.

We recognize the same kind of conclusion we found very allusively in his *Questions on Physics*: if we continually divide a magnitude according to an irrational ratio, on the one hand the division is infinite and there will always remain something to be divided. But what if the division is supposed to be fully achieved? Would there be any remainder to be divided? In a sense, there would, but it is a *new* sense: all parts would have been divided, but all the rational *points* would remain undivided. Thus, the focus has shifted from undivided *parts* to undivided *points*, or, as one could say more exactly, *uncut* points.

This sheds a new light on the initial question of VI.3: if Oresme, quite traditionally, agrees with the fact that the continuum is “divisible in always divisible things”, he doesn’t understand it in the traditional way. If the process of division is fully achieved, as it is when projected in the past, there is no remaining *part* to be divided. Once again, his first corollary states clearly: “any part of the continuum has been divided (*quelibet pars continui fuit divisa*).” However, there is an infinite number of *points* where no division is ever felt: this is why, even in this strange case, something still remains “*undivided*”, uncut. But this corresponds to a totally new understanding of the continuum, a novelty confirmed by the other corollaries.

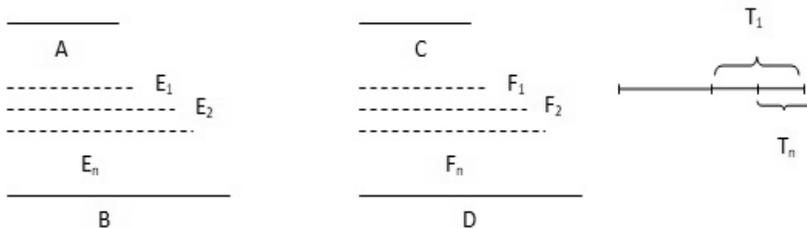
Indeed, Oresme immediately draws a strange conclusion from this first imagination: “From this it follows that, if a portion of prime matter is given – a portion that, according to Aristotle, has existed since eternity –, then it was so divided that no part remains undivided, and yet, in the future, it can be divided in an infinite number

¹⁶⁴ “Primum est, quod quodlibet continuum, verbi gratia linea, potest dividi in duo incommensurabilia et ex isto sequitur quod, si aliqua linea sit divisa in duo talia, quorum unum sit sicut dyiameter et reliquum sicut costa, et iterum quelibet istarum partium in duo talia et sic in infinitum, et si ista linea fuisset divisa secundum omnia ista puncta ymaginata (super que potest fieri divisio talis isto modo), quod adhuc remansisset dividendum et cum hoc fuissent infinita puncta, super que non fuisset divisio, quia super nullum punctum dividens eam in duo commensurabilia cecidisset divisio, et hoc patet. Iterum ex isto sequitur, quod demonstrata una portions materie prime, que fuit ab eterno secundum Aristotelem, quod ipsa fuit taliter divisa, quod nulla pars remanet indivisa, et tamen infinitis poterit dividi aliter quam umquam fuerit divisa prius”, Nicole Oresme, *Questiones super geometriam Euclidis*, 127.

of ways different from those by which it has been already divided.”¹⁶⁵ Two very different ideas about the continuum are clearly distinguished here: the first, in terms of *parts*, the second, in terms of *points*.

The second imagination confirms this analysis and demonstrates once more how profound Oresme was about mathematical continuity. Here, Oresme states:

- (2) “I suppose one line A and one another B double to A, and a line C equal to A and D equal to B. Then between A and B let us imagine a line incommensurable



to both, then between any of the other lines and this [last] one, another line incommensurable to both, and so on infinitely. And in the same way, let there be between C and D commensurable lines and so on, infinitely. In the same way, let's imagine an hour divided in instants in two equal parts, and similarly those equal parts in two and so on infinitely.”¹⁶⁶

Oresme doesn't go any further, but now we can guess what he was talking about when speaking of a “continuous increase” of magnitude A. Let's call E_n and F_n any magnitude respectively greater than A and smaller than B, and greater than C and smaller than D. Let's also divide proportionally one hour and call T_n any part of it. The set of all magnitudes between A and B is a scale along which a variable magnitude could increase from A to B taking *continually* the value of a magnitude incommensurable to both A and B. Therefore, the increase is *continuous* in the sense that for any increase from A to, say, E_1 however small, during a small period T_1 , there is a smaller increase from A to an incommensurable line E_2 smaller than E_1 during a period T_2 smaller than

¹⁶⁵ “(...) demonstrata una portione materie prime, que fuit ab eterno secundum Aristotelem, quod ipsa fuit taliter divisa, quod nulla pars remanet indivisa, et tamen infinitis poterit dividi aliter quam umquam fuerit divisa prius”, Nicole Oresme, *Questiones super geometriam Euclidis*, 127.

¹⁶⁶ “Pono, quod a sit una linea et b sit una alia dupla ad illam et sit c una alia equalis a et d equalis b , tunc inter a et b ymaginetur una linea utrique incommensurabilis, deinde inter quamlibet aliarum et istam linea utrique incommensurabilis et sic in infinitum. Et sic eodem modo fiant linee inter c et d que sunt commensurabiles et sic in infinitum. Item ymaginetur hora dividi per instans in duas medietates et iterum quelibet medietas in duas et sic in infinitum”, Nicole Oresme, *Questiones super geometriam Euclidis*, 127-128.

T_1 . Therefore, there are no leaps during the increase from A to B. However, the situation is very paradoxical.

The paradox stems from the unification of the two sets: the one with incommensurable and the other with commensurable values. Both are *continuous* sets between the *same* terms, the magnitudes A and B (or C and D equal to them), but no element of any of them is an element of the other. The magnitude A will become greater than F_n by a *continuous* increase, without being ever equal to F_n , and in the same way the magnitude C will become greater than E_n by a *continuous* increase, without ever being equal to E_n . Each magnitude passes through holes without ever jumping...

Finally, we can try to understand the conclusions I mentioned.

If we consider the three sets, the rational values, the irrational values, and time, we have three continually divisible sets; the set of rational values and the set of irrational values are thus constituted such that there is no interval so small that it is not divisible into divisible parts. Therefore, if the magnitude has a rational value, there was an instant when it had a smaller value. And the same thing can be said of irrational values. Thus, we have a strange situation, because both increases are continuous, yet “in a certain way”, they skip values. The irrational set skips the rational values, the rational set skips the irrational ones. But if we consider separately each set, no value is skipped: there is no “instantaneous” motion. This is probably the reason why Oresme feels authorized to conclude that it *could* be argued that a continuous being *be composed of* infinite indivisibles.

3.5 Back to question VI.3: the second series of corollaries

We can now go back to the second series of corollaries in *Physics*, VI.3.¹⁶⁷ The same kind of arguments is also to be found here. Just like in the question 8 on Euclid’s geometry, Oresme first asserts that a continuum can be divided in two parts either commensurable or incommensurable.¹⁶⁸ He then adds that a rational ratio can become irrational, and conversely, by adding or subtracting an “infinitely small quantity (*infinitum modicum*)”.¹⁶⁹ He does not really explain here what he has in mind, but the proposition II.4 of his *De commensurabilitate* gives us some hints.¹⁷⁰ There, he

¹⁶⁷ Nicole Oresme, *Questiones super Physicam*, 674.

¹⁶⁸ “Primum est quod secundum quamlibet proportionem et qualitercumque potest dividi continuum in duo media vel etiam in partes commensurabiles <vel incommensurabiles>, Nicole Oresme, *Questiones super Physicam*, 674.

¹⁶⁹ “Qualibet proportione rationali data per infinitum modicum fieret irrationalis, aut e converso, addendo vel diminuendo”, Nicole Oresme, *Questiones super Physicam*, 674.

¹⁷⁰ “Conclusio quarta. Nulla est circuli tam parva portio in qua talia duo mobilia non coniungantur in posterum et in qua non fuerint [in preterito] aliquando coniuncta”, Nicole Oresme,

suggests applying the side of a square on its diagonal as many times as necessary for it to exceed it.¹⁷¹ Obviously, the excess of added sides upon the diagonal is smaller than the diagonal. If we now apply again this excess to the diagonal as many times as necessary for it to exceed once again the diagonal, we will obtain a new smaller excess. Oresme concludes: “Proceeding thus infinitely, the excess by which the diagonal is divided would be diminished infinitely (*in infinitum diminueretur*) so that, in the whole time, no part of the diagonal would remain undivided (*nulla pars remanet toto tempore indivisa*).”¹⁷²

Therefore, the excess is understood as a *variable* magnitude, continuously decreasing one division after the other, in such a way that no part remains undivided, and thus “tending” to *non quantum*. What Oresme is asking in the corollary of VI.3 is the reverse process: this “infinitely small quantity” is now *added* to any magnitude. For example, added to the diagonal of a square, it makes a very small increase from a magnitude incommensurable with the side to a commensurable one: an irrational ratio has become rational by what one could call an *infinitesimal increase*. Once again, this reversal of a subtracting process to an additive process is no exception in Oresme’s works.¹⁷³

We can now examine the most astonishing corollary of those two series, obviously meant to be spectacular. Oresme now wants to prove that during the time of the increase (of a magnitude), two contradictory propositions will be continuously (*continue*) true: “These are commensurable”, and “These are not commensurable.”¹⁷⁴ Going back to the case given above, an increase from A to B through incommensurable values will be continuous, as we saw, just like an increase from C to D through commensurable values. Now, if we “mix” those two increases by considering a growing

Tractatus de commensurabilitate vel incommensurabilitate motuum celi, edited by E. Grant (Madison: University of Wisconsin Press, 1971), 252.

¹⁷¹ “Ergo nulla pars circuli restabit quin aliquando ad ymaginationem sit divisa per hunc modum sicut qui replicaret costam quadrati super dyametrum quousque excederet, et iterum absunderet ilium excessum secundum et replicaret ut prius et acciperet tertium excessum, et sic procederet in infinitum. Tunc in infinitum diminueretur ille excessus secundum cuius quantitatem semper divideretur dyameter, igitur nulla pars dyametri remanet toto tempore indivisa; et ita est quodammodo in proposito”, Nicole Oresme, *Tractatus de commensurabilitate*, 254.

¹⁷² See the footnote above.

¹⁷³ Question 2 of his Questions on the geometry of Euclid is another very important case. Nicole Oresme, *Questiones super geometriam*, 103-106. See also: Mazet, “La théorie des séries de Nicole Oresme”.

¹⁷⁴ “Ex quo sequitur tertio quod continue per illud tempus augmenti utrumque contradictorium erit verum continue, id est sine intermissione temporis, supposito quod instans sit aliquid verum, et contradictoria sunt illa: hec sunt commensurabilia, hec non sunt commensurabilia. Ex hoc possunt haberi multe ymaginationes de mixtione et aliis, convertendo ymaginationem de successivo ad permanens. Et ita etiam si *a* sit unisonus et *b* sonus continue intendatur, tunc continue erit concordia et continue erit discordia, et sic de aliis”, Nicole Oresme, *Questiones super Physicam*, 674.

line taking *all* the values, commensurable and incommensurable, thus allowing for those infinitely small increases defined above between the commensurable and the incommensurable, we obtain the paradoxical situation where, on the one hand, the magnitude is commensurable continuously and without interruption, and at the same time continuously incommensurable. Of course, at any instant, the growing line is either commensurable or incommensurable to the given and static lines. But if one considers the *whole* period of increasing, the growing line is *continuously* commensurable *and* incommensurable; truth is now in the state of Schrödinger's cat.¹⁷⁵

Oresme has thus defined a continuity of higher order: an increase is continuous in a "first order" continuity, or "*sine intermissione*", when it is the reverse process of the classical geometrical infinite division. But it is of a "second order" continuity when the set of all possible increases includes the "infinitely small" increase defined above between commensurable and incommensurable magnitudes. The Schrödinger-like state induced can even be heard: if a sound is *continuously* intensified – with a second-order continuity – then it will be *at the same time* but *continuously* in concord *and* discord with another given sound...¹⁷⁶

3.6 Oresme's solution to Zeno's paradox

In question VI.3, the first two *argumenta quod non* are explicitly Zeno's paradoxes.¹⁷⁷ According to the second one, if continuity was composed of continually divisible parts, then the quicker mobile would not reach the slower one. The continuity of magnitude seems in contradiction to the continuity of motion. After his long and profound study on continuity, and his new understanding of a continuous increase and decrease, Oresme can answer this objection in a very straightforward way.¹⁷⁸

¹⁷⁵ Oresme's reasoning could be compared to the Dirichlet function, where $f(x)$ equals 1 if x is a rational number and 0 if x is not rational.

¹⁷⁶ Nicole Oresme, *Questiones super Physicam*, 674.

¹⁷⁷ "Et arguitur quod non, quia sequitur quod nulla magnitudo finita posset pertransiri tempore finito. Patet statim, quia pertransiretur medietas, deinde medietas residui, deinde medietas secundi residui; et sic semper, <si> quodlibet residuum sit divisibile. Secundo, sequitur quod mobile velox non possit attigere mobile tardum. Verbi gratia: sit a velox, b tardum precedens; tunc, quando a venerit in puncto c ubi nunc est b , adhuc non erit coniunctum b , quia b erit ulterius motum propter hoc quod movetur continue; et tunc iterum, quando a venit in d ubi nunc est b , adhuc non attingeret b , quia b excedit d . Et sic argueretur semper, si quelibet pars spatii est divisibilis; ergo a nunquam attingeret b . Et ille sunt due rationes Zenonis; et una prius fuit facta et quattuor habent parvam apparentiam", Nicole Oresme, *Questiones super Physicam*, 671.

¹⁷⁸ "Ad secundam, dicitur quod mobile velox attingit mobile tardum, sed numquam dum distans, sed in primo instanti in quo non distabunt. Immo bene probat ratio quod non est ultimum instans in quo distant, sed quandocumque distant, adhuc distabunt semper; et sic in infinitum. Tamen quia hoc est semper diminuendo, totum pertransitur isto tempore habente etiam infinitas partes", Nicole Oresme, *Questiones super Physicam*, 677.

He distinguishes two relations between the mobiles: to be mutually distant, and not to be mutually distant. The argument proves that there is no last instant when the two mobiles are distant, because of the infinite divisibility. However, he adds, “because this is always diminishing, the total will be passed through during this period having also an infinite number of parts.”¹⁷⁹ The “period” he is talking about is the one during which the two mobiles are mutually distant. This period does not have a last instant, but is, however, limited by the first instant when the two mobiles *are not* mutually distant. Thus, if we suppose D to be the initial distance between the two mobiles at the beginning of the motion, Oresme is stating that during this period, D will decrease until it vanishes: “the total is passed through (*totum pertransitur*).” This requires the new mathematical methods Oresme has just introduced, in particular, the *complete* exhaustion. It doesn’t immediately require what I called the second order continuum, but the formalization of the idea an “infinitely small increase” between the commensurable and incommensurable, a quasi-punctual increase (or decreasing) really gives a mathematical feeling of what the continuity of motion is.

Thus, we see that, although he remains traditional in his general conclusions, Oresme totally renews the meaning of them by distinguishing *undivided parts* and *uncut points*. He never advocates the atomization of the continuum, but his fine-grained mathematical analysis of the continuum is really tantamount to such an atomization. The geometrical demonstrations that end the *De Configurationibus* illustrate the way those logical and ontological reflections beg imperceptible et effective mathematical techniques. In particular, proposition III.11 really is a mathematical variation on a Zenonian theme, turning the logical paradox into an ability to measure mathematically the finite space spanned by a mobile during a never-ending motion whose velocity is continuously decreasing.¹⁸⁰

Conclusion

As we saw, Oresme’s understanding of the continuity of motion is very ambivalent: he has a tendency to assert vigorously the continuity of motion, but another tendency to atomize this continuity. On the one hand, he defines continuity as an *essential* property of real motion by contrast with apparent motion. His analysis of cartoon-like discrepancies even reveals his psychological subtlety. But on the other hand, the way he understands this continuity, ontologically and mathematically, is tantamount to a very original kind of atomization of the continuum: Oresme’s notion of an *absolutely* successive being makes it possible for him to understand motion as a continuous whole “composed” or “generated” by an infinite number of atoms of motion, just like a continuous line that would be generated by an infinite number of indivisible points. This does not mean, of course, that the continuum really is *composed* of indivisibles,

¹⁷⁹ See the preceding note.

¹⁸⁰ Nicole Oresme, *De Configurationibus*, 424-426.

since as Oresme repeatedly asserts, indivisibles are not beings, as the mathematician wrongly imagines. In the case of motion, the ontological paradoxes implied by such a “being” could be solved thanks to the *condicio* theory. The mathematical counterpart of this ontological analysis is Oresme’s original method of complete exhaustion, and his ability to calculate the summation of different series. To the infinite divisibility of the continuum, he adds a new kind of property: the existence, for any division of the continuum, of an infinite number of uncut points. Consequently, a line being given, the set of rational points and the set of irrational points are both continuous, and the union of the two sets is a continuum of higher order. In such a way, a growing line taking successively all these values passes through infinitely smaller increases, the increase from a rational/irrational value to an irrational/rational one. Thus, we can see how subtle Oresme was when analyzing the continuity of motion, and why, in the course of those analyses, he had the feeling of meeting difficulties only comparable to the mysteries of his religion.

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